MATH 3063 CALCULUS I Project 1 Name:
PROF. PAUL BAILEY December 2, 2008

Due Friday, Wednesday, October 15, 2008.

Write all solutions neatly, in complete sentences. The statement of the problem should always be copied onto a blank sheet of $8\frac{1}{2} \times 11$ computer paper, followed by the solution. Staple this sheet to the front of your solutions.

Definition 1. Let $D \subset \mathbb{R}$ be an interval and let $f : D \to \mathbb{R}$. Let $a \in D$. We say that f is *continuous* at a if for every $\epsilon > 0$ there exists $\delta > 0$ such that for every $x \in D$, we have

$$|x - a| < \delta \quad \Rightarrow \quad |f(x) - f(a)| < \epsilon.$$

Problem 1. Let

$$f : \mathbb{R} \to \mathbb{R} \quad \text{be given by} \quad f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q}; \\ 0 & \text{otherwise.} \end{cases}$$

Apply the definition to show that f is continuous at 0.

Theorem 1. Intermediate Value Theorem (IVT)
Let $f : [a, b] \to \mathbb{R}$ be continuous. If $f(a)f(b) < 0$, then there exists $c \in (a, b)$ such that $f(c) = 0$.

Problem 2. Let $f : [0, 1] \to [0, 1]$ be continuous.
Apply the Intermediate Value Theorem to show that there exists $c \in [0, 1]$ such that $f(c) = c$.

Definition 2. Let $A \subset \mathbb{R}$.
We say that A is *globally discrete* if

$$\text{there exists } \epsilon > 0 \text{ such that for every } a \in A, (a - \epsilon, a + \epsilon) \cap A = \{a\}.$$

We say that A is *locally discrete* if

$$\text{for every } a \in A \text{ there exists } \epsilon > 0 \text{ such that } (a - \epsilon, a + \epsilon) \cap A = \{a\}.$$

We say that A is *indiscrete* if A is not locally discrete.
We say that A is *bounded* if there exists $a, b \in \mathbb{R}$ such that $A \subset [a, b]$.

Problem 3. Which of the following sets are 1) bounded; 2) globally discrete; 3) locally discrete; 4) indiscrete? Describe why.

(a) $A = \mathbb{Q}$

(b) $B = \mathbb{Z}$

(c) $C = [2, 5]$

(d) $D = [2, 5] \cap \mathbb{Q}$

(e) $E = [2, 5] \cap \mathbb{Z}$

(f) $F = \{x \in \mathbb{R} \mid x = \frac{1}{n} \text{ for some } n \in \mathbb{N}\}$

Problem 4. Answer the following questions. Explain your reasoning.

(a) Is every finite set bounded?

(b) Is every globally discrete set locally discrete?

(c) Is every finite set globally discrete?

(d) Is every infinite set indiscrete?

(e) Is every bounded infinite set indiscrete?