CATEGORY THEORY Lesson 0422
DR. PAUL L. BAILEY Wednesday, April 22, 2020

Read through the first few pages of Gallian Chapter 32. Then, think about these problems.

Let E/F be a field extension. Let $\text{Aut}(E)$ denote the set of all automorphisms of E. Let $\text{Aut}(E/F)$ denote the subgroup of $\text{Aut}(E)$ consisting of those automorphisms we fix F pointwise:

$$\text{Aut}(E/F) = \{\phi \in \text{Aut}(E) \mid \phi(x) = x \text{ for all } x \in F\}.$$

I do not call this $\text{Gal}(E/F)$ unless E/F is a normal extension.

Let $H \leq \text{Aut}(E/F)$ be a subgroup of $\text{Aut}(E/F)$. Let

$$\text{Fix}(H) = \{x \in E \mid \phi(x) = x\}.$$

Problem 1. Let E/F be a finite field extension. Show that $\text{Fix}(H)$ is a subfield of E which contains F.

Problem 2. Let E/F be a finite field extension. Let K be a subfield of E which contains F.
Let $G = \text{Aut}(E/F)$ and $H = \text{Aut}(E/K)$. Clearly $H \leq G$.

(a) Does $\text{Fix}(H) = L$?

(b) How does $\text{Aut}(K/F)$ relate to G and H?