MATH 1525 CALCULUS I Project 2 Name:
PROF. PAUL BAILEY April 13, 2009

Due Thursday, April 2, 2008.

Write all solutions neatly, in complete sentences. The statement of the problem should always be copied onto a blank sheet of $8\frac{1}{2} \times 11$ computer paper, followed by the solution. Staple this sheet to the front of your solutions. Never express a numerical answer using decimal notation.

Problem 1. Let $f(x) = x^2 + 9$. Find the equation of the unique line through the origin with positive slope which is tangent to the graph of f.

Problem 2. Let $f(x) = 1 - x^2$ and $g(x) = (x-2)^2$. Find the equation of the unique line which is tangent to the graphs of f and g.

Problem 3. Find the area of a circle centered at the origin which is tangent to the parabola $y = 1 - x^2$.

Problem 4. Let $f(x) = cx - x^3$, where $c \in \mathbb{R}$ is positive. Then there exist $a, b \in \mathbb{R}$ with $a < b$ such that f has a local minimum at $x = a$ and a local maximum at $x = b$.

Let m be the slope of the line through $(a, f(a))$ and $(b, f(b))$. Find c such that $m = 1$.

Problem 5. Let $a, b, c \in \mathbb{R}$ with $a < b < c$, and let f be a cubic polynomial with zeros at a, b, and c. The average of the zeros is

$$w = \frac{a+b+c}{3}.$$

Show that f has an inflection point at $(w, f(w))$.