Name:

Calculus I (Math 1525)
Midterm Exam I (Remake)

Professor Paul Bailey
Due Friday, October 24, 2008

The examination contains five problems which are worth 20 points each, plus a bonus problem worth an additional 20 points. You may use your book or your notes, but you may not share information with your classmates, or ask anyone for help.

You must sign the below following statement to receive credit.

I have have not received help from any other person in the completion of this examination, nor have I discussed any of these problems with a single living soul.

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus	Total Score

Problem 1. (Computation)
Compute the following values.

(a) Let $A = [1, 9], B = (3, 5]$, and $C = \{1, 7, 9\}$. Find $(A \setminus B) \setminus C$.

(b) Find the domain of the function $f(x) = \dfrac{\sqrt{4 - 36x^2}}{x^5}$.

(c) Find $\displaystyle\lim_{\theta \to 0} \frac{\sin 8\theta}{2\theta}$.

(d) Find $\displaystyle\lim_{h \to 0} \frac{\sqrt{5a + h} - \sqrt{5a}}{h}$.

(e) Find the average rate of change of $f(t) = \sin t$ on the interval $[0, 2\pi/3]$.

Problem 2. (Continuity)
Let $f(x) = x^3 - 3x^2 + 3x - 1$, and let $a = 1$. Let $\epsilon > 0$. Find $\delta > 0$ (which depends on ϵ) such that

$$|x - a| < \delta \quad \Rightarrow \quad |f(x) - f(a)| < \epsilon.$$

Problem 3. (Derivatives)
Let

$$f(x) = \sin(x) \quad \text{and} \quad g(x) = x^2 - 2x + 1.$$

(a) Find $f'(x)$ and $g'(x)$.

(b) Find $(fg)'(x)$.

(c) Find $\left(\frac{f}{g}\right)'(x)$.

(d) Find $(f \circ g)'(x)$.

Problem 4. (Transformations)
Let $f(x) = x^4 - 5x^2 + 4$.

(a) Sketch the graph of f by finding its x and y intercepts and noting that it is an even function.

(b) Find conditions on k such that the equation $f(x) = k$ has zero, one, two, three, or four solutions. (Hint: you can do this without calculus by imagining vertical shifts of the graph of f by k, and applying the quadratic formula on $f(x) - k$. Or, you can also do this with calculus.)

Problem 5. (Tangent Parabolas)
Let $a, b > 0$ and let $f(x) = a^2 - x^2$ and $g(x) = (x - b)^2$ such that the graph of f is tangent to the graph of g, that is, such that their graphs intersect in exactly one point.

(a) Sketch the graph of f and g in the same picture.

(b) Find a relationship between a and b.

(c) Find the point of intersection.

(d) Find the x-intercept of the common tangent line.

Problem 6. (Bonus)
Let $f(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial which is tangent to the line $y = x$ at the origin and does not intersect this line elsewhere. Find a, b, and c. Explain your reasoning.
(Hint: it may help to compute $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$.)