Calculus I Worksheet 0 - Set Operations

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Two sets are *disjoint* if their intersection is empty. Write your answer either as a finite set, an interval, the union of disjoint intervals, or the union of a finite set and one or more disjoint intervals.

Exercise 1. Consider the following sets of natural numbers.
$A = \{n \in \mathbb{N} \mid n \leq 25\}$
$E = \{n \in A \mid n \text{ is even}\}$
$O = \{n \in A \mid n \text{ is odd}\}$
$P = \{n \in A \mid n \text{ is prime}\}$
$S = \{n \in A \mid n \text{ is a square}\}$
Compute the following sets.
(a) P
(b) S
(c) $S \cup P$
(d) $E \cap S$
(e) $(P \cup S) \cap O$
(f) $(O \setminus P) \cap (O \setminus S)$
(g) $(O \cap S) \times (E \cap S)$

Exercise 2. Consider the following intervals of real numbers.
$A = [0, 10]$
$B = (4, 12)$
$C = (-5, 7]$
$D = [-3, 13)$
$E = (5, 15]$
Compute the following sets.
(a) $A \cup B$
(b) $A \cap B$
(c) $(A \cup B) \setminus (A \cap B)$
(d) $(C \cup D) \setminus B$
(e) $(C \cap D) \cup B$
(f) $C \setminus D$
(g) $A \cup E$
(h) $A \cap E$
(i) $D \setminus A$

Exercise 3. Compute the following sets of real numbers.
(a) $A = [1, 20] \cap \mathbb{Z}$
(b) $B = (5, 10) \cap A$
(c) $C = [8, 11] \cap A$
(d) $D = (B \cup C) \setminus (B \cap C)$
(e) $E = [3, 10] \setminus D$
(f) $F = (5, 18) \setminus E$

Answer 1. Answers to Exercise 1.
(a) $\{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
(b) $\{1, 4, 9, 16, 25\}$
(c) $\{1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 17, 19, 23, 25\}$
(d) $\{4, 16\}$
(e) $\{1, 3, 5, 7, 9, 11, 13, 17, 19, 23\}$
(f) $\{15, 21\}$
(g) $\{(1, 4), (1, 16), (9, 4), (9, 16), (25, 4), (25, 16)\}$

Answer 2. Answers to Exercise 2.
(a) $[0, 12)$
(b) $(4, 10]$
(c) $[0, 4] \cup (10, 12]$
(d) $(-5, 4] \cup [12, 13)$
(e) $[-3, 12)$
(f) $(-5, -3)$
(g) $[0, 15]$
(h) $(5, 10]$
(i) $[-3, 0) \cup (10, 13)$

Answer 3. Answers to Exercise 3.
(a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
(b) $\{6, 7, 8, 9\}$
(c) $\{8, 9, 10, 11\}$
(d) $\{6, 7, 10, 11\}$
(e) $[3, 6) \cup (6, 7) \cup (7, 10)$
(f) $\{6, 7\} \cup [10, 18)$