CHAPTER 3

Linear Systems

3A Linear Systems in Two Dimensions

3-1 Using Graphs and Tables to Solve Linear Systems

3-2 Using Algebraic Methods to Solve Linear Systems

3-3 Solving Systems of Linear Inequalities

3-4 Linear Programming

MULTI-STEP TEST PREP

3B Linear Systems in Three Dimensions

3-5 Linear Equations in Three Dimensions

3-6 Solving Linear Systems in Three Variables

Lab Explore Parametric Equations

Ext Parametric Equations

MULTI-STEP TEST PREP

Whooping It Up!

You can use linear systems to plan a fund-raiser in which calendars featuring endangered whooping cranes are sold.

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Chapter Project Online
KEYWORD: MB7 ChProj

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. equation
2. inequality
3. solution set
4. slope

A. an equation whose solutions form a line on a coordinate plane
B. steepness of a line given as a ratio of rise over run
C. a mathematical statement using $>$, $<$, $\geq$, or $\leq$
D. a mathematical statement that says two expressions are equal
E. the set of values that make a statement true

Least Common Multiple

Find the least common multiple, or LCM, for each pair of numbers.

5. 3, 18 **6.** 28, 8 **7.** 8, 36 **8.** 15, 27

Slopes of Parallel and Perpendicular Lines

State whether the linear equations in each pair are parallel, perpendicular, or neither.

9. $\begin{cases} y = 5x - 4 \\ y = -\frac{1}{5}x - 4 \end{cases}$ **10.** $\begin{cases} 5x - 10y = 3 \\ y = \frac{1}{2}x - 6 \end{cases}$ **11.** $\begin{cases} x - y = 3 \\ x + y = -4 \end{cases}$ **12.** $\begin{cases} 2x - 3y = -4 \\ 3y - x = 5 \end{cases}$

Evaluate Expressions

Evaluate each expression for the given values of the variables.

13. $1.5x + 3y$ for $x = 8$, $y = 14$
14. $5x - \frac{3}{4}y$ for $x = 6$, $y = -4$
15. $4x - \sqrt{2}y$ for $x = 0.25$, $y = \sqrt{2}$
16. $-\frac{75x}{3y}$ for $x = 1$, $y = \frac{1}{3}$

Solve Multi-Step Equations

Solve each equation.

17. $8x + 19 = -5$
18. $5x + 4 = 25 - 2x$
19. $9x - (x + 12) = -13$
20. $-3(4x - 5) - 1 = 20$

Solve Equations with Fractions

Solve each equation.

21. $\frac{1}{4}x + \frac{2}{3}x = 8$
22. $\frac{2}{5}x + \frac{1}{6} = -4$
23. $x + \frac{1}{2} = -\frac{1}{5}$
24. $-\frac{1}{2} = 3x - \frac{1}{3}x$

CHAPTER 3

Study Guide: Preview

Where You've Been

Previously, you

- graphed linear equations.
- graphed linear inequalities.
- solved linear equations.
- studied three-dimensional figures such as cubes and prisms.

In This Chapter

You will study

- graphing systems of linear equations.
- graphing systems of linear inequalities.
- solving systems of linear equations.
- the three-dimensional coordinate system.

Where You're Going

You can use the skills in this chapter

- to solve more complicated systems of equations.
- to understand linear systems in other classes, such as Chemistry, Physics, and Economics.
- outside of school to organize fund-raisers, plan a trip, or spend money wisely.

Key Vocabulary/Vocabulario

constraint	restricción
elimination	eliminación
feasible region	región factible
linear programming	programación lineal
linear system	sistema lineal
substitution	sustitución
system of equations	sistema de ecuaciones
system of linear inequalities	sistema de desigualdades lineales
three-dimensional coordinate system	sistema de coordenadas tridimensional

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What does the word **eliminate** mean? What might the *elimination* method refer to when solving mathematical equations?
2. **Constraint** refers to a restriction or limitation. What might a mathematical *constraint* refer to?
3. The word **feasible** means "capable of being done or used." Give examples of sentences that use the word *feasible*. Then discuss what a *feasible region* might refer to.
4. What can you say about a **three-dimensional coordinate system** from its name? If x and y are used for the first two, which letter would be a logical choice for the third coordinate?

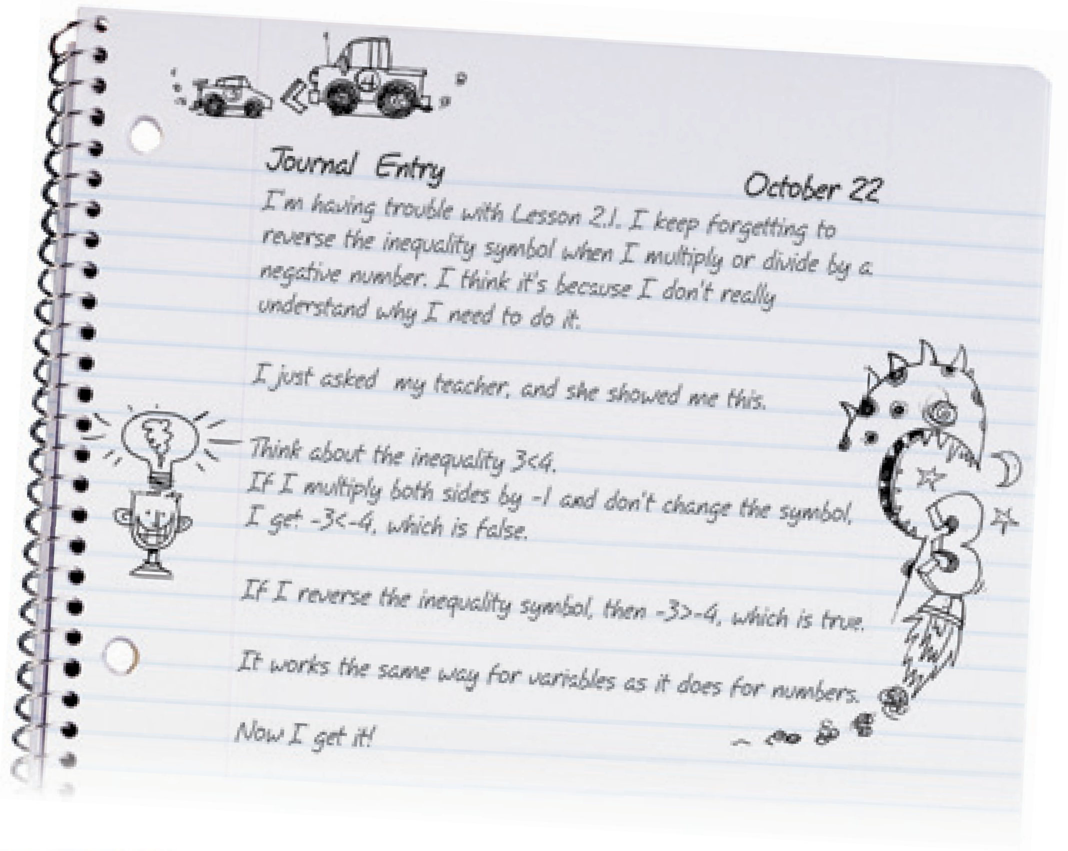
Reading and Writing Math

Writing Strategy: Keep a Math Journal

Keeping a math journal will help you improve your writing and reasoning skills. By expressing yourself in a journal, you can make sense of confusing or frustrating math situations.

You can use your journal to reflect on what you learned in class, write out any troubles you are having, summarize important concepts and vocabulary, or express your thoughts about a particular topic. Most importantly, though, a math journal helps you see your progress as you continue through Algebra 2.

Journal Entry: Read the entry a student made in his journal.



Try This

Begin a math journal. Use these ideas to begin your journal entry each day for this week. Be sure to date and number each page.

- What I already know about this lesson is . . .
- What I am unsure about in this lesson is . . .
- The skills I need in order to finish this lesson are . . .
- What trouble spots did I find? How did I handle these difficulties?
- What I enjoyed/did not enjoy about this lesson was . . .

3-1 Using Graphs and Tables to Solve Linear Systems

Objectives

Solve systems of equations by using graphs and tables.

Classify systems of equations, and determine the number of solutions.

Vocabulary

system of equations
linear system
consistent system
inconsistent system
independent system
dependent system

Who uses this?

Winter sports enthusiasts can use systems of equations to compare the costs of renting snowboards. (See Example 4.)

A **system of equations** is a set of two or more equations containing two or more variables. A **linear system** is a system of equations containing only linear equations.

Recall that a line is an infinite set of points that are solutions to a linear equation. The solution of a system of equations is the set of all points that satisfy each equation.

On the graph of the system of two equations, the solution is the set of points where the lines intersect. A point is a solution to a system of equations if the x- and y-values of the point satisfy both equations.

EXAMPLE 1 Verifying Solutions of Linear Systems

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

A $(2, 4); \begin{cases} x - 2y = -6 \\ 2x + y = 8 \end{cases}$

$$\begin{array}{r|l} \multicolumn{2}{c}{x - 2y = -6} \\ \hline (2) - 2(4) & -6 \\ -6 & -6\ ✔ \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{2x + y = 8} \\ \hline 2(2) + (4) & 8 \\ 8 & 8\ ✔ \end{array}$$

Substitute 2 for x and 4 for y in each equation.

Because the point is a solution of both equations, it is a solution of the system.

B $(3, 2); \begin{cases} 2x + 3y = 12 \\ 8x - 6y = 24 \end{cases}$

$$\begin{array}{r|l} \multicolumn{2}{c}{2x + 3y = 12} \\ \hline 2(3) + 3(2) & 12 \\ 12 & 12\ ✔ \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{8x - 6y = 24} \\ \hline 8(3) + 6(2) & 24 \\ 36 & 24\ ✘ \end{array}$$

Substitute 3 for x and 2 for y in each equation.

Because the point is not a solution of both equations, it is not a solution of the system.

CHECK IT OUT! **Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.**

1a. $(4, 3); \begin{cases} x + 2y = 10 \\ 3x - y = 9 \end{cases}$ **1b.** $(5, 3); \begin{cases} 6x - 7y = 1 \\ 3x + 7y = 5 \end{cases}$

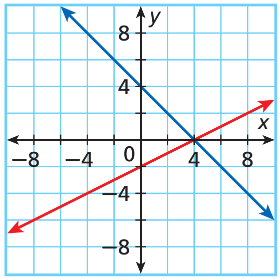
Recall that you can use graphs or tables to find some of the solutions to a linear equation. You can do the same to find solutions to linear systems.

EXAMPLE 2 Solving Linear Systems by Using Graphs and Tables

Use a graph and a table to solve each system. Check your answer.

A $\begin{cases} x + y = 4 \\ 2y + 4 = x \end{cases}$

Solve each equation for y. $\begin{cases} y = -x + 4 \\ y = \frac{1}{2}x - 2 \end{cases}$



On the graph, the lines appear to intersect at the ordered pair $(4, 0)$.

Make a table of values for each equation. Notice that when $x = 4$, the y-value for both equations is 0.

$y = -x + 4$

x	y
1	3
2	2
3	1
4	0

$y = \frac{1}{2}x - 2$

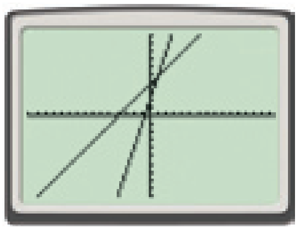
x	y
1	$-\frac{3}{2}$
2	-1
3	$-\frac{1}{2}$
4	0

The solution to the system is $(4, 0)$.

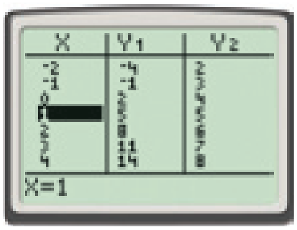
B $\begin{cases} 3x - y = -2 \\ x - y = -4 \end{cases}$

Solve each equation for y. $\begin{cases} y = 3x + 2 \\ y = x + 4 \end{cases}$

Use your graphing calculator to graph the equations and make a table of values. The lines appear to intersect at $(1, 5)$. This is confirmed by the table of values.
The solution to the system is $(1, 5)$.



Check Substitute $(1, 5)$ in the original equations to verify the solution.



$$\begin{array}{r|l} 3x - y = & -2 \\ \hline 3(1) - (5) & -2 \\ -2 & -2 \checkmark \end{array} \qquad \begin{array}{r|l} x - y = & -4 \\ \hline (1) - (5) & -4 \\ -4 & -4 \checkmark \end{array}$$

Helpful Hint

To enter the equations into your calculator, let Y_1 represent $y = 3x + 2$ and let Y_2 represent $y = x + 4$.

CHECK IT OUT! **Use a graph and a table to solve each system. Check your answer.**

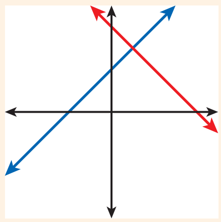
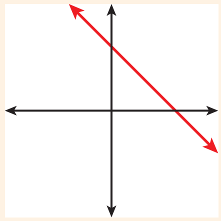
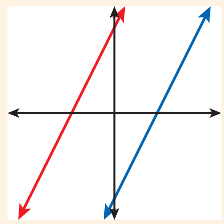
2a. $\begin{cases} 2y + 6 = x \\ 4x = 3 + y \end{cases}$ **2b.** $\begin{cases} x + y = 8 \\ 2x - y = 4 \end{cases}$ **2c.** $\begin{cases} y - x = 5 \\ 3x + y = 1 \end{cases}$

The systems of equations in Example 2 have exactly one solution. However, linear systems may also have infinitely many or no solutions. A **consistent system** is a set of equations or inequalities that has at least one solution, and an **inconsistent system** will have no solutions.

