MATH 1525 CALCULUS I — PROF. PAUL BAILEY — Project 2 — December 2, 2008 — Name:

Due Thursday, November 20, 2008.

Write all solutions neatly, in complete sentences. The statement of the problem should always be copied onto a blank sheet of $8\frac{1}{2} \times 11$ computer paper, followed by the solution. Staple this sheet to the front of your solutions.

Let $P, Q \in \mathbb{R}^2$. Denote the distance between P and Q by $d(P, Q)$.

Let $A, B \subset \mathbb{R}^2$ be nonempty. The *distance* between A and B, denoted $d(A, B)$, is the largest nonnegative real number such that

$$d(A, B) \leq d(P, Q) \text{ for all } P \in A \text{ and } Q \in B.$$

Example 1. Let $A = \{(1, 2)\}$ and $B = \{(5, 1)\}$. Then $d(A, B) = \sqrt{(5-1)^2 + (1-2)^2} = \sqrt{17}$.

Example 2. Let $A = \{(0, 4)\}$ and let B denote the x-axis. Then $d(A, B) = 4$.

Example 3. Let $A = \{(x, y) \in \mathbb{R}^2 \mid y = 2x + 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 \mid y = 3x - 5\}$. Since A and B are nonparallel lines, they intersect, so $d(A, B) = 0$.

Example 4. Let $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 \mid (x - 6)^2 + y^2 = 4\}$. Then $d(A, B) = 3$.

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Let $P \in \mathbb{R}^2$ be a point not on the graph of f. Let $A = \{P\}$ and $B = \{(x, y) \in \mathbb{R}^2 \mid y = f(x)\}$. Let Q be point on the graph of f such that $d(A, B) = d(P, Q)$. Show that the line through Q tangent to the graph of f is perpendicular to the line through P and Q.

Problem 2. Let

$$A = \{(x, y) \mid y = 3x - 5\} \quad \text{and} \quad B = \{(x, y) \in \mathbb{R}^2 \mid y = 3x + 3\}.$$

Find $d(A, B)$.

Problem 3. Let

$$A = \{(x, y) \mid y = (x - 2)^2 + (y - 3)^2 = 4\} \quad \text{and} \quad B = \{(x, y) \in \mathbb{R}^2 \mid (x + 5)^2 + (y + 5)^2 = 9\}.$$

Find $d(A, B)$.

Problem 4. Let $a \in \mathbb{R}$. Let

$$A = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\} \quad \text{and} \quad B = \{(0, b)\}.$$

The distance between A and B is a function of b.
Find $\rho(b) = d(A, B)$.

Problem 5. Let $m \in \mathbb{R}$. Let

$$A = \{(x, y) \mid y = x^2 + 9\} \quad \text{and} \quad B = \{(x, y) \in \mathbb{R}^2 \mid y = mx\}.$$

The distance between A and B is a function of the slope m.
Find $\rho(m) = d(A, B)$ (Hint: express ρ in three cases).