AP Calculus AB | Responses 03/24
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Question 1. Why do we use e^x more than $\exp x$ for the exponential function?

Answer. These are the same function, so I assume you are asking, why do most people and the book use the notation e^x instead of $\exp x$?

The best answer to this is probably that it is traditional. Euler was one of the earlier mathematicians to explore this function with a complex domain, and he used this notation:

`https://en.wikipedia.org/wiki/ContributionsofLeonhardEulertomathematics`

But it could also be emphasis; to emphasize that we have a function, and that its domain and range are important, we write $\exp x$. Also, this mirrors the notation for logarithm.

Let a be a positive real number.

The exponential function base a is

$$\exp_a : \mathbb{R} \to (0, \infty) \quad \text{given by} \quad \exp_a(x) = \exp(x \log(a)) = a^x.$$

The logarithm function base a is

$$\log_a : (0, \infty) \quad \text{given by } \log_a(x) = y \Leftrightarrow \exp_a(y) = x.$$

Note the symmetry in the notation. □

Question 2. So e just signifies exponential growth? What else was I supposed to get from the document? I understand the rules and the way exponents work, but in what other way do they relate to e?

Answer. The first question addressed by the document is, "What does a^x mean if x is irrational?"

So first we say what a^x means when x is rational, and this is motivated by the rules of what it means when x is a positive integer.

Next we say "if x_n is a sequence of rational numbers that converges to x, then a^x is the limit of the a^{x_n}'s as n goes to infinity.

The second question addressed by the document is, "what is e". Well, it once we know what e^x is, we define $e = e^1$, such that e^x is in fact the x^{th} power of e (whether x is an integer, rational, or irrational).

One approach to saying what e mean is to consider the balance of an account, with interest compound periodically. As I attempted to explain in the document, the formula is

$$A_k(t) = A_0 \Big(1 + \frac{r}{k}\Big)^{kt},$$

where A_0 is the initial deposit, r is the annual rate, k is the number of periods per year, and $A_k(t)$ is the amount of money in the account after t years.

Then the exponential function produces *continuously compounded interest* by letting the frequency of compounding to to infinity:

$$e^{rt} = \lim_{k \to \infty} A_k(t).$$

Our edition of Thomas doesn't do this; instead, Thomas defines things in this order:

- $\log x = \ln x = \displaystyle\int_1^\infty \frac{1}{t}\, dt$
- $\exp x = e^x$ is the inverse function of log
- $\exp_a x = a^x = \exp(x \log a)$
- $\log_a x$ is the inverse function of $\exp_a$

□

Question 3. Will compound interest be anywhere on the AP or our tests?

Answer. Noncontinuous interest is not on the AP exam, but continuously compounded interest (an application of e^x and differential equations) may be. □

Question 4. When are we going to start reviewing for the AP?

Answer. When we are done with new material. □