AP Calculus AB
Dr. Paul L. Bailey

Homework 0428a
Tuesday, April 28, 2020

Problem 1. The functions f and g have continuous second derivatives. The table below gives values of the functions and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

First, find the slope of the line.

We know $k'(x) = f'(g(x))\,g'(x)$.

Then $k'(3) = f'(g(3))\,g'(3)$

$= f'(6)(2)$

$= 5 \cdot 2 = 10$

So $y = m(x - x_0) + y_0$ where $x_0 = 3$, $y_0 = k(3) = f(g(3)) = f(6) = 4$

So $\boxed{y = 10(x-3) + 4}$

~~maximize~~

$$\cancel{\frac{d}{dn}\;\frac{1}{n+1}\;\frac{1}{2n}}$$

Problem 1 (continued). The functions f and g have continuous second derivatives. The table below gives values of the functions and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

We know $h'(x) = \dfrac{g'f - gf'}{f^2}$

So $h'(1) = \dfrac{g'(1)f(1) - g(1)f'(1)}{(f(1))^2}$

$= \dfrac{8(-6) - 2(3)}{(-6)^2}$

$= \dfrac{-48-6}{36} = -\dfrac{54}{36} = -\dfrac{3}{2}$

(c) Evaluate $\displaystyle\int_1^3 f''(2x)\,dx$.

We use FTC. We know

$\int f''(2x)\,dx = \frac{1}{2}f'(2x) + C$

So $\displaystyle\int_1^3 f''(2x)\,dx = \frac{1}{2}\left[f'(2(3)) - f'(2(1))\right]$

$= \frac{1}{2}\left[f'(6) - f'(2)\right]$

$= \frac{1}{2}\left[5+2\right] = \frac{7}{2}$

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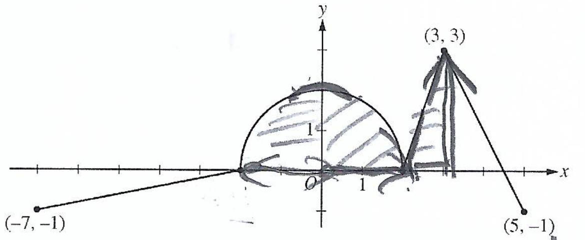
Homework 0428b
Tuesday, April 28, 2020

Problem 1. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure below.



Graph of g'

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

(a) Find $g(3)$ and $g(-2)$.

We know $g(0) = 5$,

so $f(b) = f(a) + \int_a^b f'(x)\,dx$

So $g(3) = g(0) + \int_0^3 g'(x)\,dx$

$= 5 + \frac{1}{4}(\pi 2^2) + \frac{1}{2}(1)(3)$

$= 5 + \pi + \frac{3}{2} = \frac{13}{2} + \pi$

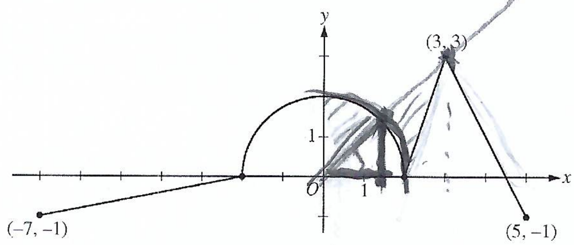
$g(-2) = g(0) - \int_{-2}^{0} g'(x)\,dx = \boxed{5 - \pi}$

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

We know g has a point of inflection where g' changes direction, that is, it changes from increasing to decreasing or vice versa.

This occurs at $\boxed{x = 0, 2, 3}$

Problem 1 (continued). The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure below.



Graph of g'

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

We set $h'(x) = 0$ and solve for x.

So $h'(x) = g'(x) - x = 0$.

We want $g'(x) = x$.

This occurs at $x = 3$, and

$(2\cos\frac{\pi}{4}, 2\sin\frac{\pi}{4}) = (\sqrt{2}, \sqrt{2})$

so $x = \sqrt{2}$

We see that h' changes sign from positive to negative at $x = \sqrt{2}$, so g changes from increasing to decreasing at $x = \sqrt{2}$, so we have a local maximum there.

At $x = 3$, h' does not change sign.

So, it is not a local extremum.

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Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
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3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

(a) $k(3) = f(g(3)) = f(6) = 4$
$k'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$3 : \begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$

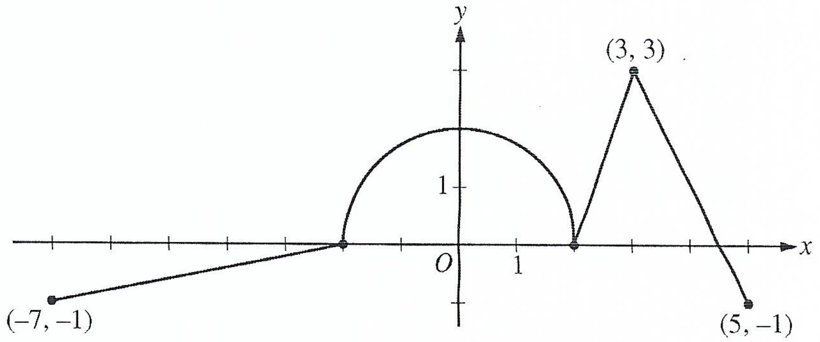
(b) $$h'(1) = \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} = \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2}$$

$3 : \begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$

(c) $$\int_1^3 f''(2x)\,dx = \frac{1}{2}\left[f'(2x)\right]_1^3 = \frac{1}{2}[f'(6) - f'(2)] = \frac{1}{2}[5 - (-2)] = \frac{7}{2}$$

$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$ $g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$	$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$
(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.	$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$
(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$ On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$. On this interval, $g'(x) = x$ when $x = \sqrt{2}$. The only other solution to $g'(x) = x$ is $x = 3$. $h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$ $h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$ Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.	$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$