You can classify linear systems by comparing the slopes and y-intercepts of the equations. An **independent system** has equations with different slopes. A **dependent system** has equations with equal slopes and equal y-intercepts.

Know it! Note

Classifying Linear Systems

EXACTLY ONE SOLUTION	INFINITELY MANY SOLUTIONS	NO SOLUTION
		
Consistent, independent The graphs are intersecting lines with different slopes.	Consistent, dependent The graphs are coinciding lines; they have the same slope and same y-intercept.	Inconsistent The graphs are parallel lines; they have the same slope but different y-intercepts.

EXAMPLE 3

Classifying Linear Systems

Classify each system and determine the number of solutions.

A $\begin{cases} 2x + y = 3 \\ 6x = 9 - 3y \end{cases}$

Solve each equation for y. $\begin{cases} y = -2x + 3 \\ y = -2x + 3 \end{cases}$ *The equations have the same slope and y-intercept and are graphed as the same line.*

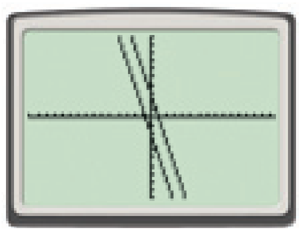
The system is dependent with infinitely many solutions.

B $\begin{cases} 3x + y = 3 \\ 2 + y = -3x \end{cases}$

Solve each equation for y. $\begin{cases} y = -3x + 3 \\ y = -3x - 2 \end{cases}$ *The equations have the same slope but different y-intercepts and are graphed as parallel lines.*

The system is inconsistent and has no solution.

Check A graph shows parallel lines.



Remember!

The slope-intercept form of a linear equation makes comparing slopes and y-intercepts easy.

$y = mx + b$

Slope $\quad$ y-intercept

CHECK IT OUT!

Classify each system and determine the number of solutions.

3a. $\begin{cases} 7x - y = -11 \\ 3y = 21x + 33 \end{cases}$ **3b.** $\begin{cases} x + 4 = y \\ 5y = 5x + 35 \end{cases}$

EXAMPLE 4 *Winter Sports Application*

Big Dog Snowboard Co. charges $15 for equipment rental plus $35 per hour for snowboarding lessons. Half-Pipe Snowboards, Inc. charges $40 for equipment rental plus $25 per hour for lessons. For what number of hours is the cost of equipment and lessons the same for each company?

Step 1 Write an equation for the cost of equipment rental and lessons at each company.

Let x represent the number of hours and y represent the total cost in dollars.

Big Dog Snowboard Co.: $y = 35x + 15$

Half-Pipe Snowboards, Inc.: $y = 25x + 40$

Because the slopes are different, the system is independent and has exactly one solution.

Step 2 Solve the system by using a table of values.

Use increments of $\frac{1}{2}$ to represent 30 min.

When $x = 2\frac{1}{2}$, the y-values are both 102.5. The cost of equipment rental and a $2\frac{1}{2}$-hour snowboard lesson is $102.50 at either company. So the cost is the same at each company for $2\frac{1}{2}$ hours.

$y = 35x + 15$

x	y
1	50
$1\frac{1}{2}$	67.5
2	85
$2\frac{1}{2}$	102.5
3	120

$y = 25x + 40$

x	y
1	65
$1\frac{1}{2}$	77.5
2	90
$2\frac{1}{2}$	102.5
3	115

CHECK IT OUT!

4. Ravi is comparing the costs of long distance calling cards. To use card A, it costs $0.50 to connect and then $0.05 per minute. To use card B, it costs $0.20 to connect and then $0.08 per minute. For what number of minutes does it cost the same amount to use each card for a single call?

THINK AND DISCUSS

1. Explain how to find the number of solutions of a system of equations using only a graph.

2. Explain why a system of equations whose graphs are distinct parallel lines has no solution.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give information about or examples of each solution type.

	Exactly One Solution	Infinitely Many Solutions	No Solution
Example			
Graph			
Slopes			
y-intercepts			

3-1 Exercises

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Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. Vocabulary A system of equations with no solution is __?__. (*consistent* or *inconsistent*)

SEE EXAMPLE 1
p. 182

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

2. $(3, 3)$ $\begin{cases} 2x - y = 3 \\ y + x = 6 \end{cases}$

3. $(1, -3)$ $\begin{cases} y - 4x = -7 \\ 5x + y = -6 \end{cases}$

4. $(-2, 2)$ $\begin{cases} 5y - 5x = 10 \\ 3x + 10 = 2y \end{cases}$

5. $(-1, 4)$ $\begin{cases} y = 3 - x \\ 6x + 2y = 2 \end{cases}$

SEE EXAMPLE 2
p. 183

Use a graph and a table to solve each system. Check your answer.

6. $\begin{cases} y + x = 5 \\ 3x - 5y = -1 \end{cases}$

7. $\begin{cases} 3y + 6x = 3 \\ x - y = -7 \end{cases}$

8. $\begin{cases} y - x = 0 \\ 8x + 4y = -24 \end{cases}$

9. $\begin{cases} x - y = -1 \\ 4x - 2y = 2 \end{cases}$

SEE EXAMPLE 3
p. 184

Classify each system and determine the number of solutions.

10. $\begin{cases} 7x + y = 13 \\ 28x + 4y = -12 \end{cases}$

11. $\begin{cases} 2x - 3y = -15 \\ 3y - 2x = 15 \end{cases}$

12. $\begin{cases} 8y - 24x = 64 \\ 9y + 45x = 72 \end{cases}$

13. $\begin{cases} 2x + 2y = -10 \\ 4x + 4y = -16 \end{cases}$

SEE EXAMPLE 4
p. 185

14. Aquariums Marco is draining his two aquariums. The tanks are the same size. One tank has 7 in. of water in it. It is being drained at a rate of 1 in./min. The other tank has 5 in. of water in it. It is being drained at a rate of 0.5 in./min. After how many minutes will the tanks contain the same amount of water?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–18	1
19–22	2
23–26	3
27	4

Extra Practice

Skills Practice p. S8
Application Practice p. S34

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

15. $(-2, 2)$ $\begin{cases} x + y = 0 \\ 7y - 14x = 42 \end{cases}$

16. $(-3, -5)$ $\begin{cases} 2y - 6x = 8 \\ 4y = 8x + 4 \end{cases}$

17. $(3, 2)$ $\begin{cases} y = 2 \\ y + 8 = 6x \end{cases}$

18. $(6, 1)$ $\begin{cases} y = 8x + 2 \\ x - 3y = 3 \end{cases}$

Use a graph and a table to solve each system. Check your answer.

19. $\begin{cases} 2 + y = x \\ x + y = 4 \end{cases}$

20. $\begin{cases} 4y - 2x = 4 \\ 10x - 5y = 10 \end{cases}$

21. $\begin{cases} 12x + 4y = -4 \\ 2x - y = 6 \end{cases}$

22. $\begin{cases} y = 10 - x \\ 3x - 3y = 0 \end{cases}$

Classify each system and determine the number of solutions.

23. $\begin{cases} 24x - 27y = 42 \\ -9y + 8x = 14 \end{cases}$

24. $\begin{cases} \frac{3}{2}x + 9 = y \\ 4y - 6x = 36 \end{cases}$

25. $\begin{cases} 7y + 42x = 56 \\ 25x - 5y = 100 \end{cases}$

26. $\begin{cases} 3y = 2x \\ -4x + 6y = 3 \end{cases}$

27. Business Jamail and Wanda sell home theater systems. Jamail earns a base salary of $2400 per month, plus $100 for each system he sells. Wanda earns a base salary of $2200 per month, plus $120 for each system she sells. How many systems do Jamail and Wanda have to sell before they earn the same amount of money?

Determine if the given ordered pair is a solution of the system of equations. If it is not, give the correct solution.

28. $(4, 2)$ $\begin{cases} y - x = 2 \\ 2x + y = 8 \end{cases}$

29. $(-1, 2)$ $\begin{cases} 3x + y = -1 \\ 8x + 6 = -y \end{cases}$

30. $(7, 2)$ $\begin{cases} x + y = 9 \\ 4y + 4 = x \end{cases}$

31. $(0, 6)$ $\begin{cases} 3x + 4y = -9 \\ y = 2x + 6 \end{cases}$

32. Bicycling Roberto is competing in a bicycle race. He has traveled 12 mi and is maintaining a steady speed of 15 mi/h. Alexandra is competing in the same race but got a flat tire. She has traveled 8 mi and is maintaining a steady speed of 18 mi/h.

a. Write and graph a system of equations that could be used to model the situation.

b. How long will it take Alexandra to catch Roberto?

c. How many miles into the race will they be when they meet?

LINK

Aviation

When in the air, pilots must follow certain flight-path restrictions to avoid collisions. The direction the airplane travels determines its required altitude. For example, airplanes traveling west fly at even-numbered thousands of feet.

33. Aviation Lynn is piloting a plane at an altitude of 10,000 feet. She begins to descend at a rate of 200 feet per minute. Miguel is flying a different plane at an altitude of 5000 feet. At the same time that Lynn begins to descend, Miguel begins to climb at a rate of 50 feet per minute.

a. Write and graph a system of equations that could be used to model the situation.

b. In how many minutes will the planes be at the same altitude?

c. What will that altitude be?

34. Multi-Step Juan is comparing the cell phone plans shown in the advertisement.

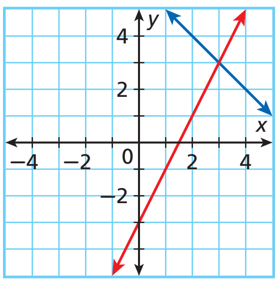
a. Write and graph a system of equations that could be used to model the situation.

b. For what number of minutes of use do the plans cost the same?

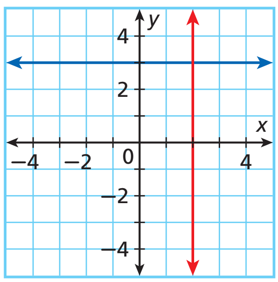
c. If Juan expects to use the phone for about 2 hours a month, which plan should he choose? Explain.

Write equations for each system graphed below. Then classify the system and find the solution set.

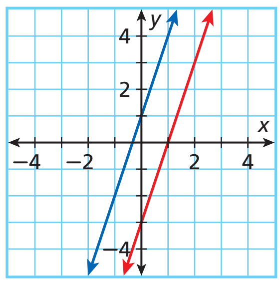
35.



36.



37.



Graphing Calculator **The graphing calculator can find points of intersection. Use the intersect feature of a graphing calculator by pressing** 2nd CALC TRACE**, then select 5: Intersect to find the solution to each system of equations. Round each solution to the nearest thousandth.**

38. $\begin{cases} y = -7x + 11 \\ y = 5x - 13 \end{cases}$

39. $\begin{cases} y = 4x + 5 \\ y = 12x + 7 \end{cases}$

40. $\begin{cases} 43 + y = 27x \\ 18x - y = -15 \end{cases}$

41. $\begin{cases} 32x = 121 + y \\ 45x + y = 97 \end{cases}$

MULTI-STEP TEST PREP

42. This problem will prepare you for the Multi-Step Test Prep on page 212.

The fuel tank of a compact truck holds 17 gallons. On a full tank, the truck can travel about 408 miles in the city or 476 miles on the highway. The fuel tank of a compact car holds 14 gallons. On a full tank, the car can travel about 364 miles in the city or 490 miles on the highway.

a. Compare the fuel use of the car and the truck in miles per gallon.

b. How many hours of highway driving are required for each to empty its tank? Assume an average speed of 60 miles per hour.

c. If the car travels at a constant 60 miles per hour on the highway, at what speed must the truck travel if it is to empty its tank at the same time as the car?

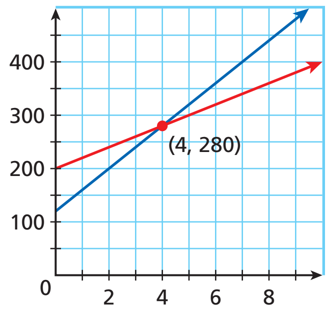
43. **Critical Thinking** One equation in a linear system is $x + 2y = 4$. What is an equation that would cause the system to have an infinite number of solutions? no solutions? one solution?

44. **Estimation** Use a graph to estimate the solution of the following system.

$$\begin{cases} y = 1.25x - 4 \\ y = -1.4x + 5 \end{cases}$$

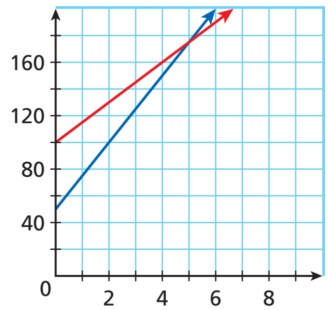
45. **Critical Thinking** How would you classify a system of equations that is composed of two lines with different slopes and the same y-intercepts? What is the solution to the system?

