

Due Date: Friday, October 3, 2008.

Write your solutions neatly on separate pieces of paper and attach this sheet to the front.

In Problem 1, recall that  $(123)_5$  is the numeral 123 in base 5, which is 38 in base 10. Thus,  $(123)_5 = (38)_{10}$ .

In the constructions of Problem 3 and 4, describe each step, and draw all steps with a straight-edge and compass, labeling each point significant for the construction. Explain why your construction works. You may use propositions one through five in the notes (when constructing a midpoint or a perpendicular via these propositions, you may use a ruler or protractor to get a more accurate picture).

**Problem 1.** Solve the following equations for the positive integers  $n$  and  $b$ .

(a)  $n = (13425)_b = (4115)_{2b}$

(b)  $n = (1234)_b = (532)_{2b-1}$

(See Eves Problem Study 1.8.)

**Problem 2.** A *Pythagorean triple* is an ordered triple  $(a, b, c)$  of positive integers such that  $a^2 + b^2 = c^2$ .

(a) Show that there exists a Pythagorean triple  $(a, b, c)$  for every integer  $a \geq 3$ .

(b) Show that there exist only finitely many Pythagorean triples  $(a, b, c)$  for each integer  $a \geq 3$ .

(See Eves Problem Study 3.6 and discussion on pp. 81-82)

**Problem 3.** Given line segment  $\overline{AB}$  of length 11 and  $\overline{CD}$  of length 3, construct a point  $C$  on  $\overline{AB}$  such that  $|CB| = x$ , where  $x$  is a solution to the quadratic equation

$$x^2 - 11x + 9 = 0.$$

State the exact value of  $x$ . (See Eves Problem Study 3.10a and discussion on pp. 88-89)

**Problem 4.** Given two points  $A$  and  $B$ , construct a point  $Z$  such that  $\angle BAZ = \angle ABZ = 75^\circ$ .