AP CALCULUS AB
DR. PAUL L. BAILEY

Homework 0505a
Tuesday, May 5, 2020

Problem 1. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

We know it moves left when $v < 0$.

Let $\theta = \frac{\pi}{6}t$. So solve $\cos\theta = 0$:

then $\theta = \frac{\pi}{2} + k\pi$ where k is an integer.

So $\frac{\pi}{6}t = \frac{\pi}{2} + k\pi$

so $t = 3 + 6k$

in $[0,12]$, $t = 3$ or $t = 9$.

sign chart

$+$ on $(0,3)$, $-$ on $(3,9)$, $+$ on $(9,12)$; marks at 0, 3, 9, 12

$v < 0$ in interval $(3,9)$.

(b) Write, [redacted] an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

Integral of velocity is displacement.

$$\int_0^6 |v(t)|\,dt = \int_0^6 \left|\cos\tfrac{\pi}{6}t\right| dt$$

$$= \int_0^3 \cos\tfrac{\pi}{6}t\,dt + \int_3^6 \left(-\cos\tfrac{\pi}{6}t\right) dt$$

$$= \frac{6}{\pi}\sin\frac{\pi}{6}t\Big|_0^3 - \frac{6}{\pi}\sin\frac{\pi}{6}t\Big|_3^6$$

$$= \frac{6}{\pi} + \frac{6}{\pi} = \frac{12}{\pi}$$

If $F' = f$

then

$\int f(at)\,dt$

$= \frac{F(at)}{a} + C$

$u = at \Rightarrow \frac{1}{a}du = dt$

Problem 1 (continued). For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$$a(t) = -\sin\left(\frac{\pi}{6}t\right)$$

$$a(4) = -\sin\left(\frac{2}{3}\pi\right) < 0$$

$$v(4) = \cos\left(\frac{\pi}{6}\cdot 4\right) < 0$$

Since v and a have same sign, speed is increasing.

(d) Find the position of the particle at time $t = 4$.

$$x(t) = x(0) + \int_0^t \cos\left(\frac{\pi}{6}x\right) dx$$

$$= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}x\right)\right]_0^t$$

$$= -2 + \frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)$$

$$x(4) = -2 + \frac{6}{\pi}\sin\left(\frac{2}{3}\pi\right)$$

$$= -2 + \frac{6}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{3\sqrt{3}}{\pi} - 2}$$

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Homework 0505b
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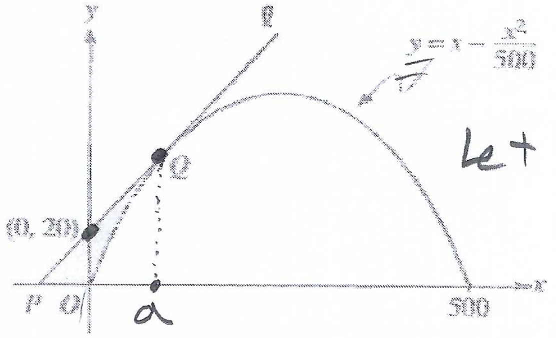
Problem 1. The line ℓ is tangent to the graph of $y = x - \dfrac{x^2}{500}$ at the point Q, as shown in the figure below.



Let $f(x) = x - \dfrac{x^2}{500}$.

(a) Find the x-coordinate of the point Q.

Let $Q = (a, f(a)) = \left(a, a - \dfrac{a^2}{500}\right)$

We know slope $\ell = \dfrac{f(a) - 20}{a - 0} = \dfrac{a - \frac{a^2}{500} - 20}{a} = 1 - \dfrac{a}{500} - \dfrac{20}{a}$

Also, $f'(x) = 1 - \dfrac{x}{250}$

So $f'(a) = 1 - \dfrac{a}{250} = 1 - \dfrac{a}{500} - \dfrac{20}{a}$.

$$\frac{a}{250} = \frac{a}{500} + \frac{20}{a}$$

$$2a^2 = a^2 + 10000$$

So $a^2 = 10000$. Thus $a = 100$.

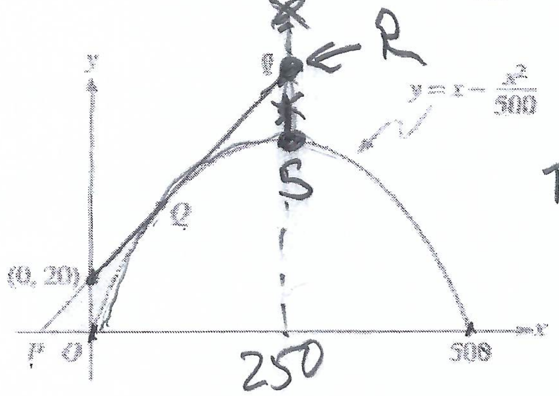
(b) Write an equation for the line ℓ.

Slope $\ell = f'(a) = 1 - \dfrac{a}{250} = 1 - \dfrac{100}{250} = \dfrac{3}{5}$.

Point is $(0, 20)$

So $\ell(x) = \dfrac{3}{5}x + 20$.

Problem 1 (continued). The line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure below.



Tree is 50 ft tall.

(c) Suppose that graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along the line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

Find R.

$$f(x) = x - \frac{x^2}{500}$$

$$\ell(x) = \frac{3}{5}x + 20$$

Note $f(250) = 250 - \frac{250^2}{500}$

$= 250 - \frac{250}{2} = \frac{250}{2} = 125$

and $\ell(250) = \frac{3}{5}(250) + 20$

$= 170$

The top of the tree is at $f(250) + 50 = 175$.

This is higher than the light,

so, the light hits the tree.