AP CALCULUS AB
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Homework 0506c
Wednesday, May 6, 2020

Problem 1. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

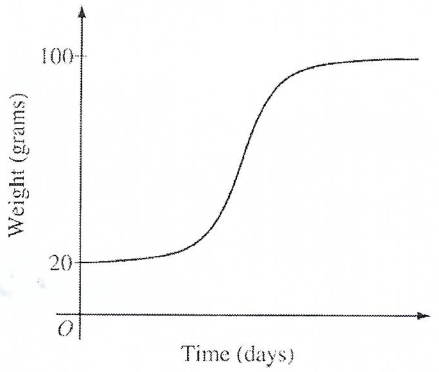
(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

$$\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = \frac{60}{5} = 12 \frac{g}{day}$$

$$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = \frac{30}{5} = 6 \frac{g}{day}$$

The bird is gaining weight faster when it weighs 40g.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



$$\frac{d^2B}{dt^2} = \frac{d}{dt}\frac{dB}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{5}(100-B)\right)$$

$$= \frac{1}{5}\left(-\frac{dB}{dt}\right)$$

$$= \frac{1}{5}\left(-\frac{1}{5}(100-B)\right)$$

$$= -\frac{1}{25}(100-B)$$

So for $B < 100$, the graph of B is concave down.

Since $B < 100$ all t, B is concave down all t, unlike the graph in the picture.

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$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\int \frac{dB}{100-B} = \int \frac{1}{5}\,dt$$

$$-\ln(100-B) = \frac{1}{5}t + C$$

$B(0)=20 \Rightarrow$ $$-\ln(80) = \frac{1}{5}\cdot 0 + C \Rightarrow C = -\ln(80)$$

So $$-\ln(100-B) = \frac{1}{5}t - \ln(80)$$

$$\ln(100-B) = \ln(80) - \frac{1}{5}t$$

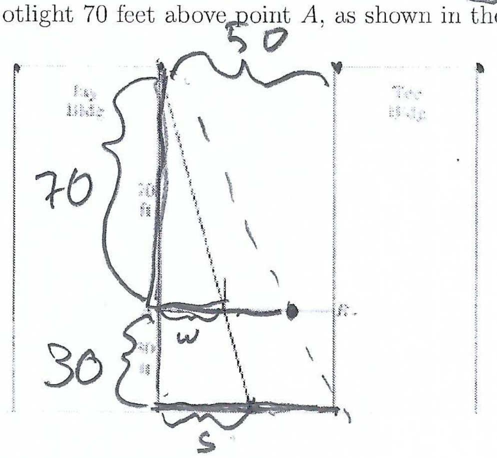
So $$100 - B = e^{\ln(80) - \frac{1}{5}t}$$

$$= e^{\ln(80)}\, e^{-\frac{1}{5}t}$$

$$100 - B = 80\, e^{-\frac{1}{5}t}$$

so $$B = 100 - 80\, e^{-\frac{1}{5}t}$$

Problem 1. A tight rope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker,walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



(a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

Let w = walker's position and s = shadow's position,

Know $\frac{dw}{dt} = 2 \frac{ft}{sec}$. Choose $\frac{ds}{dt}$. We use similar triangles.

$$\frac{w}{70} = \frac{s}{100}$$

$$\frac{1}{70}\frac{dw}{dt} = \frac{1}{100}\frac{ds}{dt}$$

So $\frac{1}{35} = \frac{1}{100}\frac{ds}{dt}$

Thus,

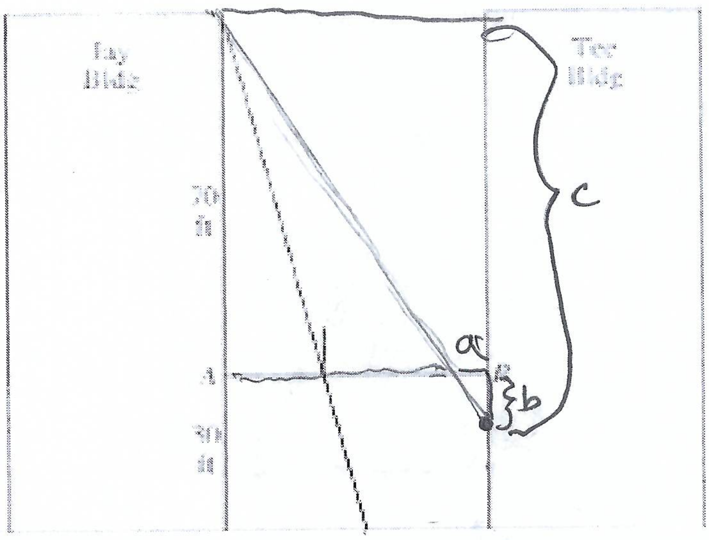
$$\frac{ds}{dt} = \frac{100}{35} = \boxed{\frac{20}{7}\frac{ft}{sec}}$$

(b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

Know $\frac{w}{70} = \frac{s}{100}$

So $w = 70 * \frac{50}{100} = \boxed{35 \text{ ft}}$

Problem 1. A tight rope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



(c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units of measure.)

Let $a = 50 - w$ $\quad \frac{da}{dt} = -\frac{dw}{dt} = -2$

$b = 30 - y$ $\quad \frac{db}{dt} = \frac{dc}{dt} = -\frac{dy}{dt}$

$c = 100 - y$

By similar Δ's,

$$\frac{b}{a} = \frac{c}{50}$$

Note $b = c +$

So $50b = ac$

$$50\frac{db}{dt} = \frac{da}{dt}c + a\frac{dc}{dt}$$

$$-50\frac{dy}{dt} = -2(100-y) - (50-w)\frac{dy}{dt}$$

$$-w\frac{dy}{dt} = 2y - 200$$

$$\frac{dy}{dt} = \frac{200-2y}{w} = \frac{200}{40} - \frac{25}{40}$$

$$= \frac{175}{40} = \frac{35}{8}\ \frac{ft}{sec}$$

when $a = 40$ what is y?

$$\frac{b}{10} = \frac{b+70}{50}$$

$50b = 10b + 700$

$4b = 70$

$b = \frac{70}{4} = \frac{35}{2}$

$y = 30 - \frac{35}{2}$

$= \frac{25}{2}$

$w = 40$