Calculus I Worksheet 3 - Polynomial Functions

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A *polynomial function* is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_i \in \mathbb{R}$ and $a_n \neq 0$. The *degree* of $f(x)$ is $\deg(f) = n$. The real numbers a_i are the *coefficients* of $f(x)$. The *leading coefficient* of $f(x)$ is a_n. The *constant coefficient* of $f(x)$ is a_0.

The *zeros* of $f(x)$ are the *real* and *complex* solutions to the equation $f(x) = 0$.

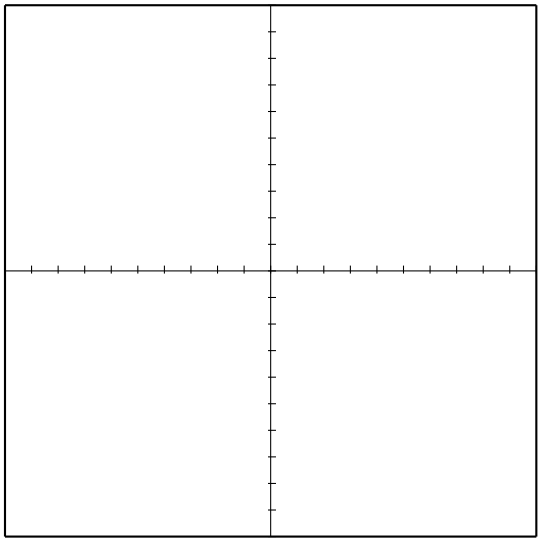
The *y-intercept* of $f(x)$ is the point $(0, a_0)$.

The *x-intercepts* of $f(x)$ are the points $(r, 0)$, where r is a *real* zero of $f(x)$.

The *shape* of $f(x)$, which determines the behavior of the function near $\pm\infty$, is

(a) $+|+$ if n is even and $a_n > 0$;
(b) $-|-$ if n is even and $a_n < 0$;
(c) $-|+$ if n is odd and $a_n > 0$;
(d) $+|-$ if n is odd and $a_n < 0$.

Find the degree, leading coefficient, constant coefficient, zeros, intercepts, and shape of $f(x) = y$. Use the intercepts and the shape to sketch the graph of $f(x)$.



Polynomial: $y = \sqrt{5} - 2$

Degree:

Leading Coefficient:

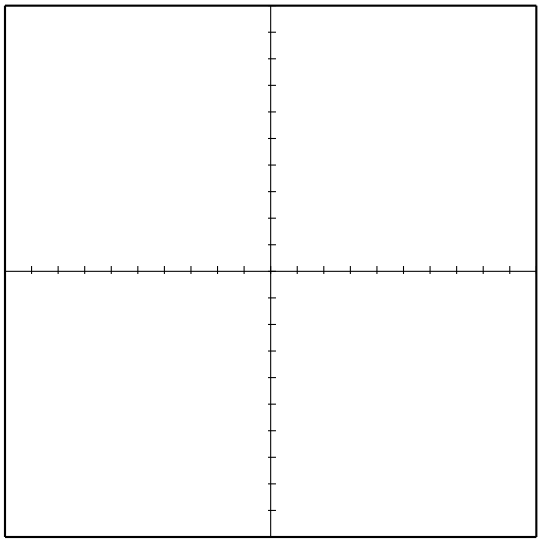
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = 8 - 2x^2$

Degree:

Leading Coefficient:

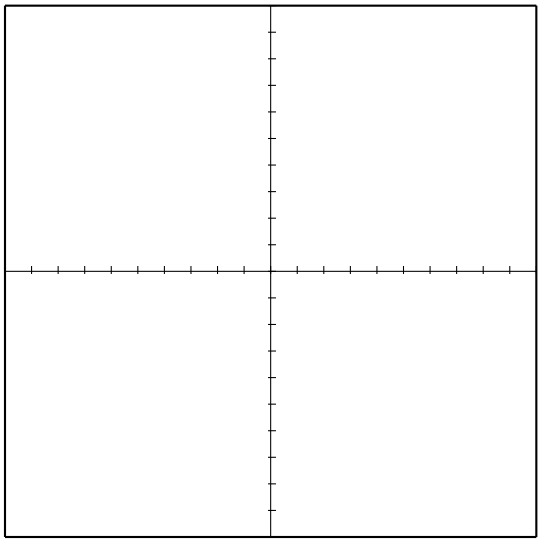
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = 7 + 8x - 3x^2$

Degree:

Leading Coefficient:

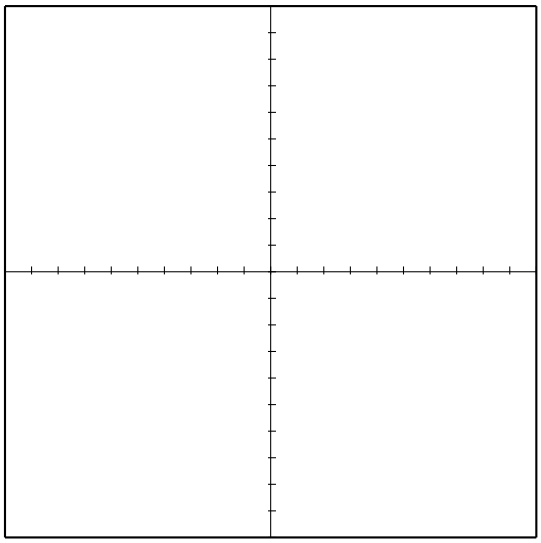
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = x^3 - 9x$

