AP Calculus AB — Homework 0504
Dr. Paul L. Bailey — Monday, May 4, 2020

Problem 1. Let $f : [0, 5] \to \mathbb{R}$ be given by $f(x) = x^3 - 3x + 1$.

(a) Sketch the graph of f as follows. Let $g(x) = x^3 - 3x + 2$. Factor it. Sketch it by first plotting its zeros. Now $f(x) = g(x) - 1$. Shift the graph of g down by one.

(b) What is the hypothesis of the Intermediate Value Theorem (IVT)? Does f satisfy it? Show that there exists $c \in (0, 5)$ such that $f(c) = 0$.

Problem 1 (continued)**.** Let $f : [0, 5] \to \mathbb{R}$ be given by $f(x) = x^3 - 3x + 1$.

(c) What is the hypothesis of the Extreme Value Theorem (EVT)? Does f satisfy it? Find $c_1, c_2 \in [0, 5]$ such that f has an absolute minimum at c_1 and an absolute maximum at c_2.

(d) What is the hypothesis of the Mean Value Theorem (MVT)? Does f satisfy it? What is the conclusion of MVT? Find $c \in (0, 5)$ such that f satisfies the conclusion of the MVT at $x = c$.

Problem 2. Let $f : [0, 5] \to \mathbb{R}$ be the piecewise defined function given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \in [0, 1) \ ; \\ 3 - (x - 2)^2 & \text{if } x \in [1, 5] \ . \end{cases}$$

For $x \in (0, 1) \cup (1, 5)$, the derivative of f is

$$f'(x) = \begin{cases} 2x & \text{if } x \in [0, 1) \ ; \\ -2(x - 2) & \text{if } x \in [1, 5] \ . \end{cases}$$

(a) Sketch the graph of f.

(b) Does f satisfy the hypothesis of IVT on $[0, 5]$? Show that there exists $c \in (0, 5)$ such that $f(c) = 0$.

Problem 2 (continued)**.** Let $f : [0,5] \to \mathbb{R}$ be the piecewise defined function given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \in [0,1)\ ; \\ 3 - (x-2)^2 & \text{if } x \in [1,5]\ . \end{cases}$$

For $x \in (0,1) \cup (1,5)$, the derivative of f is

$$f'(x) = \begin{cases} 2x & \text{if } x \in [0,1)\ ; \\ -2(x-2) & \text{if } x \in [1,5]\ . \end{cases}$$

(c) Does f satisfy the hypothesis of EVT on $[0,5]$? Find $c_1, c_2 \in [0,5]$ such that f has an absolute minimum at c_1 and an absolute maximum at c_2.

(d) Does f satisfy the hypothesis of MVT on $[0,5]$? If so, find $c \in (0,5)$ such that f satisfies the conclusion of the MVT at $x = c$.