46. **Write About It** Describe a situation involving two hot-air balloons that could be modeled by the graph shown.



TEST PREP

47. Which of the situations below best matches the graph of the system of equations shown?



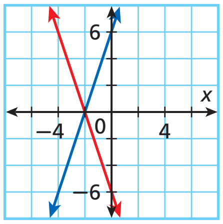
Ⓐ Kameko paid a \$25 sign-up fee plus \$50 per month for a health club membership. Maria paid a \$15 sign-up fee plus \$100 per month for her health club membership. At 5 months, they have paid the same amount for their memberships.

Ⓑ Ruby and Yuri use different companies to host their Web sites. Although the prices they pay are different, if each has 5 gigabytes of traffic per month, Ruby and Yuri pay the same amount, \$200.

Ⓒ James and Renee paid \$100 plus \$15 per day to rent a small sailboat. They kept the boat for 5 days and paid \$175.

Ⓓ Esteban and Ted belong to different country clubs, and they pay different monthly dues and different amounts to play golf. However, they've found that if they play 5 rounds of golf in a month, they each pay the same amount, \$175, in dues and golf fees.

48. Which system of equations is accurately represented on the graph?

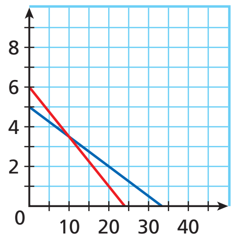


Ⓕ $\begin{cases} y = 3x - 6 \\ y = -3x + 6 \end{cases}$ Ⓗ $\begin{cases} y = 6x - 3 \\ y = -6x + 3 \end{cases}$

Ⓖ $\begin{cases} y = 3x + 6 \\ y = -3x - 6 \end{cases}$ Ⓙ $\begin{cases} y = 6x + 3 \\ y = 6x - 3 \end{cases}$

49. Adam's truck has 5 gal of fuel left in the tank and uses 0.1 gal/mi. Zoe's truck has 6 gal of fuel left in the tank and uses 0.15 gal/mi. Which of the following graphs best represents the system of equations that can be used to find the distance driven at which the trucks will have the same amount of fuel?

Ⓐ



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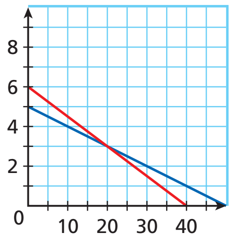

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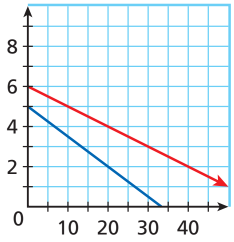

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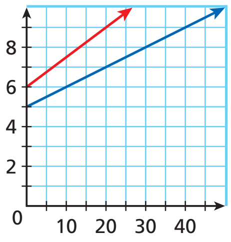


50. **Gridded Response** What is 3 times the *y*-coordinate of the solution to the following system? $\begin{cases} -x + y = 8 \\ y = 4x \end{cases}$

CHALLENGE AND EXTEND

Solve each system. Round your answer to the nearest thousandth.

51. $\begin{cases} y = 55x + 100 \\ y = 20x + 600 \end{cases}$ **52.** $\begin{cases} 20x + 5y = 135 \\ y = -50x + 32 \end{cases}$ **53.** $\begin{cases} 9x + 18y = 126 \\ 14y = -7x + 98 \end{cases}$ **54.** $\begin{cases} 0.25x - y = 2.25 \\ y = 0.75x + 3.75 \end{cases}$

55. **Business** An economist is studying a system of linear equations representing cost functions for two different products over time. She finds that the solution to the system is $(-6, -200)$. What does the solution tell her about the cost functions?

56. **Multi-Step** Brad is a farmer. He feeds his hogs 12 pounds of feed per day and has 70 pounds of feed in his barn. Cliff is also a farmer. He feeds his hogs 15 pounds of feed per day and has 100 pounds of feed in his barn.

a. In how many days will Brad and Cliff have the same amount of feed left? Use a system of equations to find your answer.

b. Does your answer to part **a** make sense?

c. How would your answer change if both Brad and Cliff receive a shipment of 100 pounds of feed on day 4?

SPIRAL REVIEW

Simplify by rationalizing each denominator. *(Lesson 1-3)*

57. $\dfrac{4}{\sqrt{12}}$ **58.** $\dfrac{1}{2\sqrt{5}}$ **59.** $\dfrac{\sqrt{6}}{\sqrt{12}}$ **60.** $\dfrac{7\sqrt{14}}{\sqrt{5}}$

Solve each equation and check your answer. *(Lesson 2-1)*

61. $\frac{5}{2}x - 1 = \frac{1}{2} + 3x$ **62.** $6(7n + 2) = (34 + 11n)3$

63. **Manufacturing** On one assembly line, 45 pounds of cheese are sliced and packaged every two hours. How many hours are needed for 900 pounds of cheese to be sliced and packaged on that assembly line? *(Lesson 2-2)*

3-2 Using Algebraic Methods to Solve Linear Systems

Objectives

Solve systems of equations by substitution.

Solve systems of equations by elimination.

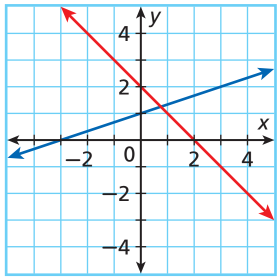
Vocabulary

substitution
elimination

Who uses this?

Zookeepers use algebraic methods to solve systems of linear equations that model mixtures of animal foods. (See Example 4.)

The graph shows a system of linear equations. As you can see, without the use of technology, determining the solution from the graph is not easy. You can use the *substitution* method to find an exact solution. In **substitution**, you solve one equation for one variable and then substitute this expression into the other equation.



EXAMPLE 1

Solving Linear Systems by Substitution

Use substitution to solve each system of equations.

A $\begin{cases} y = x + 2 \\ x + y = 8 \end{cases}$

Step 1 Solve one equation for one variable.

The first equation is already solved for y: $y = x + 2$.

Step 2 Substitute the expression into the other equation.

$x + y = 8$

$x + (x + 2) = 8$ — *Substitute $(x + 2)$ for y in the other equation.*

$2x + 2 = 8$ — *Combine like terms.*

$2x = 6$

$x = 3$

Step 3 Substitute the x-value into one of the original equations to solve for y.

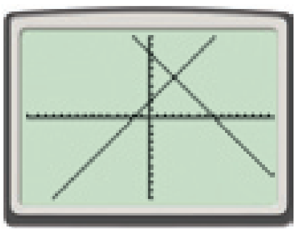
$y = x + 2$

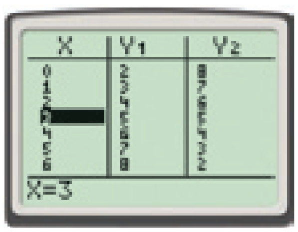
$y = (3) + 2$ — *Substitute $x = 3$.*

$y = 5$

The solution is the ordered pair $(3, 5)$.

Check A graph or table supports your answer.





Caution!

The solution to an independent system of equations is an ordered pair. Do not stop working when you have found only one value.

Use substitution to solve each system of equations.

B $\begin{cases} 2x + y = 6 \\ y - 8x = 1 \end{cases}$

Method 1 Isolate y.

$2x + y = 6$ *First equation*

$y = 6 - 2x$ *Isolate one variable.*

$y - 8x = 1$ *Second equation*

$(6 - 2x) - 8x = 1$ *Substitute the expression into the second equation.*

$6 - 10x = 1$ *Combine like terms.*

$-10x = -5$

$x = \frac{1}{2}$ *First part of the solution*

Method 2 Isolate x.

$2x + y = 6$

$x = \frac{6 - y}{2} = 3 - \frac{y}{2}$

$y - 8x = 1$

$y - 8\left(3 - \frac{y}{2}\right) = 1$

$y - 24 + 4y = 1$

$5y = 25$

$y = 5$

Substitute the value into one of the original equations to solve for the other variable.

Method 1:

$y - 8\left(\frac{1}{2}\right) = 1$ *Substitute the value to solve for the other variable.*

$y - 4 = 1$

$y = 5$ *Second part of the solution*

Method 2:

$2x + (5) = 6$

$2x = 1$

$x = \frac{1}{2}$

By either method, the solution is $\left(\frac{1}{2}, 5\right)$.

CHECK IT OUT! **Use substitution to solve each system of equations.**

1a. $\begin{cases} y = 2x - 1 \\ 3x + 2y = 26 \end{cases}$ **1b.** $\begin{cases} 5x + 6y = -9 \\ 2x - 2 = -y \end{cases}$

You can also solve systems of equations with the *elimination* method. With **elimination**, you get rid of one of the variables by adding or subtracting equations. You may have to multiply one or both equations by a number to create variable terms that can be eliminated.

EXAMPLE 2 Solving Linear Systems by Elimination

Reading Math

The elimination method is sometimes called the *addition method* or *linear combination*.

Use elimination to solve each system of equations.

A $\begin{cases} 2x + 3y = 34 \\ 4x - 3y = -4 \end{cases}$

Step 1 Find the value of one variable.

$$\begin{array}{r} 2x + 3y = 34 \\ + \; 4x - 3y = -4 \\ \hline 6x \qquad = 30 \end{array}$$

The y-terms have opposite coefficients.

Add the equations to eliminate y.

$x = 5$ *First part of the solution*

Step 2 Substitute the x-value into one of the original equations to solve for y.

$2(5) + 3y = 34$

$3y = 24$

$y = 8$ *Second part of the solution*

The solution to the system is $(5, 8)$.

Use elimination to solve each system of equations.

B $\begin{cases} 2x + 4y = -10 \\ 3x + 3y = -3 \end{cases}$

Step 1 To eliminate x, multiply both sides of the first equation by 3 and both sides of the second equation by -2.

$$\begin{aligned} 3(2x+4y) &= 3(-10) \\ -2(3x+3y) &= -2(-3) \end{aligned} \quad \rightarrow \quad \begin{aligned} 6x + 12y &= -30 \\ -6x - 6y &= 6 \\ \hline 6y &= -24 \end{aligned} \quad \textit{Add the equations.}$$

$$y = -4 \qquad \textit{First part of the solution}$$

Step 2 Substitute the y-value into one of the original equations to solve for x.

$$\begin{aligned} 3x + 3(-4) &= -3 \\ 3x - 12 &= -3 \\ 3x &= 9 \\ x &= 3 \end{aligned} \qquad \textit{Second part of the solution}$$

The solution to the system is $(3, -4)$.

Check Substitute 3 for x and -4 for y in each equation.

$$\begin{array}{r|l} \multicolumn{2}{c}{2x + 4y = -10} \\ \hline 2(3) + 4(-4) & -10 \\ -10 & -10 \ \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{3x + 3y = -3} \\ \hline 3(3) + 3(-4) & -3 \\ -3 & -3 \ \checkmark \end{array}$$

CHECK IT OUT! **Use elimination to solve each system of equations.**

2a. $\begin{cases} 4x + 7y = -25 \\ -12x - 7y = 19 \end{cases}$ **2b.** $\begin{cases} 5x - 3y = 42 \\ 8x + 5y = 28 \end{cases}$

In Lesson 3-1, you learned that systems may have infinitely many or no solutions. When you try to solve these systems algebraically, the result will be an identity or a contradiction.

EXAMPLE 3 Classifying Systems with Infinitely Many or No Solutions

Remember!

An *identity*, such as $0 = 0$, is always true and indicates infinitely many solutions.
A *contradiction*, such as $1 = 3$, is never true and indicates no solution.

Classify the system and determine the number of solutions.

$\begin{cases} 2x + y = 8 \\ 6x + 3y = -15 \end{cases}$

Because isolating y is straightforward, use substitution.

$$\begin{aligned} 2x + y &= 8 \\ y &= 8 - 2x && \textit{Solve the first equation for y.} \\ 6x + 3(8 - 2x) &= -15 && \textit{Substitute } 8 - 2x \textit{ for y in the second equation.} \\ 6x + 24 - 6x &= -15 && \textit{Distribute.} \\ 24 &= -15 \ \times && \textit{Simplify.} \end{aligned}$$

Because 24 is never equal to -15, the equation is a contradiction. Therefore, the system is inconsistent and has no solution.

CHECK IT OUT! **Classify the system and determine the number of solutions.**

3a. $\begin{cases} 56x + 8y = -32 \\ 7x + y = -4 \end{cases}$ **3b.** $\begin{cases} 6x + 3y = -12 \\ 2x + y = -6 \end{cases}$

EXAMPLE 4 *Zoology Application*

A zookeeper needs to mix feed for the prairie dogs so that the feed has the right amount of protein. Feed A has 12% protein. Feed B has 5% protein. How many pounds of each does he need to mix to get 100 lb of feed that is 8% protein?

Let a represent the amount of feed A in the mixture.

Let b represent the amount of feed B in the mixture.