Degree:

Leading Coefficient:

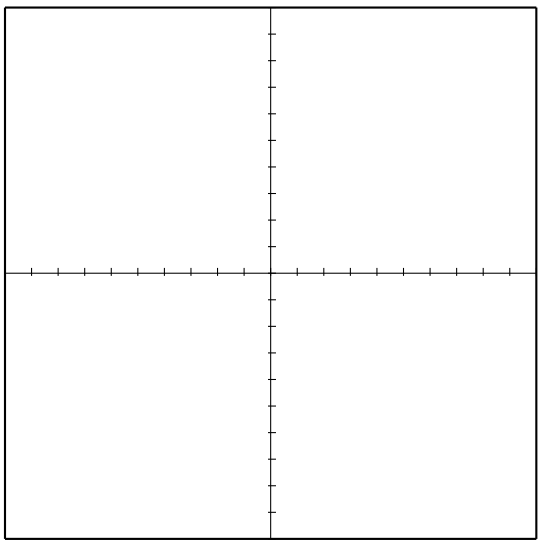
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = x^3 - 2x^2 - 4x + 8$

Degree:

Leading Coefficient:

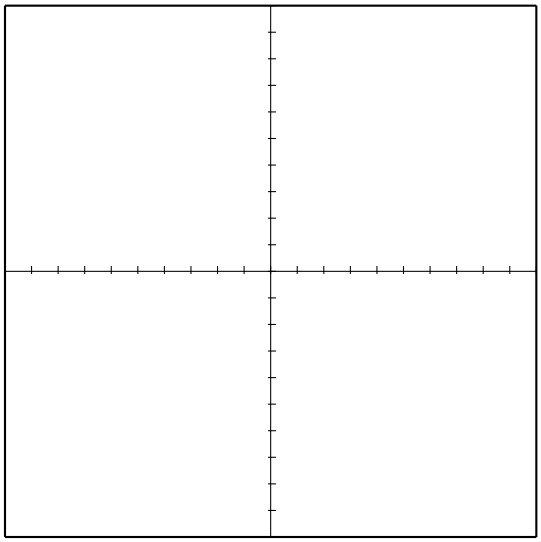
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = x^4 - 10x^2 + 9$

Degree:

Leading Coefficient:

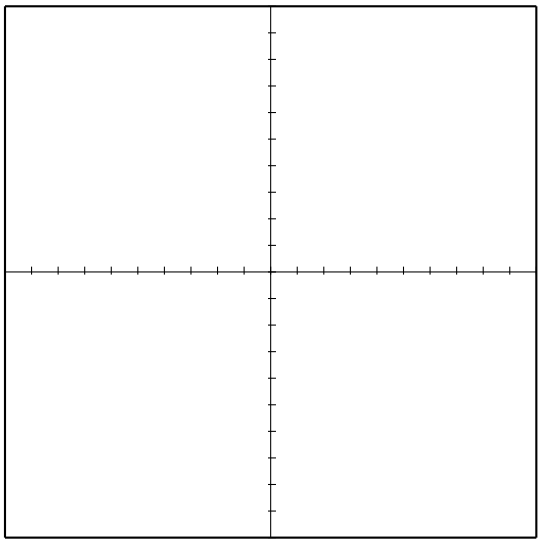
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = x^4 - 5x^3 - 3x^2 + 17x - 10$

Degree:

Leading Coefficient:

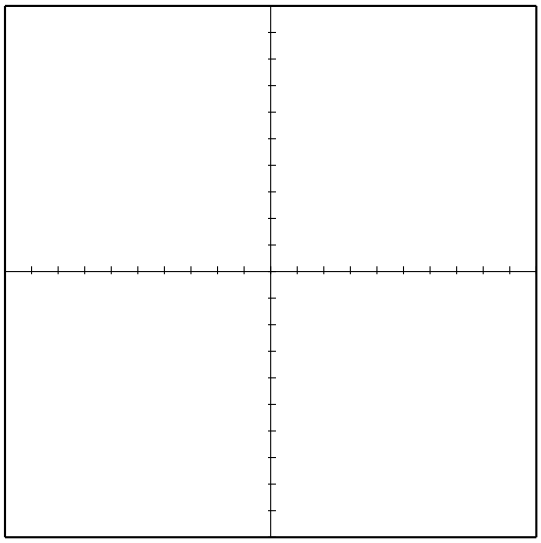
Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape:



Polynomial: $y = 6x^3 - 11x^2 - 24x + 9$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

Shape: