Name:

Calculus I (Math 1525)
PRACTICE Midterm Exam # 1

Professor Paul Bailey
Thursday, March 13, 2008

The examination contains five problems which are worth 20 points each, plus a bonus problem worth an additional 20 points. The use of books, notes, or electronic computation devices is prohibited.

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus	Total Score

Problem 1. (True/False)
Circle the letter corresponding to the best answer.

Let $f : A \to B$ be a function, and let $a \in A$. Then $f(a) \in B$.

(T) True **(F)** False

Let $f : A \to B$ be a function, and suppose that $f(a) = b_1$ and $f(a) = b_2$. Then $b_1 = b_2$.

(T) True **(F)** False

Let $f : A \to B$ be a function, and suppose that $f(a_1) = f(a_2)$. Then $a_1 = a_2$.

(T) True **(F)** False

If $f : \mathbb{R} \to \mathbb{R}$ is even, then $f(0) = 0$.

(T) True **(F)** False

If $f : [a, b] \to \mathbb{R}$ is continuous and $f(a) = -f(b)$, then $f(c) = 0$ for some $c \in [a, b]$.

(T) True **(F)** False

If f is a rational function, then f is continuous on its natural domain.

(T) True **(F)** False

If f is a continuous function, then f is differentiable.

(T) True **(F)** False

There exists a nonconstant function $f : \mathbb{R} \to \mathbb{R}$ which is both even and odd.

(T) True **(F)** False

There exists a linear function whose range is the set of positive real numbers.

(T) True **(F)** False

There exists a quadratic function whose derivative is constant.

(T) True **(F)** False

Problem 2. (Computation)
Compute the following values.

(a) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$. Find $(A \cup B) \setminus (A \cap B)$.

(b) Find $\cos \dfrac{5\pi}{12}$.

(c) Find $\lim\limits_{\theta \to 0} \dfrac{\sin 7\theta}{12}$.

(d) Find $\lim\limits_{h \to 0} \dfrac{\sqrt{x + 5h} - \sqrt{x}}{h}$.

(e) Find the average rate of change of $f(t) = t^3 - 10t + 2$ on the interval $[3, 5]$.

Problem 3. (Continuity)

(a) Find $\delta > 0$ such that

$$0 < |x - 4| < \delta \quad \Rightarrow \quad |\sqrt{x} - 2| < \frac{1}{3}.$$

(b) Let $f(x) = \dfrac{1}{x^2}$, and let $a = 1$. Let $\epsilon \in \mathbb{R}$ with $0 < \epsilon < 1$. Find $\delta > 0$ such that

$$|x - a| < \delta \quad \Rightarrow \quad |f(x) - f(a)| < \epsilon.$$

Problem 4. (Derivatives)
Find $\dfrac{dy}{dx}$.

(a) $y = \dfrac{x^2 - 1}{x^3 - 1}$

(b) $y = \sin(x^2 + x + 1)$

(c) $y = x^2\sqrt{1 + \tan^2 x}$

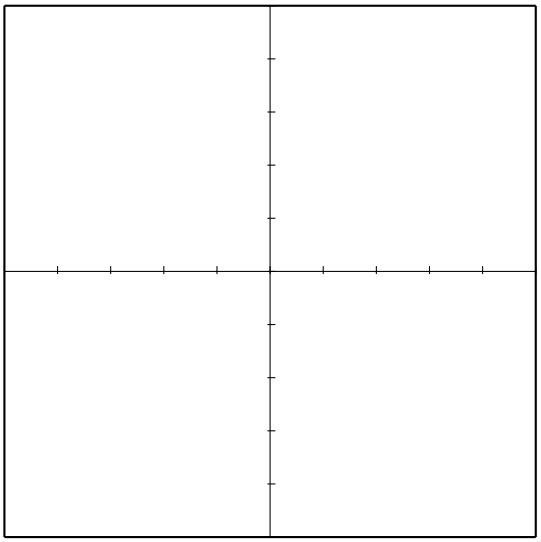
(d) $y = \cos^2 x + \sin^2 x$

(e) $y = \cos^2 x - \sin^2 x$

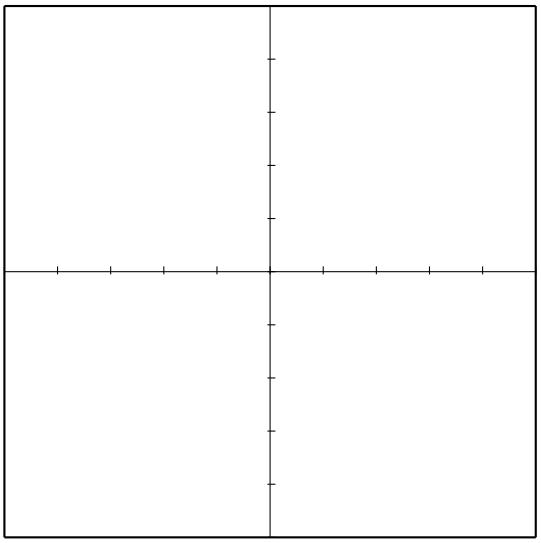
Problem 5. (Polynomials)
Let

$$f(x) = x^3 - 3x + 2.$$

(a) Find $f'(x)$, and find the zeros of $f'(x)$. Sketch the graph of $f'(x)$; label all x-intercepts and y-intercepts.



(b) Find the zeros of f. Sketch the graph of f; label all x-intercepts, y-intercepts, and points with horizontal tangents.



Theorem 1. (Intermediate Value Theorem)
Let $f : [a, b] \to \mathbb{R}$ be continuous. If $f(a)f(b) < 0$, then there exists $c \in (a, b)$ such that $f(c) = 0$.

Problem 6. (Bonus)
Prove that there exists $x \in \mathbb{R}$ such that $\cos x = x$.