AP Calculus AB | Quiz 0424 | Name:
Dr. Paul L. Bailey | Friday, April 24, 2020

Problem 1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.

Solution. Since acceleration is the derivative of velocity,

$$a(t) = \frac{dv}{dt} = -2\pi \cos(2\pi t).$$

□

(b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

Solution. The particle is at rest when $v(t) = 0$, so $1 - \sin(2\pi t) = 0$, so $\sin(2\pi t) = 1$. This occurs at $2\pi t = \frac{\pi}{2} \pm n\pi$, where n is an integer, so $t = \frac{1}{4} \pm \frac{n}{2}$. In the interval $[0, 2]$, the solutions are $t = \frac{1}{4}$ and $t = \frac{5}{4}$. □

(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

Solution. We know that position is the antiderivative of velocity, in the sense that

$$x(t) = \int v(t)\, d = \int 1 - \sin(2\pi t)\, dt = t + \frac{\cos(2\pi t)}{2\pi} + C.$$

We use that knowledge that $x(0) = 0$ to find C:

$$0 = x(0) = 0 + \frac{\cos(0)}{2\pi} + C = \frac{1}{2\pi} + C \quad \Rightarrow \quad C = -\frac{1}{2\pi}.$$

Thus

$$x(t) = t + \frac{\cos(2\pi t)}{2\pi} - \frac{1}{2\pi}.$$

□