Exam

Name___________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the derivative of the function at P_0 in the direction of u.

1) $f(x, y) = \tan^{-1} \frac{-3x}{y}$, $P_0(-7, -8)$, $u = 12i - 5j$ 1) _______

A) $\frac{456}{6565}$ B) $\frac{27}{505}$ C) $\frac{477}{6565}$ D) $\frac{393}{6565}$

Calculate the circulation of the field F around the closed curve C.

2) $F = (-x - y)i + (x + y)j$, curve C is the counterclockwise path around the circle with radius 3 centered at (3, 6) 2) _______

A) $18(1 + \pi)$ B) $18(1 + \pi) + 108$ C) 18π D) 36π

Find the potential function f for the field F.

3) $F = \frac{1}{z}i - 2j - \frac{x}{z^2}k$ 3) _______

A) $f(x, y, z) = \frac{x}{z} - 2 + C$ B) $f(x, y, z) = \frac{2x}{z} - 2y + C$

C) $f(x, y, z) = \frac{x}{z} + C$ D) $f(x, y, z) = \frac{x}{z} - 2y + C$

Evaluate. The differential is exact.

4) $\int_{(0,0,0)}^{(\pi,\pi,\pi)} -2 \sin x \cos x \, dx - \sin y \cos z \, dy - \cos y \sin z \, dz$ 4) _______

A) -2 B) 2 C) 0 D) 1

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

5) $F = (x^2 + y^2)i + (x - y)j$; C is the rectangle with vertices at (0, 0), (8, 0), (8, 5), and (0, 5) 5) _______

A) 160 B) 0 C) -160 D) 240

vector Calculus Q0416 solutions

① $f = \arctan\left(\frac{-3x}{y}\right)$ $\quad P_0 = (-7,-8)$ $\quad \vec{u} = \langle 12,-5 \rangle$

$$\nabla f = \left\langle \frac{1}{1+\left(\frac{3x}{y}\right)^2}\left(-\frac{3}{y}\right), \frac{1}{1+\left(\frac{3x}{y}\right)^2}\left(\frac{3x}{y^2}\right) \right\rangle$$

$$\nabla f(-7,-8) = \frac{1}{1+\left(\frac{21}{8}\right)^2}\left\langle \frac{3}{8}, \frac{-21}{64} \right\rangle$$

$$= \frac{1}{64+21^2}\langle 24,-21 \rangle$$

$$= \frac{1}{505}\langle 24,-21 \rangle$$

$$\begin{array}{r} 505 \\ \underline{13} \\ 1515 \\ \underline{505} \\ 6565 \end{array}$$

$$D_{\vec{u}} f = \frac{\nabla f \cdot \vec{u}}{|\vec{u}|} = \frac{1}{505 \cdot 13}\langle 24,-21 \rangle \cdot \langle 12,-5 \rangle$$

$$= \frac{288+105}{505 \cdot 13} = \frac{393}{505 \cdot 13} = \boxed{\frac{393}{6565}}$$

$\boxed{\text{Answer D}}$

#2) $\vec{F} = \langle x - y, x + y \rangle$ $\quad \vec{r}(t) = \langle 3\cos t, 3\sin t \rangle$ $[0, 2\pi]$

$\vec{v}(t) = \langle -3\sin t, 3\cos t \rangle$

$$\text{Flow} = \int_C \vec{F} \cdot \vec{v}\, dt =$$

$$= \int_0^{2\pi} \langle 3\cos t - 3\sin t, 3\cos t + 3\sin t \rangle \langle -3\sin t, 3\cos t \rangle\, dt$$

$$= \int_0^{2\pi} -9\sin t\cos t + 9\sin^2 t + 9\cos^2 t + 9\sin t\cos t \; dt$$

$$= \int_0^{2\pi} 9\, dt = \boxed{18\pi}$$

That's C

#3) $\vec{F} = \left\langle \frac{1}{z}, -2, -\frac{x}{z^2} \right\rangle$ Find potential.

$$f = \int -2\, dy = -2y + g(x,z)$$

$$\frac{\partial f}{\partial x} = \frac{\partial g}{\partial x} = \frac{1}{z} \Rightarrow g = \frac{x}{z} + h(z) \Rightarrow f = -2y + \frac{x}{z} + h(z)$$

$$\frac{\partial f}{\partial z} = -\frac{x}{z^2} + \frac{dh}{dz} = -\frac{x}{z^2} \Rightarrow h = C$$

So $\boxed{f(x,y,z) = \frac{x}{z} - 2y + C}$ That's D

#4) $$\int_{(0,0,0)}^{(\pi,\pi,\pi)} -2\sin x \cos x \, dx - \sin y \cos z \, dy - \cos y \sin z \, dz$$

If f is potential, $\int = f(B) - f(A)$.

Let $f = \int -2\sin x \cos x \, dx \Rightarrow f = \cos^2 x + g(y,z)$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial y} = -\sin y \cos z \Rightarrow g = \cos y \cos z + h(z)$$

$$\Rightarrow f = \cos^2 x + \cos y \cos z + h(z).$$

$$\frac{\partial f}{\partial z} = \frac{dh}{dz} = -\cos y \sin z \Rightarrow h = \cos y \cos z + C$$

So $f = \cos^2 x + \cos y \cos z + \cos y \cos z + C$

Now $\int = f(\pi,\pi,\pi) - f(0,0,0) = 3 - 3 = 0$ That's C

#5) $\vec{F} = \langle x^2+y^2, x-y \rangle$ on $[0,8]\times[0,5]$

Circulation $= \int_C \vec{F}\cdot\vec{v}\,dt = \iint_R \text{curl}\,\vec{F}\,dA$ by Green's Theorem Form B

$$= \int_0^5 \int_0^8 1-2y \, dx\,dy = 8\left[y - y^2\right]_0^5$$

$$= 8[-20]$$

$$= -160$$ That's, C