

## Precalculus Worksheet 4 - Rational Functions

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A *rational function* is a function of the form

$$f(x) = \frac{g(x)}{h(x)},$$

where  $g(x)$  and  $h(x)$  are polynomials. A rational function is in *lowest form* if the numerator and the denominator have no common zeros. Assume that  $f(x) = g(x)/h(x)$  is a rational function in lowest form.

The *degree* of  $f(x)$  is  $\max\{\deg(g), \deg(h)\}$ .

The *zeros* of  $f(x)$  are the zeros of  $g(x)$ ; that is, they are the solutions to  $g(x) = 0$ .

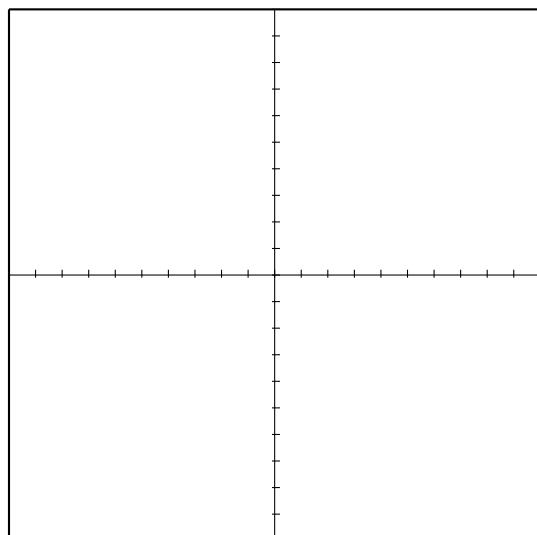
The *poles* of  $f(x)$  are the zeros of  $h(x)$ ; that is, they are the solutions to  $h(x) = 0$ .

The *y-intercept* of  $f(x)$  is the point  $(0, f(0))$ .

The *x-intercepts* of  $f(x)$  are the points  $(z, 0)$ , where  $z$  is a *real* zero of  $f(x)$ .

The *vertical asymptotes* of  $f(x)$  are the lines  $x = p$ , where  $p$  is a *real* pole of  $f(x)$ .

The *polynomial asymptote* of  $f(x)$  is the polynomial equation  $y = q(x)$ , where  $q(x)$  is the quotient when  $g(x)$  is divided by  $h(x)$  using polynomial division.



**Equation:**

$$y = \frac{6}{x-2}$$

**Degree:**

**Zeros:**

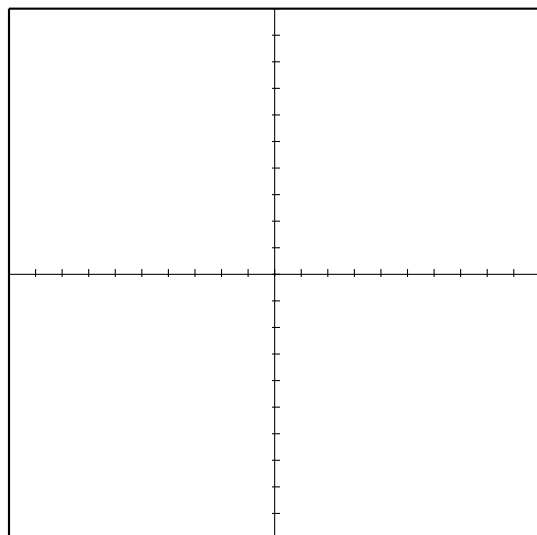
**Poles:**

**y-intercept:**

**x-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{4x+2}{3x-6}$$

**Degree:**

**Zeros:**

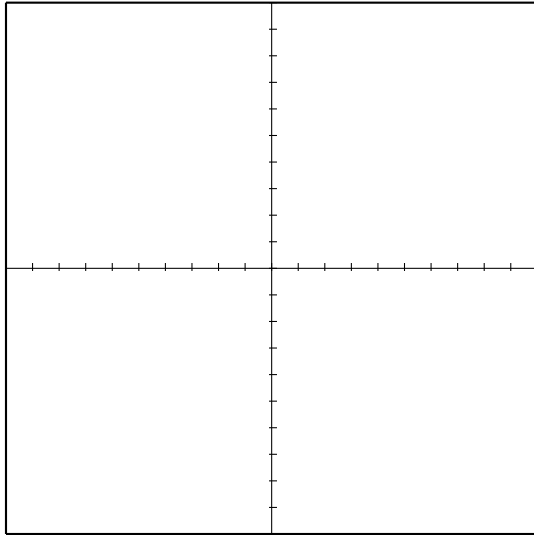
**Poles:**

**y-intercept:**

**x-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x - 5}{x^2 + x - 6}$$

**Degree:**

**Zeros:**

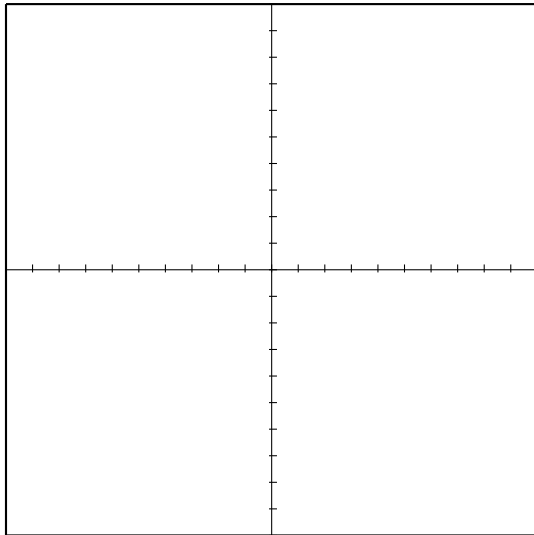
**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x^2 - x - 2}{x + 2}$$