Write one equation based on the amount of feed:

Amount of feed A	plus	amount of feed B	equals	100.
a	$+$	b	$=$	100

Write another equation based on the amount of protein:

Protein of feed A	plus	protein of feed B	equals	protein in mixture.
$0.12a$	$+$	$0.05b$	$=$	$0.08(100)$

Solve the system. $\begin{cases} a + b = 100 \\ 0.12a + 0.05b = 8 \end{cases}$

$a + b = 100$	*First equation*
$b = 100 - a$	*Solve the first equation for b.*
$0.12a + 0.05(100 - a) = 8$	*Substitute (100 − a) for b.*
$0.12a + 5 - 0.05a = 8$	*Distribute.*
$0.07a = 3$	*Simplify.*
$a \approx 42.9$	*Round to the nearest tenth.*

Substitute a into one of the original equations to solve for b.

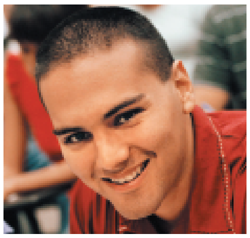
$(42.9) + b \approx 100$	*Substitute the value of a into one equation.*
$b \approx 57.1$	*Solve for b.*

The mixture will contain about 42.9 lb of feed A and 57.1 lb of feed B.

CHECK IT OUT!

4. A coffee blend contains Sumatra beans, which cost \$5/lb, and Kona beans, which cost \$13/lb. If the blend costs \$10/lb, how much of each type of coffee is in 50 lb of the blend?

Student to Student *Solving Systems*



Victor Cisneros
Reagan High School

Choosing a method to solve a system of linear equations can be confusing. Here is how I decide which method to use:

Graphing and tables— *when I'm interested in a rough solution or other values around the solution*

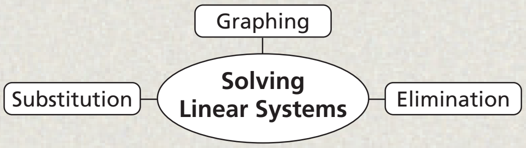
Substitution— *when it's simple to solve one of the equations for one variable (for example, solving $3x + y = 7$ for y)*

Elimination— *when variables have opposite coefficients, like $5x$ and $-5x$, or when I can easily multiply the equations to get opposite coefficients*

THINK AND DISCUSS

1. Explain which method you would use to solve the system $\begin{cases} 3x + y = 8 \\ 7x - y = 5 \end{cases}$.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, show an example of the given method of solving a linear system.



3-2 Exercises

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KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The __?__ method solves a system of linear equations by adding or subtracting equations. (*substitution* or *elimination*)

SEE EXAMPLE 1 p. 190

Use substitution to solve each system of equations.

2. $\begin{cases} x + y = 17 \\ y = x + 7 \end{cases}$
3. $\begin{cases} y = x - 19 \\ 2x - y = 27 \end{cases}$
4. $\begin{cases} 2x - y = 2 \\ 3x - 2y = 11 \end{cases}$
5. $\begin{cases} y = 3x + 5 \\ x = -3y - 5 \end{cases}$

SEE EXAMPLE 2 p. 191

Use elimination to solve each system of equations.

6. $\begin{cases} 2x + y = 12 \\ -5x - y = -33 \end{cases}$
7. $\begin{cases} 2x - 5y = -5 \\ -2x + 8y = -58 \end{cases}$
8. $\begin{cases} 2x + 6y = -8 \\ 5x - 3y = 88 \end{cases}$
9. $\begin{cases} \frac{1}{2}x + y = 4 \\ -2x - 2y = -6 \end{cases}$

SEE EXAMPLE 3 p. 192

Classify each system and determine the number of solutions.

10. $\begin{cases} 5x - y = -3 \\ 15x - 3y = -9 \end{cases}$
11. $\begin{cases} x - 2y = -8 \\ 4x = 8y - 56 \end{cases}$
12. $\begin{cases} 8x + 12y = 60 \\ 2x + 3y = -24 \end{cases}$
13. $\begin{cases} x - \frac{1}{3}y = -2 \\ 6x - 2y = -12 \end{cases}$

SEE EXAMPLE 4 p. 193

14. **Alternative Fuels** Denise owns a car that runs on a mixture of gasoline and ethanol. She can buy fuels that have 85% ethanol or 25% ethanol. How much of each type of fuel should she buy if she wants to fill her 20 gal tank with a mixture of fuel that contains 50% ethanol?

PRACTICE AND PROBLEM SOLVING

Use substitution to solve each system of equations.

15. $\begin{cases} -4y = x \\ 2x + 6y = -3 \end{cases}$
16. $\begin{cases} 12x + y = 21 \\ 18x - 3y = -36 \end{cases}$
17. $\begin{cases} y = 4x \\ 32x + 21y = 29 \end{cases}$
18. $\begin{cases} y + 1 = x \\ -2x + 3y = 2 \end{cases}$

Use elimination to solve each system of equations.

19. $\begin{cases} 4x - 9y = 26 \\ 4x - 5y = 2 \end{cases}$
20. $\begin{cases} 6x - 3y = -6 \\ -5x + 7y = 41 \end{cases}$
21. $\begin{cases} 12x - 3y = -15 \\ 8x + 8y = -58 \end{cases}$
22. $\begin{cases} 3x + y = 7 \\ -3x + 2y = 11 \end{cases}$

Independent Practice

For Exercises	See Example
15–18	1
19–22	2
23–26	3
27	4

Extra Practice

Skills Practice p. S8
Application Practice p. S34

Classify each system and determine the number of solutions.

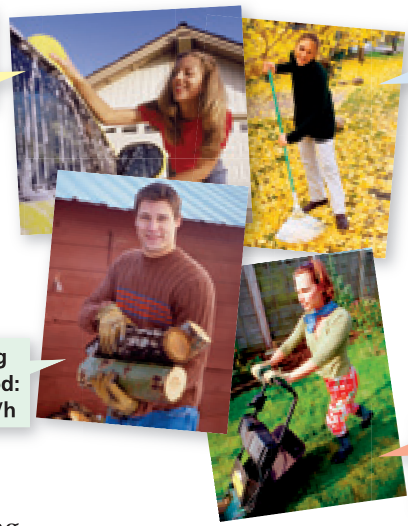
23. $\begin{cases} 4y - x = -24 \\ 3x = 12y + 72 \end{cases}$ **24.** $\begin{cases} 10x - 2y = 22 \\ 5y - 25x = 65 \end{cases}$ **25.** $\begin{cases} 4y - 3x = 32 \\ 8y - 6x = 64 \end{cases}$ **26.** $\begin{cases} -x + \frac{3}{4}y = 4 \\ 8x - 6y = -8 \end{cases}$

27. **Business** An office is printing 1200 copies of a document using two printers. During the process, printer A gets a paper jam and prints only half as many copies as printer B. Write and solve a system of equations to determine the number of copies each printer will produce.

Use substitution or elimination to solve each system of equations.

28. $\begin{cases} y + 3x = -21 \\ x = 3y + 3 \end{cases}$ **29.** $\begin{cases} y = -2x + 14 \\ 1.5x - 3.5y = 2 \end{cases}$ **30.** $\begin{cases} \frac{4}{5}y - 3x = \frac{1}{5} \\ y - x = 8 \end{cases}$ **31.** $\begin{cases} x + 5y = 5 \\ \frac{1}{5}x + 2y = -2 \end{cases}$

32. **Exercise** Ahmit gets exercise on the weekend by working around the house. On Saturday, he worked for 3 hours mowing the lawn and raking leaves. He burned 885 Calories.



a. Write a system of equations that can be used to describe the amount of time Ahmit worked and the number of Calories he burned by mowing and raking.

b. Determine how much time Ahmit spent on mowing and on raking.

33. Vanessa has collected \$2.10 in dimes and nickels.

a. **Critical Thinking** In how many different ways might Vanessa have combined dimes and nickels to total \$2.10?

b. **Critical Thinking** How does the total number of coins change as the number of dimes decreases? Explain.

c. If Vanessa has exactly 30 coins, how many of each type of coin does she have?

34. **ERROR ANALYSIS** Two students attempted to solve the system of equations $\begin{cases} 2x + 4y = 38 \\ 2x - 2y = -4 \end{cases}$ and found different answers. Which solution is incorrect? Explain the error.

A

$2x - 2y = -4$
$-2y = -4 - 2x$
$y = 2 + x$
$2x - 2(2 + x) = -4$
$2x - 4 - 2x = -4$
$0 = 0$
The system is dependent with infinitely many solutions.

B

$2x - 2y = -4$
$-2y = -4 - 2x$
$y = 2 + x$
$2x + 4(2 + x) = 38$
$2x + 8 + 4x = 38$ $y = 2 + x$
$6x = 30$ $y = 2 + 5$
$x = 5$ $y = 7$
The solution is (5, 7).

MULTI-STEP TEST PREP

35. This problem will prepare you for the Multi-Step Test Prep on page 212.

A car race is made up of 500 laps around a 0.533-mile track.

a. How many miles long is the race?

b. The course record for one lap is 14.94 seconds. Write an equation that could be used to model the distance in miles traveled by the record-setting car if it continues at the same pace.

c. If a second car travels at 125 miles per hour, use your answer to part **b** to determine the distance between the two cars when the lead car finishes the race.

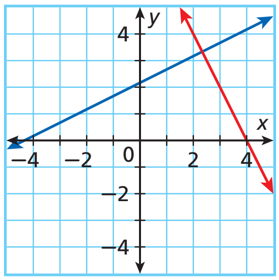
36. Multi-Step Malcolm and Owen work for a bottled water distributor. Malcolm makes \$300 per week plus \$45 for each new customer who signs a contract. Owen has more sales experience, so he earns \$325 per week plus \$60 for each new customer.

a. Write and solve a system of equations to determine the number of new customers for which Malcolm and Owen will have the same income.

b. **Critical Thinking** Is the solution reasonable? Explain.

37. Entertainment The band and the orchestra are attending a concert. The band bought 16 student tickets and 3 adult tickets for \$110.50. The orchestra bought 12 student tickets and 4 adult tickets for \$96. Find the cost of each type of ticket.

38. The graph of the system $\begin{cases} 3x - 6y = -13 \\ 6x + 3y = 24 \end{cases}$ is shown.



A student says the solution is $\left(2\frac{1}{2}, 3\frac{1}{2}\right)$.

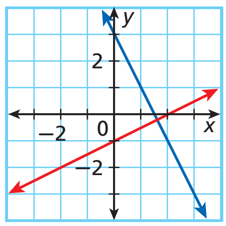
a. Find the solution algebraically to see if the student is correct.

b. **Write About It** Discuss how you can choose the most appropriate method to solve a system of equations.

39. Write About It How can you recognize a dependent system of equations by analyzing the equations?

TEST PREP

40. Which of the following systems of equations represents the graph at right?



Ⓐ $\begin{cases} y = 2x + 3 \\ y = 0.5x - 1 \end{cases}$

Ⓑ $\begin{cases} y = 2x + 3 \\ y = -0.5x - 1 \end{cases}$

Ⓒ $\begin{cases} y = -2x + 3 \\ y = -0.5x - 1 \end{cases}$

Ⓓ $\begin{cases} y = -2x + 3 \\ y = 0.5x - 1 \end{cases}$

41. An organization is holding a banquet in honor of its member of the year. Tables must be rented in order to seat all the guests. A large table seats 12 and costs \$50. A small table seats 8 and costs \$25. How many of each type of table must be rented to seat 100 guests for \$350?

Ⓕ 8 large and 3 small

Ⓖ 3 large and 8 small

Ⓗ 5 large and 5 small

Ⓙ 6 large and 2 small

42. Solve the system. $\begin{cases} x + y = 7 \\ x - y = -3 \end{cases}$

Ⓐ (2, 5) Ⓑ (5, 2) Ⓒ (−2, 5) Ⓓ (−5, 2)

43. **Short Response** What is the solution to the system of equations $\begin{cases} x - 4y = 6 \\ y = -4x + 7 \end{cases}$? Classify the system.

44. Two cyclists are competing in a 10 mi time trial. The older cyclist, who rides at a speed of 22 mi/h, is given a 1 mi head start. The younger cyclist rides at a speed of 25 mi/h. Let x represent time in hours and let y represent distance in miles. Which of the following systems of equations represents the situation?

Ⓕ $\begin{cases} y = 25 \\ y = 22 + 1 \end{cases}$ Ⓖ $\begin{cases} y = 25x + 1 \\ y = 22x \end{cases}$ Ⓗ $\begin{cases} y = 25x \\ y = 22x + 1 \end{cases}$ Ⓙ $\begin{cases} y = 25x \\ y = 22 + x \end{cases}$

CHALLENGE AND EXTEND

45. One equation in a linear system is $x + 2y = 4$. Write another equation so that the system has the unique solution $(-2, 3)$.

46. The following system has a solution. Find the solution and explain your method. How do you know you are correct?

$$\begin{cases} y - x = 4 \\ y - 1 = -2x \\ y + x - 2 = 0 \end{cases}$$

47. **Economics** A software company is considering the release of a new product. The research department has modeled the current market with a set of equations in terms of price p and quantity q.

Supply: $p = 5 + 2q$ **Demand:** $q = 100 - 4p$

a. The equilibrium price and quantity occur when supply meets demand. Find the equilibrium price and quantity for the current market.

b. **What if...?** After the release of the new product, the supply function changes to $p = 3 + 0.5q$. How will the equilibrium price and quantity change?

