I don't see any uploads in the assignments I made in Microsoft Classroom. Please try to write the solutions to these problems and upload them by tomorrow. I took out the hard part.

Problem 1. Let E/F be a finite separable extension.

(a) Show that $|\mathrm{Aut}(E/F)| \leq [E : F]$.

(b) Show that if E/F is normal, then $|\mathrm{Aut}(E/F)| = [E : F]$.

Problem 2 (Bilbo's Lemma)**.** Let E/F be a field extension. Let K be a subfield of E which contains F. Let $\alpha \in E$ be algebraic over F. Let $f \in F[X]$ be the minimum polynomial of α over F, and let $g \in K[X]$ be the minimum polynomial of α over K. Show that g divides f in $K[X]$.

Problem 3. Let E/F be a field extension. Let K be a subfield of E which contains F. Show that if E/F is normal, then E/K is normal.

Let $H \leq \mathrm{Aut}(E)$. The *fixed field* of H is

$$\mathrm{Fix}(H) = \{x \in E \mid \phi(x) = x \text{ for all } \phi \in H\}.$$

Problem 4. Let E/F be a finite separable extension. Let $H \leq \mathrm{Aut}(E/F)$. Let $K = \mathrm{Fix}(H)$. Show that K is a subfield of E which contains F.