AP CALCULUS AB
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Homework 0422c
Wednesday, April 22, 2020

Problem 1. Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table below.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

← ✓

(a) Use the data in the table to estimate the value $v'(16)$.

$v'(16) \approx$ avg ~~velocity~~ rate of change of v

$= \dfrac{240-200}{20-12} = 5 \dfrac{m}{min^2}$

Remember units!

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

We know $\int_0^{40} |v(t)|\,dt$ is the total distance traveled.

RRR $= 200(12-0) + 240(20-12) + 220(24-20) + 150(40-24)$

$= 2400 + 1920 + 880 + 2400$

$= 7600$ m

$$\begin{array}{r} 2400 \\ 2400 \\ 1920 \\ 880 \\ \hline 7600 \end{array} \qquad \begin{array}{r} 240 \\ 8 \end{array} \qquad \begin{array}{r} 150 \\ 16 \\ 300 \\ 8 \end{array}$$

Problem 1 ((continued)). Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table below.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

(c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

$$B'(t) = 3t^2 - 12t$$

$$B'(5) = 75 - 60 = 15 \frac{m}{min^2}$$

(d) Based on the model B from part **(c)**, find Bob's average velocity during the interval $0 \leq t \leq 10$.

Average Value of B on $[0, 10]$

$$= \frac{1}{10} \int_0^{10} B(t)\, dt$$

$$= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10}$$

$$= \frac{1000}{4} - 200 + 300$$

$$= 250 \cancel{-100} = 350 \frac{m}{min}$$

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

(a) $v'(16) \approx \dfrac{240 - 200}{20 - 12} = 5$ meters/min^2

1 : approximation

(b) $\int_0^{40} |v(t)|\, dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes.

$$\begin{aligned}\int_0^{40} |v(t)|\, dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

$3 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$B'(5) = 3(25) - 12(5) = 15$ meters/min^2

$2 : \begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d)
$$\begin{aligned}\text{Avg vel} &= \frac{1}{10}\int_0^{10} \left(t^3 - 6t^2 + 300\right) dt \\ &= \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10} \\ &= \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350 \text{ meters/min}\end{aligned}$$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$