SPIRAL REVIEW

Simplify each expression. Then evaluate the expression for the given value of the variable. *(Lesson 1-4)*

48. $-b^2(2b + 4) + b^5$, $b = -1$

49. $3c^2 + 1 + (5c)^2$, $c = 3$

50. $\dfrac{20 - 2x^2}{x}$, $x = -2$

51. $y^{-3}\left(\dfrac{2y}{9}\right)$, $y = -3$

Write the equation of the line in slope-intercept form that includes the points in the table. *(Lesson 2-4)*

52.

x	2	6	10	14	20	26
$f(x)$	2	4	6	8	11	14

53.

x	1	2	3	4	5	6
$f(x)$	−2	−3.5	−5	−6.5	−8	−9.5

Graph each inequality. *(Lesson 2-5)*

54. $y > -5$

55. $3x - y \le 2(x - 2)$

56. $5x + 4y > 18$

Connecting Algebra to **Geometry**

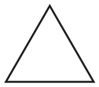
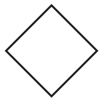
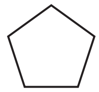
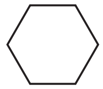
See Skills Bank page S61

Properties of Polygons

A *polygon* is a closed plane figure formed by three or more line segments. Polygons are classified by the properties of their sides and angles.

Polygons can be named by the number of sides they have.

Two angles with the same measure or segments with the same length are *congruent*. If all the sides and angles of a polygon are congruent, the polygon is *regular*. The first four regular polygons are shown.

Number of Sides	3	4	5	6
Name	Triangle	Quadrilateral	Pentagon	Hexagon
				

Here are some other ways of classifying polygons.

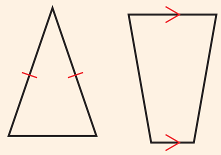
Classifying Triangles	
By Angles	
Acute	Three acute angles (greater than 0° and less than 90°)
Obtuse	One obtuse angle (greater than 90° and less than 180°)
Right	One right angle (90°)
By Sides	
Scalene	No congruent sides
Isosceles	At least two congruent sides
Equilateral	Three congruent sides

Classifying Quadrilaterals	
Types of Quadrilaterals	
Parallelogram	Two pairs of opposite parallel and congruent sides
Trapezoid	Exactly one pair of opposite parallel sides
Types of Parallelograms	
Rectangle	Four right angles
Square	Four congruent sides, 4 right angles
Rhombus	Four congruent sides

Example

Identify each polygon.

Start by counting the number of sides. Then look at the types of angles and pairs of parallel sides. Compare angles to the corner of a square: acute angles measure less than 90°; obtuse angles measure greater than 90°.

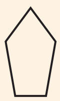
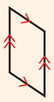
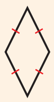
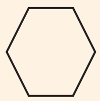


a. Two sides of the triangle are equal, so it is isosceles.

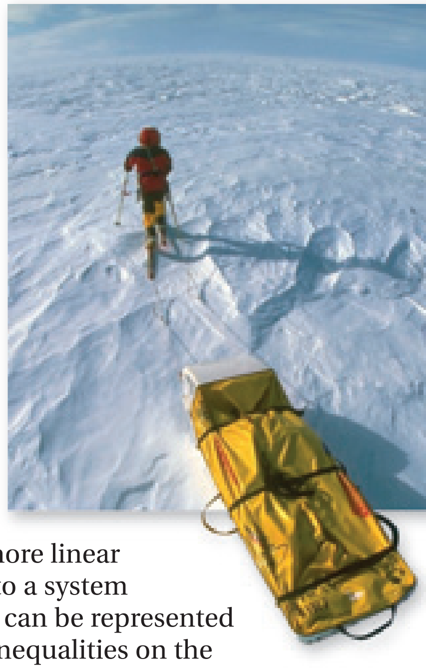
b. This quadrilateral has one pair of parallel sides. It is a trapezoid.

Try This

Identify each polygon.

1.  **2.**  **3.**  **4.**  **5.**  **6.** 

3-3 Solving Systems of Linear Inequalities



Objective
Solve systems of linear inequalities.

Vocabulary
system of linear inequalities

Who uses this?

Explorers can use systems of inequalities to determine the rates at which they must travel to avoid bad weather. (See Example 2.)

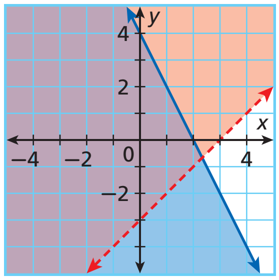
When a problem uses phrases like "greater than" or "no more than," you can model the situation using a system of linear inequalities.

A **system of linear inequalities** is a set of two or more linear inequalities with the same variables. The solution to a system of inequalities is often an infinite set of points that can be represented graphically by shading. When you graph multiple inequalities on the same graph, the region where the shadings overlap is the solution region.

EXAMPLE 1 Graphing Systems of Inequalities

Graph each system of inequalities.

A $\begin{cases} y \le -2x + 4 \\ y > x - 3 \end{cases}$



For $y \le -2x + 4$, graph the solid boundary line $y = -2x + 4$, and shade below it. For $y > x - 3$, graph the dashed boundary line $y = x - 3$, and shade above it.

The overlapping region is the solution region.

Helpful Hint

If you are unsure which direction to shade, use the origin as a test point.

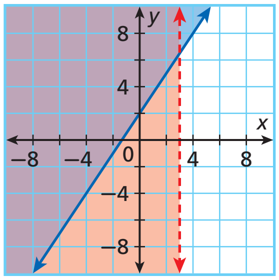
Check Test a point from each region on the graph.

Region	Point	$y \le -2x + 4$	$y > x - 3$
Left	(0, 0)	$0 \overset{?}{\le} -2(0) + 4$ $0 \overset{?}{\le} 4$ ✔	$0 \overset{?}{>} 0 - 3$ $0 \overset{?}{>} -3$ ✔
Right	(4, 0)	$0 \overset{?}{\le} -2(4) + 4$ $0 \overset{?}{\le} -4$ ✘	$0 \overset{?}{>} 4 - 3$ $0 \overset{?}{>} 1$ ✘
Top	(2, 2)	$2 \overset{?}{\le} -2(2) + 4$ $2 \overset{?}{\le} 0$ ✘	$2 \overset{?}{>} 2 - 3$ $2 \overset{?}{>} -1$ ✔
Bottom	(2, −2)	$-2 \overset{?}{\le} -2(2) + 4$ $-2 \overset{?}{\le} 0$ ✔	$-2 \overset{?}{>} 2 - 3$ $-2 \overset{?}{>} -1$ ✘

Only the point from the overlapping (left) region satisfies both inequalities.

Graph each system of inequalities.

B $\begin{cases} y \geq \frac{3}{2}x + 2 \\ x < 3 \end{cases}$



For $y \geq \frac{3}{2}x + 2$, graph the solid boundary line $y = \frac{3}{2}x + 2$, and shade above it. For $x < 3$, graph the dashed boundary line $x = 3$, and shade to the left. The overlapping region is the solution region.

Check Choose a point in the solution region, such as $(-4, 0)$, and test it in both inequalities.

$y \geq \frac{3}{2}x + 2$	$x < 3$
$0 \geq \frac{3}{2}(-4) + 2$	$-4 < 3$ ✔
$0 \geq -4$ ✔	

The test point satisfies both inequalities, and suggests that the solution region is correct.

CHECK IT OUT! **Graph each system of inequalities.**

1a. $\begin{cases} x - 3y < 6 \\ 2x + y > 1.5 \end{cases}$ **1b.** $\begin{cases} y \leq 4 \\ 2x + y < 1 \end{cases}$

EXAMPLE 2 *Expedition Application*

A polar expedition is 240 miles away from base camp, and a snowstorm is predicted to reach the area in 48 hours. The expedition will travel as far as possible by boat and then walk the remaining distance to camp before the storm hits. The explorers can navigate the boat through the ice at a rate of 12 miles per hour or walk with the equipment at a rate of 3 miles per hour. Write and graph a system of inequalities that can be used to determine how long the explorers may travel by foot or by boat to reach base camp before the storm.

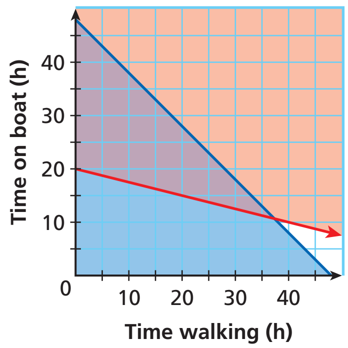
Let x represent the number of hours traveled on foot, and let y represent the number of hours traveled by boat.

The total number of hours can be modeled by the inequality $x + y \leq 48$. The number of miles covered by the explorers can be modeled by $3x + 12y \geq 240$.

The system of inequalities is $\begin{cases} x + y \leq 48 \\ 3x + 12y \geq 240 \end{cases}$.

Graph the solid boundary line $x + y = 48$, and shade below it. Graph the solid boundary line $3x + 12y = 240$, and shade above it. The overlapping region is the solution region.

Expedition Travel Time



Check Test the point $(15, 25)$ in both inequalities. This point represents traveling 15 hours by foot and 25 hours by boat.

$x + y \leq 48$	$3x + 12y \geq 240$
$(15) + (25) \leq 48$	$3(15) + 12(25) \geq 240$
$40 \leq 48$ ✔	$345 \geq 240$ ✔

CHECK IT OUT!

2. Leyla is selling hot dogs and spicy sausages at the fair. She has only 40 buns, so she can sell no more than a total of 40 hot dogs and spicy sausages. Each hot dog sells for $2, and each sausage sells for $2.50. Leyla needs at least $90 in sales to meet her goal. Write and graph a system of inequalities that models this situation.

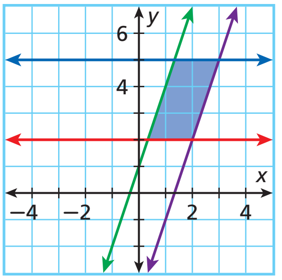
Systems of inequalities may contain more than two inequalities.

EXAMPLE 3 Geometry Application

Geometry

Graph the system of inequalities, and classify the figure created by the solution region.

$$\begin{cases} y \le 5 \\ y \ge 2 \\ y \le 3x + 1 \\ y \ge 3x - 4 \end{cases}$$



Graph the solid boundary lines $y = 5$ and $y = 3x + 1$, and shade below them. Graph the solid boundary lines $y = 2$ and $y = 3x - 4$, and shade above them. The overlapping region is the solution region.

The solution region is a four-sided figure, or quadrilateral. Notice that the boundary lines $y = 5$ and $y = 2$ are parallel, horizontal lines. The boundary lines $y = 3x + 1$ and $y = 3x - 4$ have the same slope and are also parallel. A quadrilateral with two sets of parallel sides is a parallelogram. The solution region is a parallelogram.

Remember!

Parallel lines have the same slope.

CHECK IT OUT!

Graph the system of inequalities, and classify the figure created by the solution region.

3a. $\begin{cases} x \le 6 \\ y \le \frac{1}{2}x + 1 \\ y \ge -2x + 4 \end{cases}$

3b. $\begin{cases} y \le 4 \\ y \ge -1 \\ y \le -x + 8 \\ y \le 2x + 2 \end{cases}$

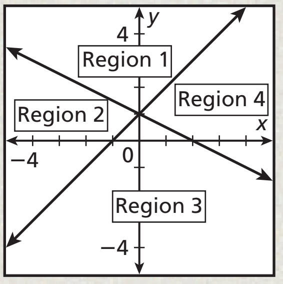
THINK AND DISCUSS

1. Explain how you know which region of the graph of a system of linear inequalities contains the solutions.

2. Find the minimum number of inequalities necessary for a triangular solution region and for a square solution region. Give examples that support your answer.

3. **GET ORGANIZED** Copy and complete the graphic organizer. For each region, write the system of inequalities whose solution it represents.

Know it! Note



3-3 Exercises

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Homework Help Online
KEYWORD: MB7 3-3
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Compare a *system of linear inequalities* with a system of linear equations.

SEE EXAMPLE 1 p. 199

Graph each system of inequalities.

2. $\begin{cases} y \geq 4x - 4 \\ y \geq 3x - 3 \end{cases}$ 3. $\begin{cases} x + y > 5 \\ x - y < -3 \end{cases}$ 4. $\begin{cases} 7x < y - 16 \\ y \leq -5x - 2 \end{cases}$ 5. $\begin{cases} 2x + 2y \leq 4 \\ 3x - y > 1 \end{cases}$

SEE EXAMPLE 2 p. 200

6. **Fund-raising** A charity is selling T-shirts in order to raise money. The cost of a T-shirt is \$15 for adults and \$10 for students. The charity needs to raise at least \$3000 and has only 250 T-shirts. Write and graph a system of inequalities that can be used to determine the number of adult and student T-shirts the charity must sell.

SEE EXAMPLE 3 p. 201

Graph the system of inequalities and classify the figure created by the solution region.

