AP CALCULUS AB	Lesson 0330
Dr. Paul L. Bailey	Sunday, March 29, 2020

Our goal for this week is to finish covering the material for the AP examination. There are two topics, both of which are covered in Thomas §9.1. These topics are:

- Slope fields
- Separable differential equations

Recall that a differential equation is an equation that involves an independent variable x, a functions y, and the derivatives of y of various orders, namely y', y'', y''', and so forth.

The order of a differential equation is the highest order of a derivative of y that appears in the equation. For example, $\frac{dy}{dx} + y = \frac{x^3}{8} - 2x$ is a first order differential equation, $\frac{d^2y}{dx^2} - ye^x = \cos x$ is a second order differential equation.

Consider the second order differential equation y'' + y = 0. It can be shown that the general solution is $y = a \sin x + b \cos x$.

Slope fields are an interesting aspect of first order differential equations. This is because, for every ordered pair (a, b), we can plug in x = a and y = b and solve for $\frac{dy}{dx}$. This will tell us the slope of any solution at a given point.

For example, consider the equation $\frac{dy}{dx} + y = \frac{x^3}{8} - 2x$. We can solve if for y' and arrive at

$$y'(x,y) = \frac{x^3}{8} - 2x - y.$$

The slope of y at (0,0) is y'(0,0) = 0, at (2,3) is is y'(2,3) = 1 - 4 - 3 = -6, and so forth. If we do this at every point, we get a real number which represents slope at every point in the plane. This is called a *slope field*.

We can draw a picture of a slope field by drawing a small, tilted line segment at a collection of points in the plane. The slope of the small line segment is the value of y'(x, y). This is a detailed picture of the slope field for $\frac{dy}{dx} + y = \frac{x^3}{8} - 2x$.

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Your assignment for today is:

- Read Thomas $\S9.1$ pages 642 646 up to Example 3 on separable equations.
- Watch the following video:

Kahn Academy Slope Fields

• Why Viruses Spread Exponentially

After your have done this, please acknowledge that by filling out the following Google Forms checkin.

0330 AP Calculus AB Checkin

I will respond to your questions and work out Friday's solutions soon.

We will have a Microsoft Teams meeting Monday at 9 AM. I will send an invite on the General channel chat. The purpose of the meeting is to discuss how to proceed going forward. Please attend if you are interested.