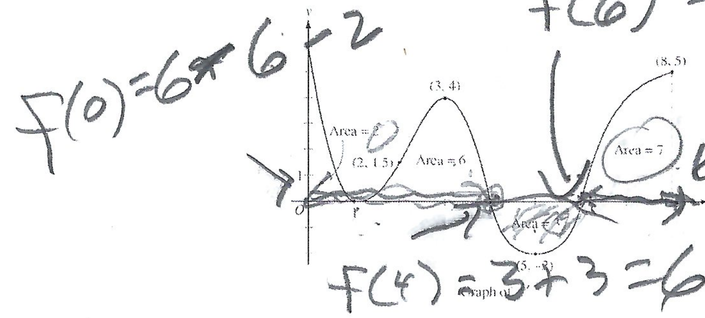
AP Calculus AB
Dr. Paul L. Bailey

Homework 0421a
Tuesday, April 21, 2020

Problem 1. The figure below shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$.

$f(6) = 4 - 7 = 3$



The graph of f' has horizonal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

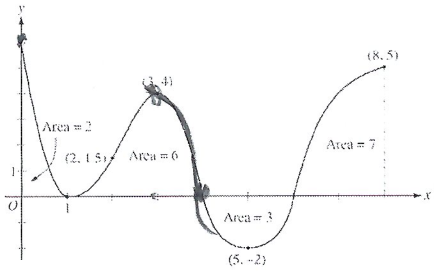
(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

We know that f has a local minimum at x if f' changes sign from negative to positive at x.
This happens at x = 6.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

Use EVT.
We know f has an absolute minimum at an endpoint or at a critical point where f' changes sign. The endpoints are 0 and 8. The pertainent critical points are 4 and 6.
We know f(8)=4.
So, $f(6) = 4 - 7 = -3$
$f(4) = -3 + 3 = 0$
$f(0) = 0 - 6 - 2 = -8$
The minimum value is -8, at x=0.

Problem 1 ((continued)). The figure below shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$.



Graph of f'

The graph of f' has horizonal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

We know f is increasing if $f' > 0$, and f is concave down if $f'' < 0$, that is, when f' is decreasing. This happens on $(0,1)$ and $(3,4)$.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

Want $g'(3)$.

Know $g'(x) = 3(f(x))^2 (f'(x))$

So $g'(3) = 3\left(-\frac{5}{2}\right)^2 (4)$

$= \boxed{75}$

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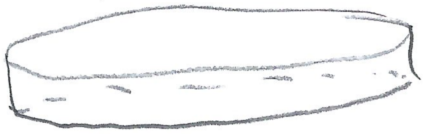
Problem 1. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?



Cheese $\frac{dh}{dt}$

$V = \pi r^2 h$

$\frac{dV}{dt} = 2000 \qquad \frac{dr}{dt} = 2.5$

$r = 100$
$h = 0.5$

$$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$$2000 = \pi\left(2(100)(2.5)\frac{1}{2} + 10000\frac{dh}{dt}\right)$$

$$= \pi\left(250 + 10000\frac{dh}{dt}\right)$$

$$\frac{dh}{dt} = \frac{\frac{2000}{\pi} - 250}{10000} = \frac{1}{5\pi} - \frac{1}{40}$$

Problem 1 (continued). Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

Now, $\frac{dV}{dt} = 2000 - 400\sqrt{t}$

$V = V_0 + \int 2000 - 400\sqrt{t}\, dt$

So $400\sqrt{t} = 2000$

$\sqrt{t} = 5$

$t = 25$

Since $\frac{dV}{dt} < 0$ for $t < 25$ and $\frac{dV}{dt} > 0$ for $t > 25$,

we know V has a local max at $t = 25$.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

$$\boxed{V = 60000 + \int_0^{25} 2000 - 400\sqrt{t}\, dt}$$