AP CALCULUS AB
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Mock Salad 0508
Friday, May 8, 2020

Problem 1. A particle moves along the x-axis. The velocity of the particle is modeled by a strictly decreasing, twice differentiable function $v(t)$ measured in meters per second. Select values of $v(t)$ at specific times t, measured in seconds, are given below. It is known at time $t = 7$, the particle's position is 3 units to the right of the origin.

t (sec)	2	3	5	7	9
$v(t)$ (m/sec)	3	1	0	-6	-8

(a) Estimate $v'(2.5)$ and $v'(6)$. Interpret the meanings in context including units.

$$v'(2.5) \approx \frac{v(3) - v(2)}{3-2} = \frac{1-3}{1} = -2 \frac{m}{s^2}$$

$$v'(6) \approx \frac{v(7) - v(5)}{7-5} = \frac{-6-0}{2} = -3 \frac{m}{s^2}$$

(b) State whether the particle is speeding up or slowing down at both $t = 2.5$ and $t = 6$.

Since velocity is decreasing, acceleration is negative. At $t=2.5$, $v>0$, so a and v have different signs, so speed is decreasing.
But at $t=6$, $v<0$, so they have the same sign, so speed is increasing.

(c) The particle's position is modeled by the function $P(t)$. Write an equation of the tangent line to the graph of P at $t = 7$. use the tangent line to approximate $P(8)$.

We know $P(7) = 3$
The velocity gives the slope of the tangent:
$v(7) = -6$. So line is

$$\boxed{L(t) = -6(t-7) + 3.}$$

$L(7) = -3.$

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(d) Is the estimate in part (c) an under approximation or over approximation of $P(8)$? Explain how you know.

$L(t) = -6(t-7) + 3$

$y = mx + b$

$y = m(x - x_0) + y_0$

$L(8) = -6(1) + 3$

$= -3$

Since $v = P'$ is decreasing, P'' is negative, so P is concave down, so $L(8)$ is an overestimate.

(e) Claire, a calculus student, uses a left Riemann sum of three subintervals to approximate $\int_2^7 v(t)\,dt$. Is her approximation an overestimate or underestimate of the actual value? Explain how you know.

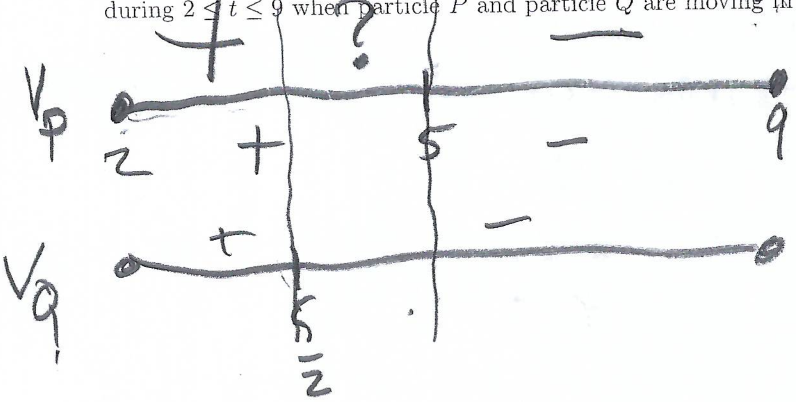
This is an overestimate because we have a left Riemann sum of a decreasing function.

$\boxed{Q(t) = 4 + 5t - t^2}$

(f) Another particle Q is also moving along the x-axis. Let $Q(x) = 4 + 5x - x^2$. State open interval(s) during $2 \le t \le 9$ when particle P and particle Q are moving in the same direction.

$Q'(t) = 5 - 2x = 0$

$\Rightarrow t = \frac{5}{2}$



So V_p and V_q have same sign on $(2, \frac{5}{2}) \cup (5, 9)$