Question 1. How do you use chain rule with integral problems?

Answer. Substitution is running the chain rule in reverse. See R0414b.

Question 2. How am I supposed to integrate a modulus?

Answer. If

## $f(x) = \begin{cases} f_1(x) & \text{if } x < a \\ f_2(x) & \text{if } x \ge a \end{cases}$

then

$f'(x) = \left\{ \begin{array}{l} \\ \end{array} \right.$	$\int f_1'(x)$	if $x < a$
	$f_2'(x)$	$\text{if } x \ge a$

In particular, if f(x) = |x|, then

$$f(x) = \begin{cases} -x & \text{if } x < 0\\ x & \text{if } x \ge 0 \end{cases}$$

then

$$f'(x) = \begin{cases} -1 & \text{if } x < 0\\ 1 & \text{if } x \ge 0 \end{cases}$$

Does that answer you question?

**Question 3.** How do you take the integral of  $x^{-1}$ ?

Answer. If we use the power rule, we get  $\int x^{-1} dx = \frac{x^{-1+1}}{-1+1} = \frac{1}{0}$ . Well that's not right. According to Thomas Section 7.2, the *definition* of  $\ln x$  is

$$\ln x = \int_1^x t^{-1} dt$$

 $\int \frac{1}{x} \, dx = \ln x + C.$ 

So  $\frac{d}{dx} \ln x = \frac{1}{x}$ , so

Question 4. what is the integral of In(x)?

Answer. Note that

$$\frac{d}{dx}(x\ln x - x) = \frac{d}{dx}x\ln x - \frac{d}{dx}x = (\ln x + \frac{x}{x}) - 1 = \ln x.$$

Thus

$$\int \ln x \, dx = x \ln x - x + C.$$

This can be derived using "Integration by Parts", a topic which is a BC topic. It's relatively easy, though, and you can read about it in Thomas Section 8.2.  $\hfill \square$ 

Question 5. Can you invert a sphere without making an incision on the surface?

Answer. You may like this:

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