7. $\begin{cases} x \geq 9 \\ y \geq -18 \\ x \leq 13 \\ y \leq -4 \end{cases}$ 8. $\begin{cases} y \leq 7 \\ 2x - y \leq 3 \\ x + 2y \geq -6 \end{cases}$ 9. $\begin{cases} x \leq -1 \\ y \leq 3x + 2 \\ y \geq -3x - 10 \end{cases}$ 10. $\begin{cases} y \geq x \\ y \leq x + 6 \\ x \leq 6 \\ x \geq -2 \end{cases}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–14	1
15	2
16–19	3

Extra Practice
Skills Practice p. S8
Application Practice p. S34

Graph each system of inequalities.

11. $\begin{cases} 5x - y > 0 \\ y < x \end{cases}$ 12. $\begin{cases} 3y \geq 2x - 3 \\ y \geq 3x + 8 \end{cases}$ 13. $\begin{cases} x + y > 5 \\ -2x + y \leq 2 \end{cases}$ 14. $\begin{cases} y > 4 \\ x + 4y \geq 8 \end{cases}$

15. **Music** A musician is releasing a new CD. The record company will manufacture the basic CD plus a special promotional version to distribute to radio stations. No more than 10,000 CDs will be made, and the number of promotional CDs will be at most 20% of the number of basic CDs. Write and graph a system of inequalities that describes the possible number of each type of CD.

Graph the system of inequalities and classify the figure created by the solution region.

16. $\begin{cases} x \geq 0 \\ -\frac{1}{3}x + y \geq -4 \\ \frac{1}{3}x + y \leq -1 \end{cases}$ 17. $\begin{cases} y \leq 2.5 \\ y \geq -0.5 \\ y \leq -x + 8 \\ y \leq 2x + 4 \end{cases}$ 18. $\begin{cases} y \leq x + 6 \\ y \geq x + 1 \\ y \leq -x + 6 \\ y \geq -x - 1 \end{cases}$ 19. $\begin{cases} y \leq x \\ y \leq -x + 2 \\ y \geq 0 \end{cases}$

20. **Sports** In 2003, LaDainian Tomlinson led the National Football League in yards from scrimmage, a combination of rushing yards and receiving yards. He had a total of 2370 yards from scrimmage, including 1645 rushing yards. The runner-up, Jamal Lewis, had fewer yards from scrimmage but more rushing yards. Write and graph a system of inequalities that models the possible rushing and receiving yardage for Jamal Lewis.

Geometry Write a system of linear inequalities whose solution region forms the given shape.

21. a rectangle 22. a square 23. a right triangle 24. a trapezoid

MULTI-STEP TEST PREP

25. This problem will prepare you for the Multi-Step Test Prep on page 212.

Most race cars are subject to various size and weight restrictions, depending on their classification. Champ cars must weigh a minimum of 1565 pounds without the driver. Formula One cars must weigh at least 1322.77 pounds with the driver.

a. Write a system of linear inequalities that could be used to compare the possible weights of Champ cars and Formula One cars without drivers.

b. Identify a reasonable domain and range for the system.

c. Graph the system.

26. **Multi-Step** Frostbite is a dangerous condition where skin freezes because of exposure to cold temperatures and wind. People can develop frostbite in 10 to 30 minutes under conditions modeled by the system $\begin{cases} w \geq 2.4t + 23 \\ w \leq 1.4t + 43 \end{cases}$, where t represents temperature (°F) and w represents wind speed (mi/h).

a. Identify a reasonable domain and range for the system.

b. Graph the solution region for the system of inequalities.

c. If the temperature is 15°F and the wind speed is 55 mi/h, can a person develop frostbite in 10 to 30 minutes? Explain.

27. **Income Tax** Brian and Maria are married and file their taxes jointly. Currently, they have a combined income within the 25% tax bracket, and Maria earns at least \$2000 more per year than Brian. Use the data in the table to write and graph a system of inequalities that models their possible incomes.

2003 Tax Rate Schedule (Married Filing Jointly)

Income	Tax Rate
\$14,000 to \$56,800	15%
\$56,801 to \$114,650	25%
\$114,651 to \$174,700	28%

Graph the solution region for the system of inequalities shown. Then identify three points in the solution region.

28. $\begin{cases} -5y < 2x \\ 5y \geq 2x - 20 \end{cases}$

29. $\begin{cases} y + 7 > 0 \\ y < 2x + 5 \\ y < -3x + 4 \end{cases}$

30. $\begin{cases} y \geq -8 \\ x + 2y < 4 \\ x > -6 \end{cases}$

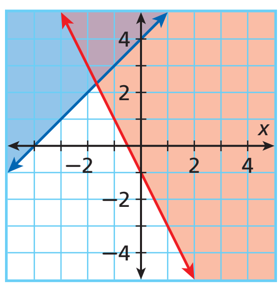
31. $\begin{cases} \frac{1}{2}x + 3y \leq 2 \\ x - y > 3 \end{cases}$

32. **Critical Thinking** If the boundary lines in a system of inequalities are parallel, what are the possible solution regions?

33. **Write About It** Is it possible for a system of two inequalities to have no solution? Explain.

TEST PREP

34. Which of the following systems of inequalities describes the graph shown?



Ⓐ $\begin{cases} y > x + 4 \\ y < -2x - 1 \end{cases}$

Ⓑ $\begin{cases} y > x + 4 \\ y > -2x - 1 \end{cases}$

Ⓒ $\begin{cases} y \leq x + 4 \\ y \geq -2x - 1 \end{cases}$

Ⓓ $\begin{cases} y \geq x + 4 \\ y \geq -2x - 1 \end{cases}$

35. A manufacturing company produces gizmos. It costs at least \$300 plus \$1.25 per gizmo to produce the items. Each gizmo sells for \$2.50 at most. Which of the following systems of inequalities, where c is the cost and n is the number of gizmos produced or sold, has a solution region that can be used to represent the company's possible profit?

Ⓕ $\begin{cases} c \le 2.5n \\ c \le 300 + 1.25n \end{cases}$ Ⓗ $\begin{cases} c \ge 2.5n \\ c \le 300 + 1.25n \end{cases}$

Ⓖ $\begin{cases} c \le 2.5n \\ c \ge 300 + 1.25n \end{cases}$ Ⓙ $\begin{cases} c \ge 2.5n \\ c \ge 300 + 1.25n \end{cases}$

36. Which of the tables below contains possible solutions to the following system of inequalities? $\begin{cases} 3x - 12y > 8 \\ x + 5y > -5 \end{cases}$

Ⓐ

x	y
1	−2
2	0
3	0
4	−2

Ⓑ

x	y
1	−2
2	1
3	1
4	−2

Ⓒ

x	y
1	−1
2	−1
3	0
4	0

Ⓓ

x	y
1	−2
2	−2
3	−1
4	−1

CHALLENGE AND EXTEND

37. Write a system of linear inequalities whose solution region is a pentagon.

38. In the following system of inequalities, is there a value of m that will cause the system to have no solution? If so, find the value. If not, explain. $\begin{cases} y > -3x + 2 \\ y < mx - 3 \end{cases}$

39. Kira is investing \$30,000 divided between two separate simple interest accounts. One pays 5% and has very low risk, and the other pays 7% and has slightly higher risk. What is the least she can invest in the riskier account and still earn at least \$1900 after one year?

SPIRAL REVIEW

Find the additive and multiplicative inverse for each number. *(Lesson 1-2)*

40. 7 **41.** $-\frac{3}{4}$ **42.** 2.48 **43.** -1

Write the equation of each line. *(Lesson 2-4)*

44. through $(2, -7)$ and $(1, 1)$

45. with slope 0 through $(3, -3)$

46. through $(1, -1)$ and $(0, 0)$

47. with slope $-\frac{1}{3}$ through $(9, 6)$

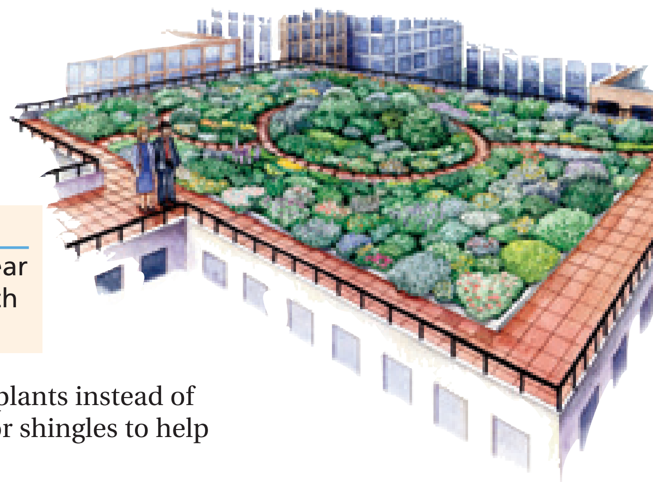
48. perpendicular to $y = 4x - 1$ and through $(-2, 4.5)$

49. parallel to $y = -x - 7$ and through $(3, 2)$

50. The youth baseball league charted the number of players and coaches in each age bracket. Find the correlation coefficient to the nearest thousandth and the equation of the line of best fit. *(Lesson 2-7)*

Players	32	18	55	37	50	86
Coaches	3	2	6	5	6	10

3-4 Linear Programming



Objective
Solve linear programming problems.

Vocabulary
linear programming
constraint
feasible region
objective function

Who uses this?
Landscape architects can use linear programming to determine which plants to plant on a green roof.

Green roofs are roofs covered with plants instead of traditional materials like concrete or shingles to help lower heat and improve air quality.

The plants landscape architects choose might depend on the price, the amount of water they require, and the amount of carbon dioxide they absorb.

Linear programming is a method of finding a maximum or minimum value of a function that satisfies a given set of conditions called *constraints*. A **constraint** is one of the inequalities in a linear programming problem. The solution to the set of constraints can be graphed as a **feasible region**.

EXAMPLE 1 Graphing a Feasible Region

Gillian is planning a green roof that will cover up to 600 square feet. She will use two types of plants: blue lagoon sedum and raspberry red sedum. Each blue lagoon sedum will cover 1.2 square feet. Each raspberry red sedum will cover 2 square feet. Each plant costs \$2.50, and Gillian must spend less than \$1000. Write the constraints, and graph the feasible region.

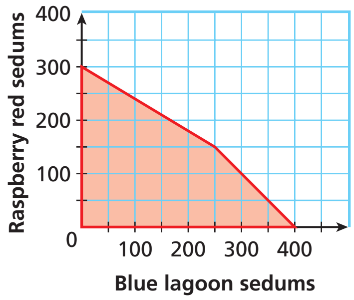
Let $b =$ the number of blue lagoon sedums, and
$r =$ the number of raspberry red sedums.

Write the constraints:

$$\begin{cases} b \geq 0 \\ r \geq 0 \\ 1.2b + 2r \leq 600 \\ 2.50b + 2.50r \leq 1000 \end{cases}$$

The number of plants cannot be negative.
The combined area is less than or equal to 600 ft^2.
The combined cost is less than or equal to \$1000.

Graph the feasible region. The feasible region is a quadrilateral with vertices at $(0, 0)$, $(400, 0)$, $(250, 150)$, and $(0, 300)$.



Check A point in the feasible region, such as $(100, 100)$, satisfies all of the constraints. ✔

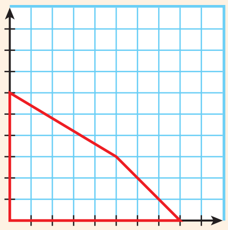
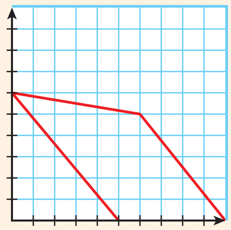
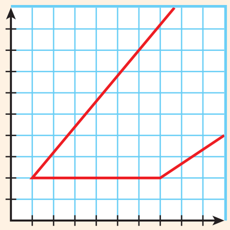
CHECK IT OUT!

1. Graph the feasible region for the following constraints.

$$\begin{cases} x \geq 0 \\ y \geq 1.5 \\ 2.5x + 5y \leq 20 \\ 3x + 2y \leq 12 \end{cases}$$

In most linear programming problems, you want to do more than identify the feasible region. Often you want to find the best combination of values in order to minimize or maximize a certain function. This function is the **objective function**.

The objective function may have a minimum, a maximum, neither, or both depending on the feasible region.

Bounded and Unbounded Regions	
Bounded Feasible Region	**Unbounded Feasible Regions**
	 
Objective function has both a minimum and a maximum value.	Objective function has either a maximum value or a minimum value but not both.

More advanced mathematics can prove that the maximum or minimum value of the objective function will always occur at a vertex of the feasible region.

Know it! Note

The Vertex Principle of Linear Programming

If an objective function has a maximum or minimum value, it must occur at one or more of the vertices of the feasible region.

EXAMPLE 2

Solving Linear Programming Problems

One of Gillian's priorities for the green roof is to help control air pollution. To do this, she wants to maximize the amount of carbon dioxide the plants on the roof absorb. Use the carbon dioxide absorption rates and the data from Example 1 to find the number of each plant Gillian should plant.

Helpful Hint

Check your graph of the feasible region by using your graphing calculator.

Be sure to change the variables to x and y.

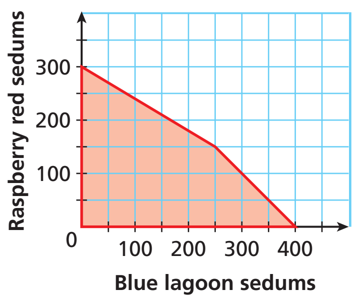
Step 1 Let $C =$ the number of pounds of carbon dioxide absorbed.

