AP Calculus AB — Lesson 03/25
Dr. Paul L. Bailey — Tuesday, March 24, 2020

A *differential equation* is an equation involving a function and its derivatives. Usually, the function is y and the independent variable is x. *Solving* a differential equation means finding the function y which, when plugged into the equation, makes it true.

For example, $\frac{dy}{dx} = 3x^2$ is a differential equation. A solution is $y = x^3$, because if $y = x^3$, then $\frac{dy}{dx} = 3x^2$ is true.

The *general solution* is the set of all solutions. For $\frac{dy}{dx} = 3x^2$, the general solution is $y = x^3 + C$.

A *particular solution* is any one of the solutions. For $\frac{dy}{dx} = 3x^2$, a particular solution is $y = x^3 + 7$.

Note that in the case of $y = x^3 + C$, as the C changes, the graph of y moves up or down. Indeed, if you plot any point, it is possible to move the graph of $y = x^3$ up or down until the graph passes through that point.

For example, notice that of all the general solutions to $\frac{dy}{dx} = 3x^2$, only one has a graph which passes through the point $(1, 5)$.

An *initial condition* for a differential equation is a given value for y at a given x.

Example 1. Find the unique particular solution to the differential equation $\frac{dy}{dx} = 3x^2$ whose initial condition is $y(1) = 5$.

Solution. We know that if $\frac{dy}{dx} = 3x^2$, then $y = x^3 + C$ for some C. Since $y(1) = 5$, we have $5 = 1^3 + C$, so $C = 4$. Thus the particular solution is $y = x^3 + 4$. □

We will learn more about differential equations in Chapter 9.

Your assignment for today is to read Thomas §7.5 through Example 5, pages 502 through 507, then work on the following problems. You may wish to read a couple examples, try a problem, then read the rest of the examples. But please look at all 5 of them because that will help you.

- Thomas §7.5 # 1, 2, 3, 4

Also, please attempt the following problem I gave to my College Algebra class at Southern Arkansas University (for fun?). You can do this using properties of logs and exponentials; you don't need a calculator to get the half life. We'll go over it tomorrow.

Problem 1. If a radioactive material has a half-life of h, and the amount at time zero is A_0, then the amount at time t is

$$A(t) = A_0 \left(\frac{1}{2}\right)^{t/h}.$$

In 1992, an evil dictator smuggled in 9.6 kilograms of radioactive delusium. After 12 years (in 2004), conquering armies found that only 0.3 kilograms remained. How much delusium actually existed when the intelligence report was forged in 2001?

(a) Find h; how long does it take for half of the material to decay?

(b) Write the function $A(t)$ using the values for A_0 and h.

(c) Use $A(t)$ to answer the question.

When you are done, fill out the Google Forms Checkin:

0325 AP Calculus AB Checkin