AP Calculus AB — Responses 03/26

Dr. Paul L. Bailey — Tuesday, March 25, 2020

Problem 1. If a radioactive material has a half-life of h, and the amount at time zero is A_0, then the amount at time t is

$$A(t) = A_0\left(\frac{1}{2}\right)^{t/h}.$$

In 1992, an evil dictator smuggled in 9.6 kilograms of radioactive delusium. After 12 years (in 2004), conquering armies found that only 0.3 kilograms remained. How much delusium actually existed when the intelligence report was forged in 2001?

(a) Find h; how long does it take for half of the material to decay?

(b) Write the function $A(t)$ using the values for A_0 and h.

(c) Use $A(t)$ to answer the question.

Solution. We use the equation

$$A(t) = A_0\left(\frac{1}{2}\right)^{t/h},$$

where t is the number of years since 1991. Thus, $A(0) = A_0 = 9.6$. It is also given that $A(12) = 0.3$. We use this to solve for h:

$$\begin{aligned}
A(12) = A_0(1/2)^{12/h} \quad &\Leftrightarrow \quad 0.3 = 9.6(1/2)^{12/h} \\
&\Leftrightarrow \quad \frac{0.3}{9.6} = (1/2)^{12/h} \\
&\Leftrightarrow \quad \frac{9.6}{0.3} = 2^{12/h} \\
&\Leftrightarrow \quad 32 = 2^{12/h} \\
&\Leftrightarrow \quad 2^5 = 2^{12/h} \\
&\Leftrightarrow \quad 5 = 12/h \qquad \text{because exponential functions are injective} \\
&\Leftrightarrow \quad h = \frac{12}{5}.
\end{aligned}$$

Thus, we now have the complete formula for the function amount function:

$$A(t) = 9.6\left(\frac{1}{2}\right)^{5t/12}.$$

Thus

$$A(9) = 9.6\left(\frac{1}{2}\right)^{45/12} \approx 0.7135,$$

certainly not enough to build a bomb. □

Question 1. What is delusium?

Answer. Delusium is an imaginary radioactive substance, believed in by deluded people who accepted the claim that the evil dictator was developing "weapons of mass destruction". The moral of the story: always ask why. □

Question 2. Under the Radioactivity section, can the equation also be applied for exponential growth if k is positive?

Answer. The equation in that section is

$$y = y_0 e^{-kt}, \quad k > 0.$$

Since k is positive, $-k$ is negative, so this is better referred to as "exponential decay". □

Problem 2. If $2\dfrac{dy}{dx} + 5 = x + \cos x$, what is y?

Solution. This is a differential equation; our goal is to solve it. To solve it means, find a function y which satisfies the equation. We could find a particular solution, or we could find the general solution. The general solution is the set of all possible solutions.

In this case, first we solve for $\dfrac{dy}{dx}$ and get

$$\frac{dy}{dx} = \frac{1}{2}(x + \cos x - 5).$$

Integrate both sides with respect to x and get

$$y = \int \frac{dy}{dx}\,dx = \int \frac{1}{2}(x + \cos x - 5)\,dx = \frac{1}{2}\Big(\frac{x^2}{2} + \sin x - 5x\Big) + C.$$

This is the general solution. □

Question 3. I understand how to do differential equations but I didn't know how to do the question "If 2dy/dx+5=x+cosx. what is y?" because there was no y in the equation and usually there is a y and an x to use to solve for C but there wasn't. I don't know if this was a typo but if it isn't, much help is needed. Thank you

Answer. There is a y in $\dfrac{dy}{dx}$. "Solving" a differential equation means, find the function y (as a function of x). □

Question 4. What do you mean by finding y in the first problem? Is half-life going to be on the AP?

Answer. Exponential growth and decay are in the syllabus, and half-life is an excellent example of exponential decay. So yes, there is a possibility it will appear. □

Question 5. I don't understand the previous question because I don't understand the information required to solve the problem.

Answer. If you get stuck on a problem, please email me, or connect with me and/or the class via the chat feature in Windows Teams. That's what we are here for. You are not alone. □