Write the objective function:

$C = 1.4b + 2.1r$

Step 2 Recall the constraints and the graph from Example 1.

$$\begin{cases} b \geq 0 \\ r \geq 0 \\ 1.2b + 2r \leq 600 \\ 2.50b + 2.50r \leq 1000 \end{cases}$$



Step 3 Evaluate the objective function at the vertices of the feasible region.

(b, r)	1.4*b* + 2.1*r*	*C*(lb)
(0, 0)	1.4(0) + 2.1(0)	0
(0, 300)	1.4(0) + 2.1(300)	630
(250, 150)	1.4(250) + 2.1(150)	665
(400, 0)	1.4(400) + 2.1(0)	560

The maximum value occurs at the vertex (250, 150).

Gillian should plant 250 blue lagoon sedums and 150 raspberry red sedums to maximize the amount of carbon dioxide absorbed.

CHECK IT OUT!

2. Maximize the objective function $P = 25x + 30y$ under the following constraints. $\begin{cases} x \geq 0 \\ y \geq 1.5 \\ 2.5x + 5y \leq 20 \\ 3x + 2y \leq 12 \end{cases}$

EXAMPLE 3 *Problem-Solving Application*

PROBLEM SOLVING

Brad is an organizer of the Bolder Boulder 10K race and must hire workers for one day to prepare the race packets. Skilled workers cost \$60 a day, and students cost \$40 a day. Brad can spend no more than \$1440. He needs at least 1 skilled worker for every 3 students, but only 16 skilled workers are available. Skilled workers can prepare 25 packets per hour, and students can prepare 18 packets per hour. Find the number of each type of worker that Brad should hire to maximize the number of packets produced.

1 Understand the Problem

The **answer** will have two parts—the number of skilled workers and the number of students that will be hired.

List the important information:

- Skilled workers cost \$60 per day. Students cost \$40 per day.
- Brad can spend no more than \$1440.
- Skilled workers can prepare 25 packets per hour. Students can prepare 18 packets per hour.
- Brad needs at least 1 skilled worker for every 3 students.
- Only 16 skilled workers are available.

2 Make a Plan

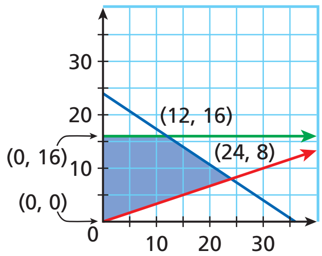
Let x = the number of students and y = the number of skilled workers. Write the constraints and objective function based on the important information.

$$\begin{cases} x \geq 0 & \textit{The number of workers cannot be negative.} \\ y \geq 0 & \\ 40x + 60y \leq 1440 & \textit{Cost of labor must be no more than \$1440.} \\ y \geq \frac{1}{3}x & \textit{At least 1 skilled worker for every 3 students} \\ y \leq 16 & \textit{Only 16 experienced workers are available.} \end{cases}$$

Let P represent the number of packets prepared each hour. The objective function is $P = 18x + 25y$.

3 Solve

Graph the feasible region, and identify the vertices. Evaluate the objective function at each vertex.



$P(0, 0) = 18(0) + 25(0) = 0$

$P(0, 16) = 18(0) + 25(16) = 400$

$P(12, 16) = 18(12) + 25(16) = 616$

$P(24, 8) = 18(24) + 25(8) = 632$

The objective function is maximized at $(24, 8)$, so Brad should hire 24 students and 8 skilled workers.

4 Look Back

Check the values $(24, 8)$ in the constraints.

$x \geq 0$	$y \geq 0$	$y \leq 16$
$24 \geq 0$ ✔	$8 \geq 0$ ✔	$8 \leq 16$ ✔
	$y \geq \frac{1}{3}x$	$40x + 60y \leq 1440$
	$8 \geq \frac{1}{3}(24)$	$40(24) + 60(8) \leq 1440$
	$8 \geq 8$ ✔	$1440 \leq 1440$ ✔

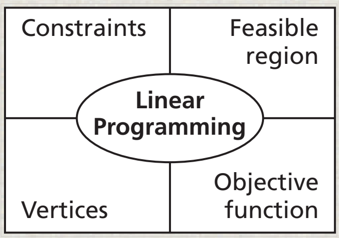
CHECK IT OUT!

3. A book store manager is purchasing new bookcases. The store needs 320 feet of shelf space. Bookcase A provides 32 ft of shelf space and costs \$200. Bookcase B provides 16 ft of shelf space and costs \$125. Because of space restrictions, the store has room for at most 8 of bookcase A and 12 of bookcase B. How many of each type of bookcase should the manager purchase to minimize the cost?

THINK AND DISCUSS

1. Explain why linear programming problems often have $x \geq 0$ and $y \geq 0$ as constraints.
2. Explain why an objective function based on the constraints $\begin{cases} x + y > 0 \\ y \leq 4 \end{cases}$ will have a maximum or a minimum, but not both.
3. How can you tell whether a piece of information relates to the constraints or to the objective function?
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the given characteristic, using data from Examples 1 and 2.

Know it! Note



3-4 Exercises

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GUIDED PRACTICE

1. **Vocabulary** The inequalities in a linear programming problem are called __?__. (*constraints* or *objective functions*)

SEE EXAMPLE 1
p. 205

Graph each feasible region.

2. $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 3x + 3 \\ y \leq -x + 7 \end{cases}$

3. $\begin{cases} x \geq 0 \\ y \geq -1 \\ y \leq x + 1 \\ y \leq -\frac{1}{4}x + 6 \end{cases}$

4. $\begin{cases} x \geq -2 \\ y \leq 1 \\ y \geq 0.5x - 2 \\ y \leq -2x + 3 \end{cases}$

SEE EXAMPLE 2
p. 206

Maximize or minimize each objective function.

5. Maximize $P = 10x + 16y$ for the constraints from Exercise 2.
6. Minimize $P = 3x + 5y$ for the constraints from Exercise 3.
7. Maximize $P = 2.4x + 1.5y$ for the constraints from Exercise 4.

SEE EXAMPLE 3
p. 207

8. **Dentistry** Dr. Lee's dentist practice is open for 7 hours each day. His receptionist schedules appointments, allowing $\frac{1}{2}$ hour for a cleaning and 1 hour to fill a cavity. He charges \$40 for a cleaning and \$95 for a filling. Dr. Lee cannot do more than 4 fillings per day. Find the number of each type of appointment that maximizes Dr. Lee's income for the day.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–14	2
15	3

Extra Practice

Skills Practice p. S9
Application Practice p. S34

Graph each feasible region.

9. $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \geq 4x - 4 \\ y \leq x + 5 \end{cases}$

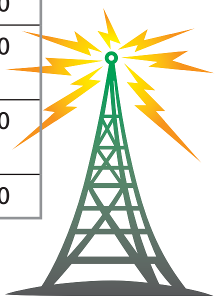
10. $\begin{cases} x \leq 0 \\ y \geq 0 \\ y \leq 9 \\ y \geq -2x - 7 \end{cases}$

11. $\begin{cases} x \geq 0 \\ x \leq 5 \\ y \geq \frac{1}{5}x - 3 \\ y \leq -x + 4 \end{cases}$

Maximize or minimize each objective function.

12. Maximize $P = -21x + 11y$ for the constraints from Exercise 9.
13. Minimize $P = -2x - 4y$ for the constraints from Exercise 10.
14. Maximize $P = x + 3y$ for the constraints from Exercise 11.
15. **Advertising** A concert tour plans to advertise upcoming tour dates. The advertising budget is \$60,000, and the tour manager will focus on prime-time television and radio commercials. She would like to have between 30 and 60 radio commercials. Use the table to find the number of prime-time television and radio commercials that will maximize the on-air time of the advertisements but will stay within the budget.

Type	Time(s)	Cost (\$)
Radio	20	400
Television (prime time)	30	1500
Television (late night)	30	1200
Newspaper	■	300



MULTI-STEP TEST PREP

16. This problem will prepare you for the Multi-Step Test Prep on page 212.

Tickets to a car race cost $25 for the upper deck and $45 for the lower deck. The track may admit no more than 160,000 spectators by order of the fire marshal.

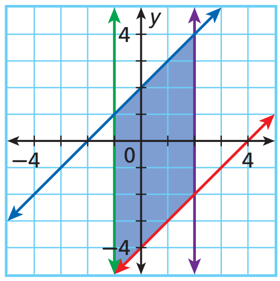
a. If the lower deck can seat no more than 60,000 fans and the upper deck can seat no more than 120,000 fans, how many of each ticket type should be sold to maximize profit?

b. How do the system and solution change if race officials expect to make an additional $60 per person in the upper deck and $30 per person in the lower deck from the sale of food and merchandise?

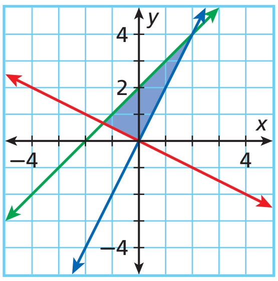
17. **Manufacturing** A camping supply company produces backpacks in two models, journey and trek. The journey model requires 4 hours of labor, and the company makes a profit of $40. The trek model requires 6 hours of labor, and the company makes a profit of $80. The distributor will accept no more than 4 trek models and 15 journey models per week. What is the minimum number of hours of labor that are required for the company to make a profit of at least $400 per week?

Geometry Given the graph of the feasible region, identify the figure and write the inequalities that represent the constraints.

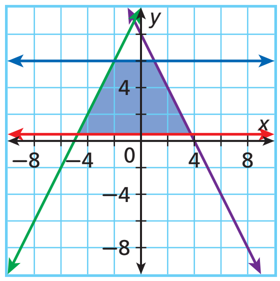
18.



19.



20.



LINK

Math History

Linear programming was developed during World War II. Also known as Operations Research (OR), the process played an important part in World War II by helping the military effectively allocate its resources.

21. **Highway Traffic** To prevent traffic jams, a city funds a courtesy patrol to aid stranded drivers on local roads. The patrol can repair a flat tire, provide the motorist with 2 gallons of gas, or call a tow truck for more serious problems. It takes 15 minutes to help a driver who is out of gas and 45 minutes to help a driver with a flat tire. The courtesy patrol driver carries 28 gallons of gas. What is the maximum number of stops for flat tires or empty gas tanks that the courtesy patrol can make in an 8-hour shift?

22. **Critical Thinking** Is it possible for a linear programming problem to have no solution? Give an example to support your answer.

23. **Nutrition** A health food store is creating smoothies with soy protein and vitamin supplements. A Soy Joy smoothie costs $2.75 and uses 2 ounces of soy and 1 ounce of vitamin supplement. A Vitamin Boost smoothie costs $3.25 and uses 3 ounces of vitamin supplement and 1 ounce of soy protein. The store has 100 ounces each of vitamin supplement and soy protein in stock. How many of each type of smoothie should the store make in order to maximize revenue?

24. **Critical Thinking** Give an example of a problem situation where the feasible region might include negative values.

25. **Write About It** Describe how to recognize when you have found the maximum or minimum value of an objective function for a given set of constraints.

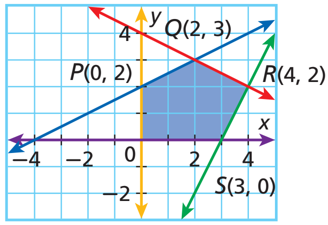
26. **Write About It** Describe how to find the coordinates of the vertices of the feasible region.

TEST PREP

27. Which point gives the minimum value of $P = -x + y$ in the feasible region shown at right?

Ⓐ P Ⓒ R

Ⓑ Q Ⓓ S



28. A feasible region has vertices $(0, 0)$, $(-1, 2)$, and $(-2, 6)$. Which of the following objective functions has a minimum value less than zero over this region?

Ⓐ $P = -4x + y - 1$ Ⓒ $P = 12x + 7y$

Ⓑ $P = -x + 3y + 2$ Ⓓ $P = -5x - y$

29. A real estate developer plans to divide a 300,000-square-foot piece of land into commercial and residential properties. Each residential property requires 2500 square feet, and each commercial property requires 30,000 square feet. The developer makes a profit of $1000 for each residential site and $20,000 for each commercial site but is limited to no more than six commercial sites because of zoning ordinances. Which of the following gives the objective function for the maximum profit of the developer?

Ⓕ $P = 2500x + 30{,}000y$ Ⓗ $P = 2500x + 1000y$

Ⓖ $P = 1000x + 20{,}000y$ Ⓙ $P = 300{,}000 - x - 6y$

CHALLENGE AND EXTEND

30. Medicine A pharmaceutical company is testing a new antibiotic on two sample strains of bacteria. To properly assess the effectiveness of the antibiotic, more than 700 viable bacteria samples must be tested, at least 400 of which must be type B. The company would like to minimize the amount of money spent on bacteria.

a. Graph the feasible region.

b. What do the points $(350, 400)$ and $(400, 350)$ represent for this problem situation?

c. Do the points satisfy the constraints? Why or why not?

