AP CALCULUS AB	Homework 0505z
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I collected some of the old AP problems that are a bit more theoretical in nature. These are good practice, please ask if you get stuck and are curious.

**Problem 1.** Let f be the function that is given by  $f(x) = \frac{ax+b}{x^2-c}$  and that has the following properties.

- (i) The graph of f is symmetric with respect to the y-axis.
- (ii)  $\lim_{x \to 2^+} f(x) = +\infty$
- (iii) f'(1) = 2
- (a) Determine the values of a, b, and c.
- (b) Write an equation for each vertical and each horizontal asymptote of the graph of f.
- (c) Sketch the graph of f in the xy-plane provided below.



**Problem 2.** Let f be the function given by  $f(x) = \ln \left| \frac{x}{1+x^2} \right|$ .

- (a) Find the domain of f.
- (b) Determine whether f is an even function, an odd function, or neither. Justify your conclusion.
- (c) At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether f(x) is a relative maximum or a relative minimum.
- (d) Find the range of f.

**Problem 3.** Let f be the function given by  $f(x) = x^3 - 6x^2 + p$ , where p is an arbitrary constant.

- (a) Write an expression for f'(x) and use it to find the relative maximum and minimum values of f in terms of p. Show that analysis that leads to your conclusion.
- (b) For what values of p does f have 3 distinct roots?
- (c) Find the value of p such that the average value of f over the closed interval [-1, 2] is 1.

**Problem 4.** A tight rope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



- (a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)
- (b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)
- (c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point *B*? (Indicate units of measure.)

**Problem 5.** A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at x = -1, and the graph of f has a point of inflection at x = -2.

- (a) Find the values of a and b.
- (b) If  $\int_0^1 f(x) dx = 32$ , what is the value of k?

**Problem 6.** (BC) Let f be a function that is everywhere differentiable and that has the following properties.

- (i)  $f(x+h) = \frac{f(x) + f(h)}{f(-x) + f(-h)}$  for all real numbers h and x.
- (ii) f(x) > 0 for all real number x.
- (iii) f'(0) = -1.
- (a) Find the value of f(0).
- (b) Show that  $f(-x) = \frac{1}{f(x)}$  for all real numbers x.
- (c) Using part (b), show that f(x+h) = f(x)f(h) for all real numbers h and x.
- (d) Use the definition of the derivative to find f'(x) in terms of f(x).