CATEGORY THEORY	Lesson $03/16$
Dr. Paul L. Bailey	Sunday, March 15, 2020

It looks like we may not be able to get together for a while, yet we would like to continue to study field theory with the goal of understanding Galois' proof of the insolvability of the general quintic. With this in mind, it is a good idea that you become better acquainted with the book, and I would like to begin by reviewing what Gallian has to say about ring theory.

Keep in mind that Gallian's definitions are slightly different from ours:

- For us, every ring has a unity, for Gallian, it does not. At UCI, we called a ring without unity a *ringlet*.
- For us, every subring has the same unity; for Gallian, this is not required. Thus for Gallian, a subring could contain a unity which is actually a non-identity idempotent in the ambient ring.
- For us, every homomorphism sends unity to unity; for Gallian, this is not required. Thus he allows for a "ring homomorphism" that sends Z to 2Z.

These differences go away when discussing fields and domains, since for Gallian, these must contain unity.

Below, I will outline the goals for the next few weeks, following Gallian, and possibly supplemented by lecture notes I have previously written (at UCI and SAU). Since my students in this class are so dedicated, it is possible we can go faster than indicated.

- Week of March 16
  - Chapter 12 Rings and Subrings (Skim)
  - Chapter 13 Integral Domains (Skim)
  - Chapter 14 Ideals and Factor Rings (Review)
  - Chapter 15 Ring Homomorphisms (Review)
  - Chapter 16 Polynomials (Review)
  - Chapter 17 Factorization of Polynomials (Review)
- Week of March 23
  - Chapter 17 Factorization of Polynomials (in  $\mathbb{Z}[X]$ )
  - Chapter 18 Divisibility in Integral Domains (ED  $\Rightarrow$  PID  $\Rightarrow$  UFD, Ascending Chain Condition)
  - Chapter 19 Vector Spaces (We only sketched this before, let's do it right now)
  - Chapter 20 Field Extensions (Review)
  - Chapter 21 Algebraic Extensions (Review)
- Week of March 30
  - Chapter 22 Finite Fields (Structure theorems)
  - Chapter 23 Geometric Constructions (Review)
  - Chapter 32 Statement of the Fundamental Theorem of Galois Theory
  - Chapter 32 Statement of the Galois Solvability Criterion
- Week of April 6
  - Concepts Regarding and Proof of the Galois Correspondence
- Week of April 13
  - Commutator subgroups, Central series
  - Nilpotent Groups (Time permitting)
  - Solvable Groups
  - Concepts Regarding and Proof of the Galois Solvability Criterion
- After that, time permitting, we can study the Sylow Theorems regarding structures of finite groups.