**Degree:**

**Zeros:**

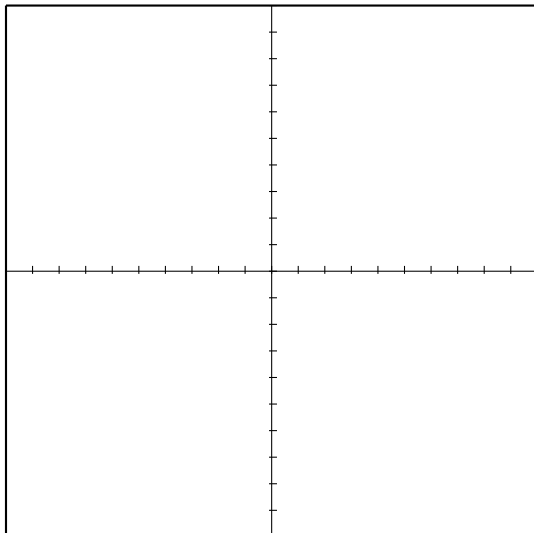
**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x^2 - 49}{x^2 - 25}$$

**Degree:**

**Zeros:**

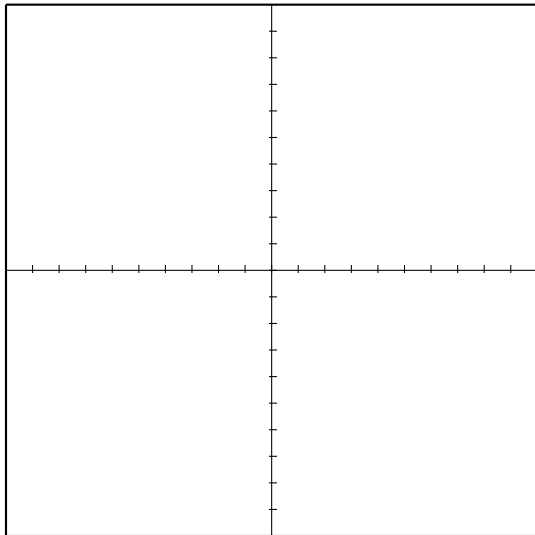
**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x^3 - x}{x^2 - 9}$$

**Degree:**

**Zeros:**

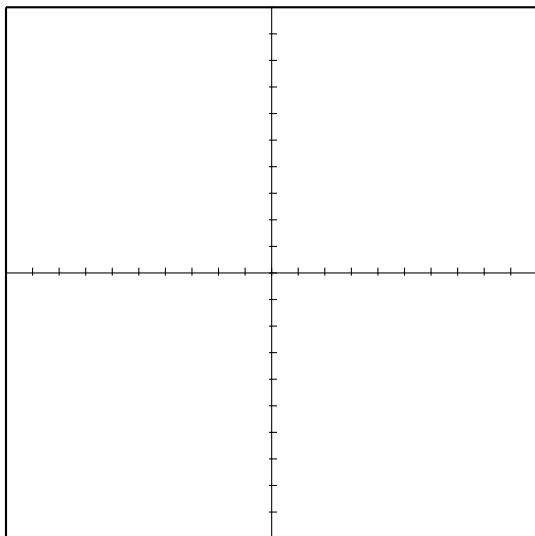
**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x^2 - 25}{x^3 - 3x^2 - 4x + 12}$$

**Degree:**

**Zeros:**

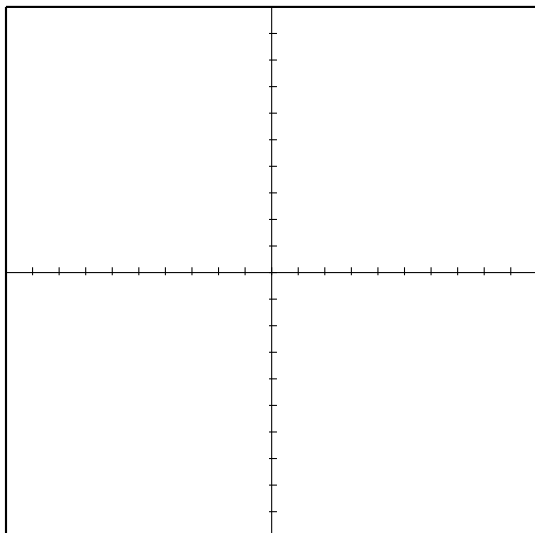
**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**



**Equation:**

$$y = \frac{x^3 - 6x + 7}{x + 1}$$

**Degree:**

**Zeros:**

**Poles:**

***y*-intercept:**

***x*-intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**