AP Calculus AB — Responses 0415

Dr. Paul L. Bailey — Tuesday, April 14, 2020

Question 1. How do you determine if a function is increasing or decreasing on an interval if the function consists of an integral?

Answer. Let $F(x) = \int_a^x f(t)\,dt$. Then $F'(x) = f(x)$.

We know that F is increasing if f is positive; that is, if the graph of f lies above the x-axis.

We know that F is decreasing if f is negative; that is, if the graph of f lies below the x-axis. □

Question 2. What is the correct notation for justifying an answer?

Answer. "Justify your answer" means "explain you reasoning". This can occasionally be done without writing in complete sentences, by why take the risk? Always write mathematics in complete sentences. If you have an equation, state its role in what you are discussing. If you are drawing a conclusion, use words to state your reasoning.

Approach writing mathematics as a lawyer writing an argument for a judge. Convince the reader not only that you know what you are talking about, but also that you are right. □

Question 3. When finding the absolute minimum and maximum, do you use the first or second derivative?

Solution. Before you look for the absolute minimum and maximum, verify the hypothesis of the Extreme Value Theorem. Your function should be *continuous* on a *closed interval.* If it is, you know it has a absolute minimum and an absolute maximum. These can *only* occur at a critical point or at an endpoint.

Let $f : [a, b] \to \mathbb{R}$ be continuous. To find the absolute extrema:

- Identify any point x in (a, b) at which $f'(x)$ does not exist; these are critical points.
- Compute $f'(x)$ and find all points x in (a, b) at which $f'(x) = 0$; these are critical points.
- Plug all of the endpoints and critical points into f and compare the values. The highest value is the maximum value of f on $[a, b]$; the lower value is the minimum value of f on $[a, b]$.

□

Question 4. When finding the relative minima and maxima, do you use the first or second derivative?

Solution. Let $f : [a, b] \to \mathbb{R}$ be continuous, and differentiable on (a, b).

If we say "f' changes from negative to positive at x", that means moving from the left to the right on the x-axis.

To find the relative extrema:

- Compute $f'(x)$ and find all points x in (a, b) at which $f'(x) = 0$; these are the critical points. We wish to classify them as minimum, maximum, or neither.
- You can use the "First Derivative Test". Suppose $f'(x) = 0$.
  - If f' changes from negative to positive at x, then f has a local minimum at x.
  - If f' changes from positive to negative at x, then f has a local maximum at x.
  - If f' does not change sign at x, then f does not have a local extremum at x.
- You can use the "Second Derivative Test". Suppose $f'(x) = 0$ and that $f''(x)$ exists.
- If $f''(x) > 0$, f is concave up at x, so f has a local minimum at x.
- If $f''(x) < 0$, f is concave down at x, so f has a local maximum at x.
- If $f''(x) = 0$, the test is inconclusive. For example, $f(x) = x^4$ has a local minimum at $x = 0$.

□

Question 5. What's the difference between volume of revolutions shell method and washer method again?

Question 6. Is it easier to use washers or shells?

Answer. Is it better to use washers or shells? It depends.

For example, let R be the region in the first quadrant above $y = 0$ and below $y = 2x - x^2$.

If you revolve this region about the x-axis, it is easier to use disks, because the integral is just

$$V = \int_0^2 \pi(2x - x^2)^2 \, dx.$$

However, if you revolve this region about the y-axis, you would have solve $y = 2x - x^2$ for y to get the radii of the washers. But if you use shells, the integral is

$$V = \int_0^2 2\pi x(2x - x^2) \, dx.$$

Basically, use the one for which you don't have to invert your equation. Typically, if you are revolving $y = f(x)$ around a horizontal line, washers are easier, but if you are revolving it around a vertical line, shells are easier.

College Board claims that shells are not on the AP examination, but I have found problems on the test that are easier with shells. □

Question 7. How do I find the domain of an ln equation?

Answer. We know that natural log is defined at x if and only if $x > 0$.

However, how do we use this to find the domain of a composition involving natural log?

For example, let $f(x) = \ln x^2$. This is defined at x if and only if $x^2 > 0$, which occurs exactly when $x \neq 0$. Thus $\operatorname{dom}(\ln x^2) = \{x \in \mathbb{R} \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

On the other hand, let $g(x) = \sqrt{\ln x}$. Here, x has to be in the domain of log, *and* $\ln x$ has to be in the domain of square root. So, $x > 0$ and $\ln x \geq 0$. The latter condition include the former, and is true when $x \geq 1$. So $\operatorname{dom}(\sqrt{\ln x}) = [1, \infty)$. □

Question 8. How do we use slope fields and what are some reasons for using them?

Answer. They can help you visualize a solution to a differential equation. Imagine the slope field and a river flowing; if you drop a leaf at a given point, follow the slopes to imagine it moving. □