Name:

Calculus I (Math 1525)
Fall 2008 Midterm Exam I

Professor Paul Bailey
Friday, October 15, 2008

This examination contains five problems which are worth 20 points each, plus a bonus problem worth an additional 20 points. The use of books, notes, or electronic computation devices is prohibited.

Your examination will look different from this, but this may give you an idea of what it will look like.

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus	Total Score

Problem 1. (True/False)
Circle the letter corresponding to the best answer.

Let $f : A \to B$ be a function, and suppose that $f(a) = b_1$ and $f(a) = b_2$. Then $b_1 = b_2$.

(T) True **(F)** False

Let $f : A \to B$ be a function, and suppose that $f(a_1) = b$ and $f(a_2) = b$. Then $a_1 = a_2$.

(T) True **(F)** False

Let $f : A \to B$ be a function, and suppose that $b \in B$. Then $b = f(a)$ for some $a \in A$.

(T) True **(F)** False

If $f : \mathbb{R} \to \mathbb{R}$ is even, then $f(0) = 0$.

(T) True **(F)** False

If $f : \mathbb{R} \to \mathbb{R}$ is odd, then $f(0) = 0$.

(T) True **(F)** False

If $f : [a, b] \to \mathbb{R}$ is continuous, $c \in (a, b)$, and $f(c) = 0$, then $f(a)f(b) < 0$.

(T) True **(F)** False

If f is differentiable at a, then f is continuous at a.

(T) True **(F)** False

There exists a nonconstant function $f : \mathbb{R} \to \mathbb{R}$ which is both even and odd.

(T) True **(F)** False

There exists an increasing function $f : \mathbb{R} \to \mathbb{R}$ such that $f'(0) = 0$.

(T) True **(F)** False

There exists a polynomial function $f : \mathbb{R} \to \mathbb{R}$ whose derivative is constant.

(T) True **(F)** False

Problem 2. (Computation)
Compute the following values.

(a) Let $A = [1, 7], B = (2, 9]$, and $C = \{1, 3, 5, 9\}$. Find $(A \cap B) \setminus C$.

(b) Find the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 4}}{(x^2 - 2x - 3)^2}$.

(c) Find $\lim\limits_{\theta \to 0} \dfrac{\sin 5\theta}{7}$.

(d) Find $\lim\limits_{h \to 0} \dfrac{\sqrt{3x + h} - \sqrt{3x}}{h}$.

(e) Find the average rate of change of $f(t) = t^3 - 7t + 6$ on the interval $[1, 4]$.

Problem 3. (Continuity)

(a) Find $\delta > 0$ such that

$$0 < |x| < \delta \quad \Rightarrow \quad |\sqrt{4 - x^2} - 2| < \frac{1}{3}.$$

(b) Let $f(x) = x^3$, and let $a = 0$. Let $\epsilon > 0$. Find $\delta > 0$ (which depends on ϵ) such that

$$|x - a| < \delta \quad \Rightarrow \quad |f(x) - f(a)| < \epsilon.$$

Problem 4. (Derivatives)
Let

$$f(x) = x^3 - 4x + 1 \quad \text{and} \quad g(x) = \sqrt{x}.$$

(a) Find $f'(x)$ and $g'(x)$.

(b) Find $(fg)'(x)$.

(c) Find $\left(\frac{f}{g}\right)'(x)$.

(d) Find the equation of the line perpendicular to the line tangent to the graph of f at the point $(2, 1)$.

Problem 5. (Transformations)
Let

$$f(x) = x^3 - x \quad \text{and} \quad g(x) = x^3 + 3x^2 + 2x + 6.$$

(a) Solve $f(x) = 0$ by factoring.

(b) Sketch the graph of f.

(c) Compute $f(x+1)$.

(d) Describe how the graph of g can be obtained from the graph of f by transformations.

(e) Sketch the graph of $g(x)$ (including the x-intercept).

Additional Problems from the "Make up take home" last semester.

Problem 6. (Computation)
Compute the following values.

(a) Let $A = [1, 9], B = (3, 5]$, and $C = \{1, 7, 9\}$. Find $(A \setminus B) \setminus C$.

(b) Find the domain of the function $f(x) = \dfrac{\sqrt{4 - 36x^2}}{x^5}$.

(c) Find $\displaystyle\lim_{\theta \to 0} \frac{\sin 8\theta}{2\theta}$.

(d) Find $\displaystyle\lim_{h \to 0} \frac{\sqrt{5a + h} - \sqrt{5a}}{h}$.

(e) Find the average rate of change of $f(t) = \sin t$ on the interval $[0, 2\pi/3]$.

Problem 7. (Continuity)
Let $f(x) = x^3 - 3x^2 + 3x - 1$, and let $a = 1$. Let $\epsilon > 0$. Find $\delta > 0$ (which depends on ϵ) such that

$$|x - a| < \delta \quad \Rightarrow \quad |f(x) - f(a)| < \epsilon.$$

Problem 8. (Derivatives)
Let

$$f(x) = \sin(x) \qquad \text{and} \qquad g(x) = x^2 - 2x + 1.$$

(a) Find $f'(x)$ and $g'(x)$.

(b) Find $(fg)'(x)$.

(c) Find $\left(\frac{f}{g}\right)'(x)$.

(d) Find $(f \circ g)'(x)$.

Problem 9. (Transformations)
Let $f(x) = x^4 - 5x^2 + 4$.

(a) Sketch the graph of f by finding its x and y intercepts and noting that it is an even function.

(b) Find conditions on k such that the equation $f(x) = k$ has zero, one, two, three, or four solutions. (Hint: you can do this without calculus by imagining vertical shifts of the graph of f by k, and applying the quadratic formula on $f(x) - k$. Or, you can also do this with calculus.)

Problem 10. (Tangent Parabolas)
Let $a, b > 0$ and let $f(x) = a^2 - x^2$ and $g(x) = (x - b)^2$ such that the graph of f is tangent to the graph of g, that is, such that their graphs intersect in exactly one point.

(a) Sketch the graph of f and g in the same picture.

(b) Find a relationship between a and b.

(c) Find the point of intersection.

(d) Find the x-intercept of the common tangent line.

Problem 11. (Bonus)
Let $f(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial which is tangent to the line $y = x$ at the origin and does not intersect this line elsewhere. Find a, b, and c. Explain your reasoning.
(Hint: it may help to compute $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$.)