

Calculus I Worksheet 2 - Quadratic Functions

Paul L. Bailey

January 31, 2008

A *quadratic function* is a polynomial of degree two.

The *normal form* of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where a , b , and c are real numbers. The graph is a *parabola* which opens upward if $a > 0$ and opens downward if $a < 0$. The y -intercept is the point $(0, f(0))$, and we see that $f(0) = c$. The zeros of the function are the values of x such that $f(x) = 0$. The *quadratic formula* says that $f(x) = 0$ if and only if

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The *discriminant* of the quadratic function is

$$\Delta = b^2 - 4ac;$$

this determines the number of real zeros. There are three cases:

- (a) if $b^2 - 4ac > 0$, there are two real zeros;
- (b) if $b^2 - 4ac = 0$, there is one real zero;
- (c) if $b^2 - 4ac < 0$, there are no real zeros.

The x -intercepts (if any) are the points $(x, 0)$, where x is a real zero.

The *shifted form* of a quadratic function is

$$f(x) = a(x - h)^2 + k,$$

where a , h , and k are real numbers. The shifted form tells how the graph of $f(x)$ is obtained from the graph of x^2 , as follows:

- (a) shift horizontally by h ;
- (b) stretch vertically by $|a|$;
- (c) reflect across the x -axis if a is negative;
- (d) shift vertically by k .

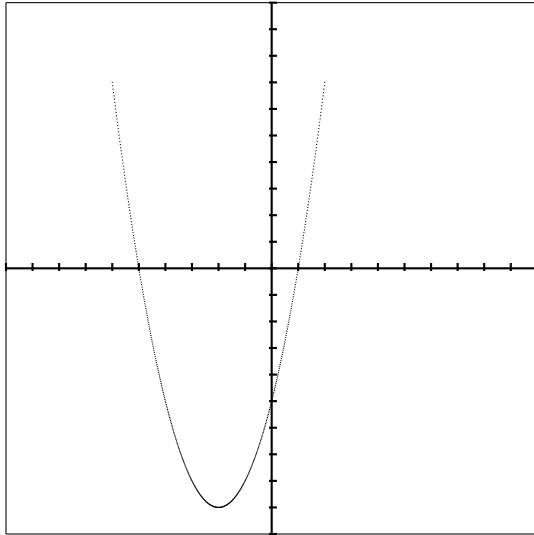
The point (h, k) where the graph turns around is called the *vertex*. Thus k is the *minimum value* of the function if $a > 0$, and is the *maximum value* of the function if $a < 0$.

We can convert from standard form to shifted form by completing the square, which leads to:

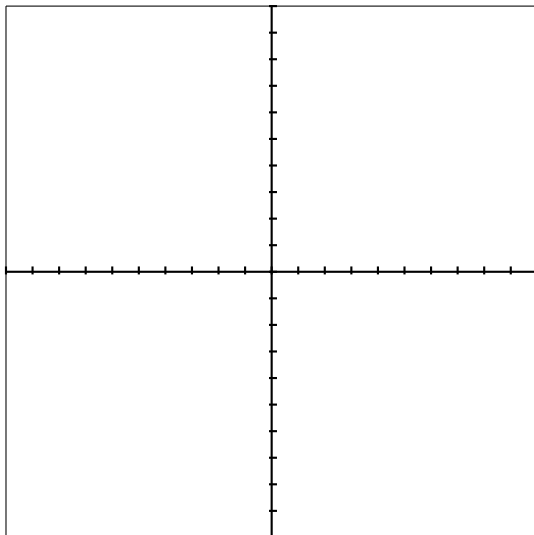
$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}.$$

We can convert from shifted form to standard form by squaring and simplifying, which leads to:

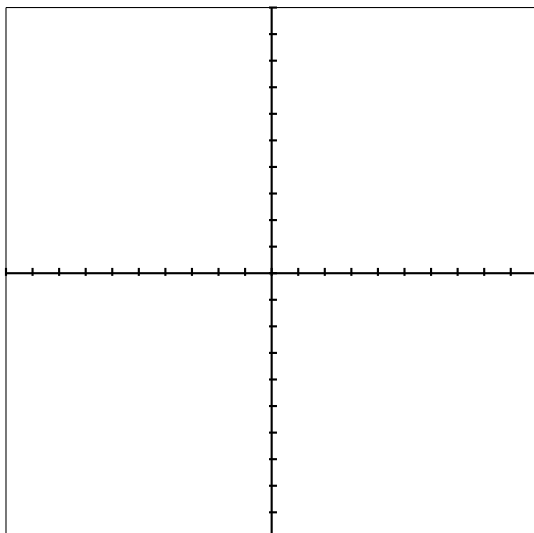
$$b = -2ah \quad \text{and} \quad c = ah^2 + k.$$



