MATH 4123 HISTORY OF MATHEMATICS Problem Set 3 Name:
PROF. PAUL BAILEY December 1, 2008

Due Date: Friday, December 5,2008.
Write your solutions neatly on separate pieces of paper and attach this sheet to the front.
Problem 4 may require some ingenuity, but is a fascinating result.

Problem 1. (Fibonacci)
Recall that the Fibonacci sequence (F_n) is defined by $F_1 = 1$, $F_2 = 1$, and $F_{n+2} = F_n + F_{n+1}$, and that $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \phi$, where $\phi = \frac{1+\sqrt{5}}{2}$.

Let $b \in \mathbb{R}$ with $b \geq 1$ and define a sequence (G_n) by $G_1 = 1$, $G_2 = 1$, and $G_{n+2} = G_n + bG_{n+1}$.

Let $c \in \mathbb{R}$ with $c \geq \phi$. Find b such that $\lim_{n\to\infty} \frac{G_{n+1}}{G_n} = c$.

Problem 2. (Tartaglia)
Recall that Tartaglia viewed the cube x^3 as $(t-u)^3$ to find solutions to cubic equations.

Let $f(x) = x^3 + 3x^2 + 6x - 8$. Find the real zero of f using Tartaglia's cube plus cosa method.

Problem 3. (Descartes)
Recall that Descartes used the concept of expanding circles and the ability to compute the number of real solutions to quadratic equations to find tangents.

Find the distance between the curve $x = y^2$ and the point $(3, 0)$ using Descartes' discriminant method.

Problem 4. (Napier)
Recall that Napier desired to find a function to convert multiplication into addition. We may use techniques of Calculus unavailable to him to see that he had very little choice. The modern definition is

$$\log x = \int_1^x \frac{dt}{t} \quad \text{and} \quad \log_b(x) = \frac{\log x}{\log b}.$$

Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function which is not constantly zero and satisfies

$$f(ab) = f(a) + f(b) \quad \text{for all } a, b \in (0, \infty).$$

Show that there exists $b \in \mathbb{R}$ such that $f(x) = \log_b(x)$.