AP Calculus AB
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Lesson 03/19
Wednesday, March 18, 2020

We have previously defined the exponential function as the limit of the compounded interest, as the frequency of compounding goes infinity, where x is the annual interest rate:

$$e^x = \lim_{n\to\infty}(1+\frac{x}{n})^n.$$

We defined the natural exponential function as

$$\exp : \mathbb{R} \to (0,\infty) \quad \text{by} \quad \exp x = e^x.$$

We defined the natural logarithm to be the inverse of exp:

$$\log : (0,\infty) \to \mathbb{R} \quad \text{by} \quad \log x = y \Leftrightarrow y = \exp x.$$

Another notation for the natural logarithm is ln:

$$\ln x = \log x \text{ for all } x \in (0,\infty).$$

We previously derived the derivative of exp using the limit definition of derivative, and found that

$$\frac{d}{dx}\exp x = \exp x, \quad \text{that is,} \quad \frac{d}{dx}e^x = e^x.$$

We derived the derivative of log using implicit differentiation, and found that

$$\frac{d}{dx}\log x = \frac{1}{x}, \quad \text{that is, } \frac{d}{dx}\ln x = \frac{1}{x}.$$

We did this rather rapidly, in order to get the derivatives of these functions under our belts (for further digestion).

Now we will revisit this topic, and follow Thomas' approach, which is quite elegant. I hope you like it.

Thomas begins with the definition

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad \text{for } x \in (0,\infty).$$

By the Fundamental Theorem of Calculus, $\frac{d}{dx}\ln x = \frac{1}{x}$. Since this is positive on the domain of $\ln x$, we see that $\ln x$ is increasing, and thus is injective, so it has an inverse. Thomas then defines e^x as the inverse function.

Your assignment for today is

- Read Thomas Section 7.2 up until the subsection on Logarithmic Differentiation.
  Be sure to understand $\int \tan x\,dx$.
- Work on the following problems from Thomas.
  1. §7.2 # 1, 4, 7, 15, 22, 24, 30, 39, 43, 54
- Acknowledge you have completed this via the Google Forms link labeled "Ticket" on my web page.

Please feel free to send me questions. If they are specific, I can give specific answers.

Responses to some of Tuesday's questions are posted under 0318. Responses to some of today's questions (will hopefully?) be posted under 0319.

Good luck!