MATH 3063 CALCULUS 1 Project 1 Name:
PROF. PAUL BAILEY February 11, 2008

Due Friday, February 15, 2008.

Write all solutions neatly, in complete sentences. The statement of the problem should always be copied onto a blank sheet of $8\frac{1}{2} \times 11$ computer paper, followed by the solution. Staple this sheet to the front of your solutions.

Definition 1. Let P, Q, and R be points in a plane. We use the following notation

- $|PQ|$ is the distance from P to Q.
- $\overline{PQ}$ is the line segment from P to Q.
- $\angle PQR$ is the angle between $\overline{PQ}$ and $\overline{QR}$.
- $\triangle PQR$ is the triangle with vertices P, Q, and R.

Problem 1. Compute $\cos 72°$, as follows.

(a) Sketch the following diagram. Let $\triangle ABC$ be an isosceles triangle with base $\overline{AC}$ and $\angle CAB = 72°$. Bisect $\angle CAB$ and let D be the point of intersection of the bisecting ray and the line segment $\overline{BC}$.

(b) Show that $\triangle CAD$ is similar to $\triangle ABC$.

(c) Assume $|AB| = 1$. Let $x = |AC|$ and $y = |CD|$. Form two equations involving x and y.

(d) Solve these equations for x.

(e) Find $\cos 72°$.

Definition 2. Let $A \subset \mathbb{R}$.
We say that A is *globally discrete* if

$$\text{there exists } \epsilon > 0 \text{ such that for every } a \in A, (a - \epsilon, a + \epsilon) \cap A = \{a\}.$$

We say that A is *locally discrete* if

$$\text{for every } a \in A \text{ there exists } \epsilon > 0 \text{ such that } (a - \epsilon, a + \epsilon) \cap A = \{a\}.$$

Problem 2. Discuss the difference between the definitions of globally discrete and locally discrete, including the following.

(a) Explain why every globally discrete set is locally discrete.

(b) Give two fundamentally different examples of globally discrete sets.

(c) Give two fundamentally different examples of locally discrete sets which are not globally discrete.