Problem 1. A tank in the shape of a cylinder of radius 5 decimeters and height 20 decimeters, which is filled with water, develops a circular hole in its bottom. The radius of the hole increases at a rate of 2 cm/minute. Water drains from the tank at the rate of 3 liters per minute per square centimeter of area of the hole. How long does it take for the tank to drain completely?

A liter is a cubic decimeter. Let t be time in minutes, starting with $t = 0$ when the hole develops. Let r be the radius of the hole and let A be its area. Let V be the volume of water in the tank.

(a) Find the initial volume of the tank.

(b) Find r as a function of t.

(c) Find A as a function of t.

(d) Find $\frac{dV}{dt}$ as a function of t (this should be negative).

(e) Find V as a function of time x by noting that

$$V(x) - V(0) = \int_0^x \frac{dV}{dt}\,dt,$$

where $V(x)$ is the volume of water in the tank at time x and $V(0)$ is the initial volume.

(f) Find time x such that $V(x) = 0$.

Optional Challenge Problems: Finding examples of things can be challenging. Try your hand at these. You don't have to turn them in.

Problem 2. (Examples) Show that these situations can exist by finding examples. Briefly justify your answer. In each case, choose whichever $a, b \in \mathbb{R}$ you like, as long as $a < b$.

(a) Construct a differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfying:

- f is increasing on $(-\infty, a)$;
- f is increasing on (b, ∞);
- f is constant on $[a, b]$.

(b) Construct a differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfying:

- f has a local maximum a;
- f has a local minimum b;
- $f(a) < f(b)$.