IVT, EVT, MVT

Intermediate Value Theorem (Short Form)

Let f be continuous on $[a,b]$. } Hypothesis.
Suppose $f(a)f(b) < 0$.

Then there exists $c \in (a,b)$ such that $f(c) = 0$.

Extreme Value Theorem

Let f be continuous on $[a,b]$.

Then there exist $c_1, c_2 \in [a,b]$ such that
f has an absolute minimum at c_1 and
f has an absolute maximum at c_2

Addendum: the absolutes extrema occur at a critical point or at an endpoint

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Mean Value Theorem

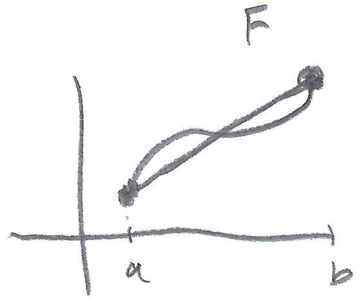
Let f be continuous on $[a,b]$ and differentiable on (a,b).
Then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Cor 1 If $f'(x) = 0$ all $x \in [a,b]$, then $f(x) = C$.

Cor 2 If $f'(x) = g'(x)$ all $x \in [a,b]$, then $f(x) = g(x) + C$.

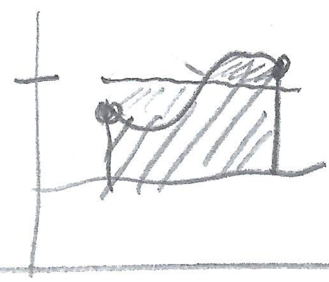
Average Rate of Change of F on $[a,b]$ is



Difference Quotient $\dfrac{F(b)-F(a)}{b-a}$

Average Value of f on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$



If $F'(x) = f(x)$ on $[a,b]$,

then

Average value of F' = Average Rate of Change of F

$$\frac{1}{b-a}\int_a^b F'(x)\,dx = \frac{F(b)-F(a)}{b-a}.$$