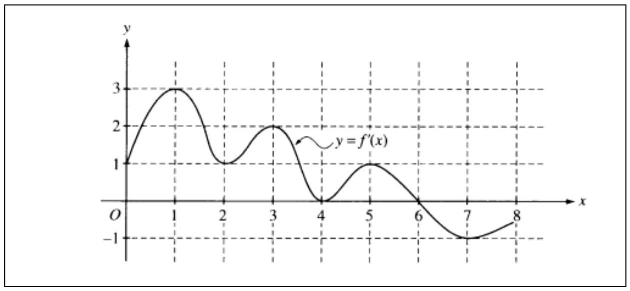
AP Calculus AB — Quiz 0409 Solutions

Dr. Paul L. Bailey — Monday, April 13, 2020

Problem 1. The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown below.



The point $(3, 5)$ is on the graph of $y = f(x)$. Find an equation of the line tangent to the graph of f at $(3, 5)$.

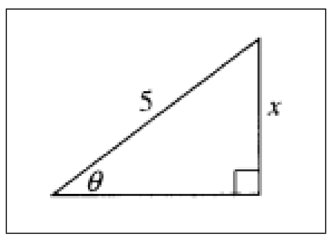
Solution. (C)

To find the slope of the line, we inspect the graph; $f'(3) = 2$, so the slope is 2, so the line in

$$\boxed{y = 2(x - 3) + 5.}$$

□

Problem 2. Consider the triangle shown below.



If θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

Solution. (E)

We know that $x = 5\sin\theta$ and that $\dfrac{d\theta}{dt} = 3$.

The cheese is $\dfrac{dx}{dt}$ when $x = 3$. When $x = 3$, the side adjacent to θ equals 4, and $\cos\theta = \dfrac{4}{5}$.

Implicitly differentiate the equation with respect to t to get

$$\frac{dx}{dt} = 5\cos\theta\frac{d\theta}{dt} = 5 \cdot \frac{4}{5} \cdot 3 = 12.$$

□

Problem 3. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

Solution. (A)

The average value of f on $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

First we compute the antiderivative by letting $u = x^3+1$, so that $du = 3x^2\,dx$, and get

$$\int x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} = \frac{2}{9}(x^3+1)^{3/2}.$$

Then

$$\texttt{Average} = \frac{1}{2}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\cdot\frac{2}{9}(x^3+1)^{3/2}\Big|_0^2 = \frac{1}{9}(27-1) = \frac{26}{9}.$$

□

Problem 4. If f is continuous on $[a,b]$ and differentiable on (a,b), which of the following could be false?

(A) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some $c \in (a,b)$.

(B) $f'(c) = 0$ for some $c \in (a,b)$

(C) f has a minimum value on $[a,b]$

(D) f has a maximum value on $[a,b]$

(E) $\int_a^b f(x)\,dx$ exists.

Solution. (B)

The function satisfies the hypothesis of the Extreme Value Theorem (EVT), so it has a global minimum and maximum, so **(C)** and **(D)** are true.

The function satisfies the hypothesis of the Mean Value Theorem (MVT), so its derivative equals its average value somewhere on the interval, so **(A)** must be true.

The function is continuous, so it is integrable, so **(E)** is true.

Thus the answer must be **(B)**. An example of such a function is $f(x) = x$; it is continuous and differentiable everywhere, but the derivative is never zero. □

Problem 5. If $f(x) = (x-1)^2\sin x$, what is $f'(0)$?

Solution. (D)

Use the product rule to get

$$f'(x) = 2(x-1)\sin x + (x-1)^2\cos x.$$

Thus

$$f'(0) = 2(-1)\sin(0) + (-1)^2\cos(0) = 1.$$

□