AP Computer Science	Project 35c - Components and Trees
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A graph is a pair  $(V, \mathcal{E})$ , where V is a set whose members are called *vertices*, and  $\mathcal{E}$  is a set whose members are called *edges*, where an *edge* is a subset of V which contains exactly two (distinct) vertices.

We view the vertices as points and the edges as line segments between them; however, this viewpoint only models the abstract notion of graph.

Two vertices are *adjacent* is they are contained in the same edge. Adjacent vertices are called *neighbors*. The *degree* of a vertex is the number of adjacent vertices.

A subgraph of  $(V, \mathcal{E})$  is a graph  $(W, \mathcal{F})$  such that  $W \subset V$  and  $\mathcal{F} \subset \mathcal{E}$ . Note that if  $(W, \mathcal{F})$  is a subgraph of (V, W), the edges in  $\mathcal{F}$  must contain only vertices that come from W.

A walk in a graph  $(V, \mathcal{E})$  is a finite sequence of at least three vertices such that their exists an edge between consecutive vertices. We say that the walk visits each of the vertices in the sequence, and that it traverses each edge between consecutive vertices. The length of the walk is the number of vertices visited, minus one. The first vertex in the sequence is called the *initial* vertex of the walk, and the last is called the terminal vertex of the walk.

The graph is *connected* if their exists a walk between any two vertices. A *component* of a graph is a maximal connected subgraph. Two vertices are *associates* if they lie in the same component.

A walk is *Eulerian* if it traverses every edge in the graph.

A trail is a walk with distinct edges. A path is a walk with distinct vertices.

A *circuit* is a walk whose initial vertex equals its terminal vertex. A *cycle* is a circuit which visits each vertex at most once, except for the initial and terminal vertex.

A *tree* is a connected graph which does not admit a cycle. Given two vertices in a tree, there is exactly one trail from one to the other. A *forest* is a graph in which each component is a tree.

- A connected graph has an Eulerian circuit if and only if every vertex has even degree.
- A connected graph has an Eulerian trail if and only if exactly zero or two vertices have odd degree.

Download the file ZCScix35\_Graph\_Gift20180423.zip if you have not already done so. This contains the complete Netbeans project to implement the low level classes for the Graph Theory project. We will add methods to this project.

**Program 1.** Create a method public List<Vertex> adjacents(Vertex v) returns a list of the vertices which share an edge with v (excluding v).

**Program 2.** Create another method public List<Vertex> associates(Vertex v) returns a list of vertices which may be visited by a walk initiating at v (including v).

**Program 3.** Create a method public boolean isConnected() which returns true if the graph is connected, and false otherwise.

**Program 4.** Create a method public int numberOfComponents() which returns the number of components. This is nonnegative and is zero if and only if the number of vertices is zero.

**Program 5.** Create a method public boolean isTree() which returns true if the graph is a tree. Create another method public boolean isForest() which returns true if the graph is a forest.