Vector Calculus

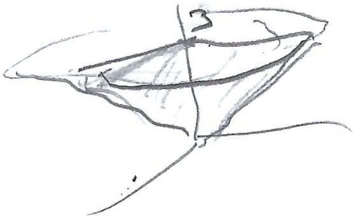
§16.6

#3 Parameterize 1^{st} octant of $z = \frac{1}{2}\sqrt{x^2+y^2}$

between $z=0$ and $z=3$

$3 = \frac{r}{2}$

$r = 6$

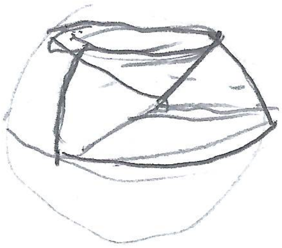


$\vec{r}(u,v) = \langle$

$\vec{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, \frac{r}{2} \rangle$

$u = r \in [0,6]$ $v = \theta \in [0, \frac{\pi}{2}]$

$\boxed{\#6}$



$x^2+y^2+z^2=4$ in 1^{st} octant

under cone $z=\sqrt{x^2+y^2}$

in spherical coord.

$\cancel{(\phi,\theta)}$ $\vec{r}(u,v) = \vec{r}(\phi,\theta)$

$= \langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi \rangle$

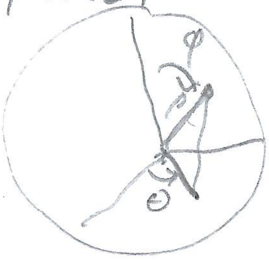
$v = \theta \in [0, \frac{\pi}{2}]$

$u = \phi \in [\frac{\pi}{4}, \frac{\pi}{2}]$

$r = \rho\sin\phi$

$x = r\cos\theta$

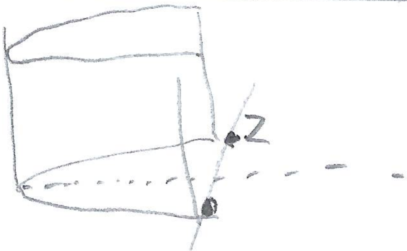
$$\boxed{\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$



#10 Parabolic cylinder

$y = x^2$

cut by $z=0$, $z=3$, $y=2$



$\vec{r}(u,v) = \langle u, u^2, v \rangle$

$u \in [-\sqrt{2}, \sqrt{2}]$

$v \in [0,3]$

#14 $x-y+2z=2$

inside $x^2+z^2=3$

$\vec{r}(u,v) = \vec{r}(r,\theta)$

$x = r\cos\theta$

$z = r\sin\theta$

$y = x-2z-2$

$\vec{r}(r,\theta) = \langle r\cos\theta, r\cos\theta - 2$

#14a Portion of plane

$$x - y + 2z = 2$$

(a) inside cylinder $x^2 + z^2 = 3$

$x = r\cos\theta$

$z = r\sin\theta$

$y = x + 2z - 2$

$= r\cos\theta + 2r\sin\theta - 2$

so $\vec{r}(u,v) = \vec{r}(r,\theta) = \langle r\cos\theta, r\cos\theta + 2r\sin\theta - 2, r\sin\theta \rangle$

$u = r \in [0, \sqrt{3}]$

$v = \theta \in [0, 2\pi]$

(b) Inside $y^2 + z^2 = 2$

$y = r\cos\theta$

$z = r\sin\theta$

$x = y - 2z + 2$

$= r\cos\theta - 2r\sin\theta + 2$

$\vec{r}(u,v) = \vec{r}(r,\theta) = \langle r\cos\theta - 2r\sin\theta + 2, r\cos\theta, r\sin\theta \rangle$

$u = r \in [0, \sqrt{2}]$

$v = \theta \in [0, 2\pi]$