AP CALCULUS AB
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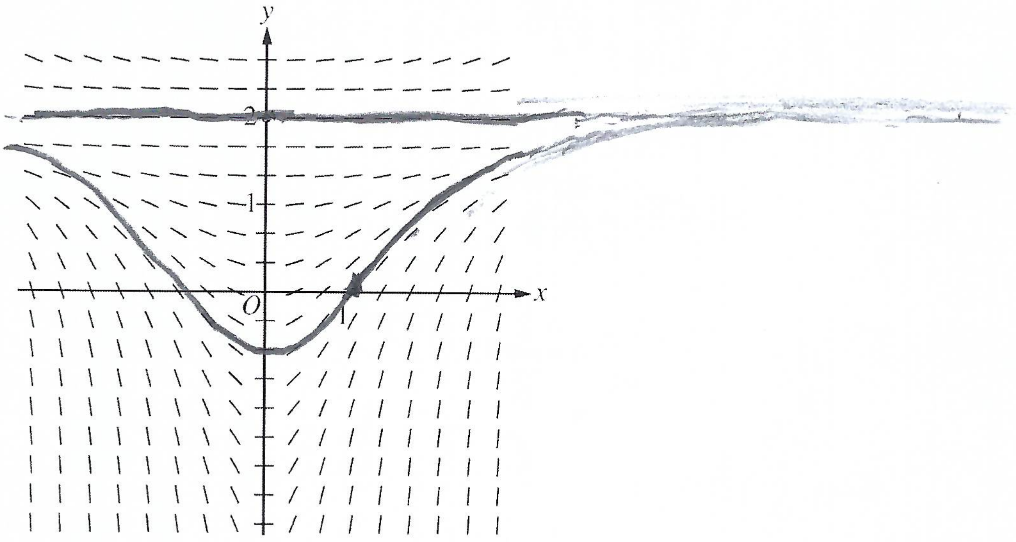
Homework 0429c
Wednesday, April 29, 2020

**Problem 1.** Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

If $y = 2$, $\frac{dy}{dx} = 0$
so – horizontal

(a) A slope field for the given differential equation is shown below.




Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial consider $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$f(1) = 0$

First get slope: plug $(1,0)$ into diff eq.

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \left.\frac{1}{3}x(y-2)^2\right|_{(1,0)}$$

$$= \frac{1}{3}(-2)^2 = \frac{4}{3}$$

$$L(x) = m(x - x_0) + y_0$$

$$= \frac{4}{3}(x-1) + 0$$

$$f(0.7) \approx L(0.7) = \frac{4}{3}(-0.3) = \boxed{-0.4}$$

**Problem 1** (continued). Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

**(c)** Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\int \frac{dy}{(y-2)^2} = \int x \frac{dx}{3}$$

$$-\frac{1}{y-2} = \frac{1}{6}x^2 + C \qquad \text{at } (1,0)$$

$$\frac{1}{2} = \frac{1}{6} + C$$

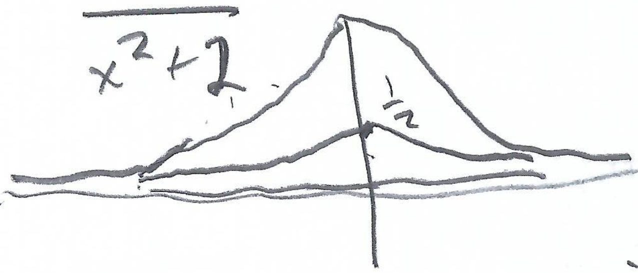
$$C = \frac{1}{2} - \frac{1}{6} = \frac{4}{12} = \frac{1}{3}$$

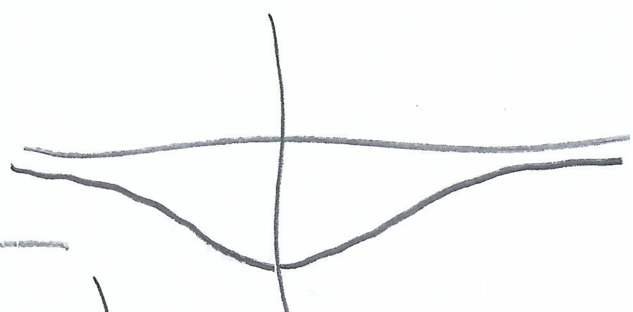
So $-\frac{1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2+2}{6}$

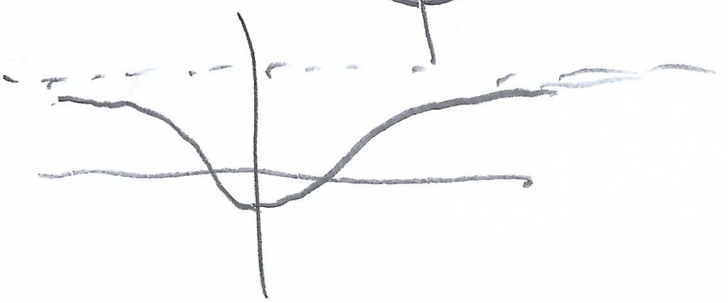
$$y-2 = -\frac{6}{x^2+2}$$

So $f(x) = 2 - \frac{6}{x^2+2}$

$\frac{1}{x^2+2}$

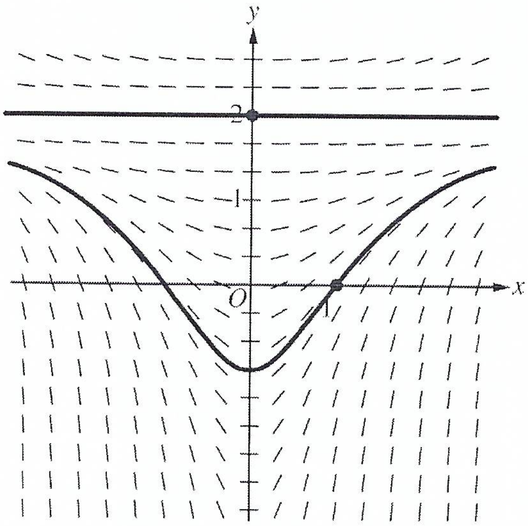




### Question 6

(a)




$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = \dfrac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x - 1)$.

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\dfrac{dy}{dx} = \dfrac{1}{3}x(y-2)^2$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2+2}{6}$$

$$y = 2 - \frac{6}{x^2+2}$$

Note: this solution is valid for $-\infty < x < \infty$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \quad\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: $0/5$ if no separation of variables

Note: max $3/5$ [1-2-0-0] if no constant of integration