MATH 4123 HISTORY OF MATHEMATICS Problem Set 2 Name:
PROF. PAUL BAILEY November 5, 2008

Due Date: Friday, November 14,2008.

This serves as the take home part of your midterm examination. Write your solutions neatly on separate pieces of paper and attach this sheet to the front.

Problem 1. Using straightedge and compass, construct an angle of $54°$. Describe each step, discussing why your construction is effective.

Problem 2. Compute the volume of a regular icosahedron inscribed in a sphere of radius 1.

Problem 3. Consider the elliptic curve given by the equation

$$y^2 = x^3 - 12x + 25.$$

Find as many rational points on this curve as you can, including all rational points that lie on a horizontal tangent. Justify your answer.

Definition 1. Let $m, n \in \mathbb{Z}$. The *least common multiple* of m and n is a positive integer $l \in \mathbb{Z}$ such that

(a) $m \mid l$ and $n \mid l$;

(b) $m \mid k$ and $n \mid k$ implies $l \mid k$.

Definition 2. Let $n \in \mathbb{Z}$ with $n \geq 2$. Set $\mathbb{Z}_n = \{r \in \mathbb{Z} \mid 0 \leq r < n\}$. Define a function

$$\rho_n : \mathbb{Z} \to \mathbb{Z}_n \quad \text{by} \quad \rho_n(a) = \text{ the remainder when } n \text{ is divided by } a.$$

We call ρ the *residue map.*

Definition 3. Let $m, n \in \mathbb{Z}$ with $m \geq 2$, $n \geq 2$. Define a function

$$\sigma_{m,n} : \mathbb{Z}_{mn} \to \mathbb{Z}_m \times \mathbb{Z}_n \quad \text{by} \quad \sigma_{m,n}(a) = (\rho_m(a), \rho_n(a)).$$

We call σ the *joint residue map.*

Problem 4. Let $m, n \in \mathbb{Z}$ with $m \geq 2$ and $n \geq 2$. Let $d = \gcd(m, n)$ and $l = \text{lcm}(m, n)$.

(a) Show that if $d = 1$, then $\sigma_{m,n}$ is bijective.

(b) Show that if $a \equiv b \pmod{l}$, then $\sigma_{m,n}(a) = \sigma_{m,n}(b)$.