Math 3063 Abstract Algebra — Prof. Paul Bailey — Project 2 — February 11, 2008 — Name:

Due Monday, February 18, 2008.

Write all solutions neatly, in complete sentences. The statement of the problem should always be copied onto a blank sheet of $8\frac{1}{2} \times 11$ computer paper, followed by the solution. Staple this sheet to the front of your solutions.

Problem 1. Let G be a group of even order. Show that G has an element of order two.

Problem 2. Let G be a group such that $g^2 = 1$ for every $g \in G$. Show that G is abelian.

Problem 3. Let G be a group and let $g, h \in G$. The *conjugate* of h by g is $g^{-1}hg$.

(a) Show that g and h commute if and only if $g^{-1}hg = h$.

(b) Show that $(g^{-1}hg)^n = g^{-1}h^n g$ (use induction).

(c) Show that if $\text{ord}(h) = n$, then $\text{ord}(g^{-1}hg) = n$.

Problem 4. Let G be a group with a unique element $g \in G$ of order two. Show that $g \in Z(G)$.

Problem 5. Let G be a group, $g \in G$, and $H \leq G$. The *centralizer* of g in H is

$$C_H(g) = \{h \in H \mid h^{-1}gh = g\}.$$

Show that $C_G(g) \leq G$.