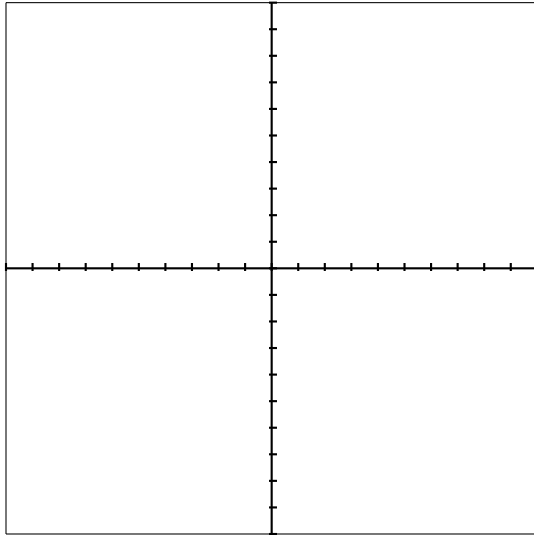
Example: $f(x) = 4x - 5 + x^2$
Normal Form: $f(x) = x^2 + 4x - 5$
Shifted Form: $f(x) = (x + 2)^2 - 9$
a: 1 **b:** 4 **c:** -5 **h:** -2 **k:** -9
Discriminant: 36
Zeros: $x = -5$ and $x = 1$
y-intercept: $(0, -5)$
x-intercept(s): $(-5, 0)$ and $(1, 0)$
Vertex: $(-2, -9)$



Problem 1: $f(x) = x^2 - 6x + 8$
Normal Form:
Shifted Form:
a: **b:** **c:** **h:** **k:**
Discriminant:
Zeros:
y-intercept:
x-intercept(s):
Vertex:



Problem 2: $f(x) = (x + 2)^2 - 5$
Normal Form:
Shifted Form:
a: **b:** **c:** **h:** **k:**
Discriminant:
Zeros:
y-intercept:
x-intercept(s):
Vertex:



Problem 3: $f(x) = 6x - x^2$

Normal Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

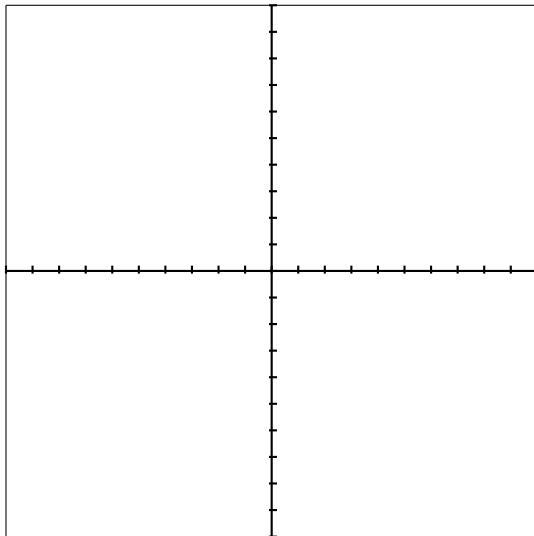
Discriminant:

Zeros:

***y*-intercept:**

***x*-intercept(s):**

Vertex:



Problem 4: $f(x) = (3x - 7)(-x + 1)$

Normal Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

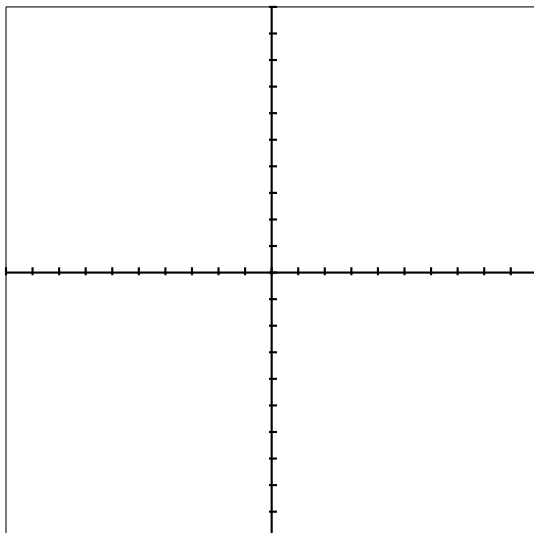
Discriminant:

Zeros:

***y*-intercept:**

***x*-intercept(s):**

Vertex:



Problem 5: $f(x) = 6 + x^2 - 4x$

Normal Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

***y*-intercept:**

***x*-intercept(s):**

Vertex: