Vector Calculus — Responses 03/26

Dr. Paul L. Bailey — Wednesday, March 25, 2020

Problem 1 (Thomas §16.2 # 33)**.** Find the field $\vec{G}$ in $\mathbb{R}^2$ with the property that at any point $(a,b) \neq (0,0)$, $\vec{G}$ is a vector of magnitude $\sqrt{a^2+b^2}$ tangent to the circle $x^2+y^2=a^2+b^2$ and pointing in the counterclockwise direction. The field is undefined at $(0,0)$.

Solution. The bit about "tangent to the circle $x^2+y^2=a^2+b^2$" just means that the tail of the arrow is (a,b) and it is tangent to the circle centered at the origin which contains this tail.

The vectors in the field are to be perpendicular to the position vector of the point. At the point (a,b), the position vector is $\langle a,b\rangle$; there are two perpendicular directions $\langle b,-a\rangle$ and $\langle -b,a\rangle$. If $a,b>0$, $\langle a,b\rangle$ lies in the upper half plane, and we can see that the counterclockwise direction direction has negative x value, so counterclockwise is $\langle -b,a\rangle$ and clockwise is $\langle b,-a\rangle$.

Then you just have to make it the right length. The length of $\langle -b,a\rangle$ is already $\sqrt{a^2+b^2}$, so it is already the right length. Thus let

$$\vec{G}(x,y)=\langle -y,x\rangle.$$

Its relationship to Figure 16.14 is $\vec{G}=\sqrt{x^2+y^2}\vec{F}$. □

Problem 2 (Thomas §16.2 #34)**.** Find the field $\vec{G}$ in $\mathbb{R}^2$ with the property that at any point $(a,b) \neq (0,0)$, $\vec{G}$ is a vector of magnitude $\sqrt{a^2+b^2}$ tangent to the circle $x^2+y^2=a^2+b^2$ and pointing in the counterclockwise direction. The field is undefined at $(0,0)$.

Solution. This is like # 33, except that the vector has unit length and goes in the opposite direction. To get unit length, divide by the length. To get the opposite direction, negate the vector. Thus

$$\vec{G}(x,y)=\frac{\langle y,-x\rangle}{\sqrt{x^2+y^2}}.$$

In this case, the relationship to Figure 16.4 is $\vec{G}=-\vec{F}$. □

Question 1. Since the equation for flux assumes the closed curve C is to be in the counterclockwise direction, how would you calculate the flux of some F over a curve C if that curve was to be in the clockwise direction?

Answer. This is a important question.

To compute these things (work, flow, flux) along a curve C, it must be an *oriented* curve. That is, you start at one end and go to the other. If you go in the opposite direction, you just get the negative of the other direction. Beyond that, the integral is independent of the parameterization of the curve.

That is, if C is an oriented curve, and $-C$ denote the same curve with the reverse orientation, then

$$\int_{-C} f = -\int_C f.$$

This is because, for any parameterization of C, the integral of the reverse orientation is obtained by simply flipping the limits of integration. □

Question 2. Are you planning on doing a live stream lesson where you review everything? I think that would be very helpful.

Answer. I am planning on holding "office hours". I will have a document viewer that I can write on. If you wish to review everything starting at the beginning of chapter 16, we can; otherwise, I will field your questions. I would like to get several of you on the same conference, so attend if you think it might be helpful.

We will start this next week and we will do it a couple times each week.

At any time, you can chat with me directly, either in the "Chat Room" channel, or (I think you can initiate it) one-on-one. □