SPIRAL REVIEW

For each function, evaluate $f(7)$ and $f\left(-\frac{1}{2}\right)$. *(Lesson 1-7)*

31. $f(x) = \dfrac{1}{2x - 3}$ **32.** $f(x) = 0.5x$ **33.** $f(x) = \dfrac{x^2 - 1}{x - 1}$

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph. *(Lesson 2-9)*

34. $(6, -3)$ **35.** $\left(\frac{1}{3}, \frac{4}{3}\right)$ **36.** $(-2.5, 0.75)$

Geometry **Graph the system of inequalities and classify the figure created by the solution region.** *(Lesson 3-3)*

37. $\begin{cases} y \le 6 \\ y - 2x \ge 0 \\ x \ge 0 \end{cases}$

38. $\begin{cases} y \ge 0 \\ y \le 2 \\ y \le x \\ x + y \le 6 \end{cases}$

CHAPTER 3

SECTION 3A

MULTI-STEP TEST PREP

Linear Systems in Two Dimensions

What a Drag! Drag racers measure fuel efficiency differently than most drivers. Instead of using miles per gallon, a drag racing team is more likely to consider gallons per second or gallons per mile to gauge a dragster's fuel consumption and performance. Suppose a drag-racing team is trying to choose a car from four possible entrants. The results of a $\frac{1}{4}$-mile test run are shown in the table.

Dragsters' Results

Dragster	Top Speed (mi/h)	Fuel Used (gal)	Time (s)
Black Dragon	288	14.2	6.1
Lucky Lady	302	15.8	5.9
Red Rocket	274	13.7	6.4
Wild Thing	318	16.5	5.4

1. Using gallons per second as a measure, which car was the most fuel efficient? Which car was the least fuel efficient?
2. Black Dragon and Red Rocket each have 18-gallon fuel tanks, and Wild Thing and Lucky Lady have 20-gallon fuel tanks. Write and graph a system of four linear equations that could be used to model the fuel remaining in these cars after t seconds.
3. After how many seconds would Black Dragon and Wild Thing have the same amount of fuel left in their tanks?
4. After how many seconds would Red Rocket and Lucky Lady have the same amount of fuel left in their tanks?
5. Suppose that Black Dragon is located at the team garage and Lucky Lady is located 1000 miles away. Use the maximum speeds of the cars to write and graph a system of linear inequalities that can be used to determine when and where the cars would meet if they travel directly toward one another.
6. Is the point $(1.5, 500)$ a possible solution to the system of linear inequalities? Explain.

READY TO GO ON?

CHAPTER 3

SECTION 3A

Quiz for Lesson 3-1 Through 3-4

3-1 Solving Linear Systems by Using Graphs and Tables

Solve each system by using a graph and a table. Check your answer.

1. $\begin{cases} 2x + y = -5 \\ x + 2y = 2 \end{cases}$

2. $\begin{cases} x + y = -1 \\ x - 2y = -4 \end{cases}$

3. $\begin{cases} x = y - 2 \\ 3x - y = 2 \end{cases}$

Classify each system and determine the number of solutions.

4. $\begin{cases} 8x - 12y = 48 \\ 3y = 2x - 4 \end{cases}$

5. $\begin{cases} 5x - 6y = 14 \\ x + 3y = 15 \end{cases}$

6. $\begin{cases} x = 2y - 10 \\ y = 5 + \frac{1}{2}x \end{cases}$

3-2 Solving Linear Systems by Using Algebraic Methods

Use substitution or elimination to solve each system of equations.

7. $\begin{cases} y = x + 3 \\ 2x + 4y = 24 \end{cases}$

8. $\begin{cases} x = 5 \\ 2x + 3y = 19 \end{cases}$

9. $\begin{cases} x - y = 5 \\ 3x - 2y = 14 \end{cases}$

10. $\begin{cases} x + 2y = 15 \\ x - 2y = -9 \end{cases}$

11. $\begin{cases} 5x - 4y = 0 \\ 8x - 4y = 12 \end{cases}$

12. $\begin{cases} 4x + 2y = 12 \\ 2x + 6y = -4 \end{cases}$

3-3 Solving Systems of Linear Inequalities

Graph each system of inequalities.

13. $\begin{cases} y - x < 3 \\ y + x < 3 \end{cases}$

14. $\begin{cases} y + x \le 0 \\ y \le 4 - x \end{cases}$

15. $\begin{cases} y \ge 2x + 3 \\ y > -x \end{cases}$

16. **Travel** Karen traveled almost 350 mi in under 7 h of highway driving. She stopped for a brief rest that was not included in her driving time. Karen averaged 60 mi/h for the first part of her trip and 50 mi/h for the second part of the trip. Write and graph a system of inequalities that can be used to determine how many hours Karen spent driving in each part of her trip.

3-4 Linear Programming

Graph each feasible region, and maximize or minimize the objective function $P = 4x + 5y$.

17. minimize; $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le x - 1 \\ y \le -\frac{1}{2}x + 4 \end{cases}$

18. maximize; $\begin{cases} x \le 2 \\ y \ge 0 \\ y \le 2x + 4 \\ y \le -3x + 9 \end{cases}$

19. **Finance** A beauty salon schedules appointments for haircuts for 30 minutes and for special services, such as tinting or curling, for 1 hour. Each haircut costs $20, and special services cost $45. The beauty salon wants to schedule no more than 4 special services each day per beautician. Find the number of each type of appointment that produces the maximum income per beautician in a workday of 8 hours at most.

3-5 Linear Equations in Three Dimensions

Objective

Graph points and linear equations in three dimensions.

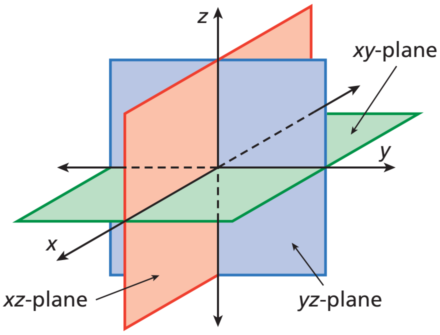
Vocabulary

three-dimensional coordinate system
ordered triple
z-axis

Why learn this?

You can participate in Geocaching, an outdoor treasure-hunting game, by using three-dimensional coordinates to pinpoint locations on Earth.

A Global Positioning System (GPS) gives locations using the three coordinates of latitude, longitude, and elevation. You can represent any location in three-dimensional space using a **three-dimensional coordinate system**, sometimes called *coordinate space*.



Each point in coordinate space can be represented by an **ordered triple** of the form (x, y, z). The system is similar to the coordinate plane but has an additional coordinate based on the ***z*-axis**. Notice that the axes form three planes that intersect at the origin.

EXAMPLE 1 Graphing Points in Three Dimensions

Graph each point in three-dimensional space.

A $A(2, 3, -2)$

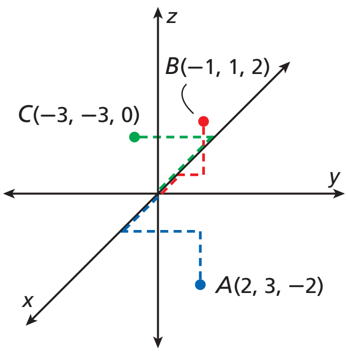
From the origin, move 2 units forward along the x-axis, 3 units right, and 2 units down.

B $B(-1, 1, 2)$

From the origin, move 1 unit back along the x-axis, 1 unit right, and 2 units up.

C $C(-3, -3, 0)$

From the origin, move 3 units back along the x-axis and 3 units left. Notice that this point lies in the xy-plane because the z-coordinate is 0.



CHECK IT OUT! **Graph each point in three-dimensional space.**

1a. $D(1, 3, -1)$ **1b.** $E(1, -3, 1)$ **1c.** $F(0, 0, 3)$

Recall that the graph of a linear equation in two dimensions is a straight line. In three-dimensional space, the graph of a linear equation is a plane. Because a plane is defined by three points, you can graph linear equations in three dimensions by finding the three intercepts.

EXAMPLE 2 Graphing Linear Equations in Three Dimensions

Helpful Hint

To find an intercept in coordinate space, set the other two coordinates equal to 0.

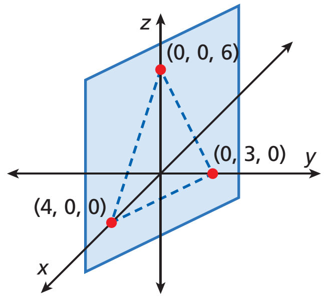
Graph the linear equation $3x + 4y + 2z = 12$ in three-dimensional space.

Step 1 Find the intercepts:

x-intercept: $3x + 4(0) + 6(0) = 12$
$x = 4$

y-intercept: $3(0) + 4y + 2(0) = 12$
$y = 3$

z-intercept: $3(0) + 4(0) + 2z = 12$
$z = 6$



Step 2 Plot the points $(4, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$. Sketch a plane through the three points.

CHECK IT OUT!

2. Graph the linear equation $x - 4y + 2z = 4$ in three-dimensional space.

EXAMPLE 3 *Technology Application*

A computer game uses a role-playing scenario in which players build civilizations. Each player begins with 100 gold coins to buy resources. The players then compete for the survival of their civilizations. Each unit of food costs 2 gold coins, wood costs 4 gold coins, and stone costs 5 gold coins.

A **Write a linear equation in three variables to represent this situation.**

Let f = units of food, w = units of wood, and s = units of stone.

Write an equation:

cost of food	+	cost of wood	+	cost of stone	+	100 gold pieces
$2f$	+	$4w$	+	$5s$	+	100

B **Use the table to find the number of units of stone each player can buy.**

Bonnie: $2(20) + 3(10) + 5s = 100$
$s = 6$

Chad: $2(15) + 3(15) + 5s = 100$
$s = 5$

Frederico: $2(40) + 3(5) + 5s = 100$
$s = 1$

LaToya: $2(25) + 3(10) + 5s = 100$
$s = 4$

Player	Units of Food	Units of Wood	Units of Stone
Bonnie	20	10	
Chad	15	15	
Frederico	40	5	
LaToya	25	10	

Bonnie can purchase 6 units of stone, Chad can purchase 5 units, Frederico can purchase 1 unit, and LaToya can purchase 4 units.

CHECK IT OUT!

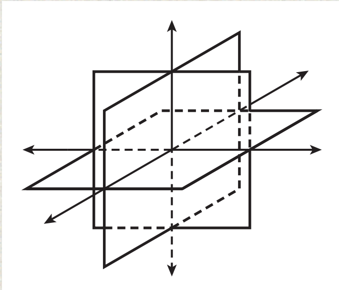
Steve purchased \$61.50 worth of supplies for a hiking trip. The supplies included flashlights for \$3.50 each, compasses for \$1.50 each, and water bottles for \$0.75 each.

3a. Write a linear equation in three variables to represent this situation.

3b. Steve purchased 6 flashlights and 24 water bottles. How many compasses did he purchase?

THINK AND DISCUSS

1. Estimate your current coordinates in three-dimensional space using the front left bottom corner of your classroom as the origin. Let 1 foot represent 1 unit.
2. Describe a plane that has only two intercepts
3. **GET ORGANIZED** Copy and complete the graphic organizer. Label each axis, plane, and line shown.



3-5 Exercises

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GUIDED PRACTICE

1. **Vocabulary** Explain the difference between the two- and three-dimensional coordinate systems.

SEE EXAMPLE 1 p. 214

Graph each point in three-dimensional space.

2. $(-3, -2, 1)$ 3. $(0, 2, 2)$ 4. $(1, 4, 5)$ 5. $(-1, 2, 4)$

SEE EXAMPLE 2 p. 215

Graph each linear equation in three-dimensional space.

6. $x + y + z = 3$ 7. $5x - 2y - 4z = 10$ 8. $1.5x + 3y - 2z = -6$

SEE EXAMPLE 3 p. 215

9. **Multi-Step** Whitney's delivery truck has a weight limit of 1.5 tons. She delivers refrigerators that weigh 225 lb, dishwashers that weigh 150 lb, and ovens that weigh 300 lb. (*Hint:* 1 ton = 2000 lb)
   a. Write a linear equation in three variables to represent this situation.
   b. Complete the table for the possible numbers of appliances the truck can hold.
   c. **Estimation** Estimate the maximum number of appliances the truck can hold.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–17	1
18–23	2
24	3

Extra Practice
Skills Practice p. S9
Application Practice p. S34

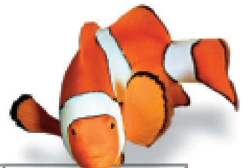
Graph each point in three-dimensional space.

10. $(2, -4, 3)$ 11. $(-1, 1, 4)$ 12. $(3, 0, 0)$ 13. $(1, -2, 0)$
14. $(-3, -3, -3)$ 15. $(5, 0, 2)$ 16. $(0, -3, 2)$ 17. $(-4, -1, 1)$

Graph each linear equation in three-dimensional space.

18. $x + y - z = -1$ 19. $2x - y + 2z = 4$ 20. $2x + \frac{1}{2}y + z = -2$
21. $5x + y - z = -5$ 22. $8x + 6y + 4z = 24$ 23. $3x - 3y + 2.5z = 7.5$

24. Aquariums Gordon has \$80 to purchase a combination of cinnamon clownfish, anemones, and hermit crabs for his aquarium. Clownfish cost \$10 each, anemones cost \$15 each, and hermit crabs cost \$2.50 each.



a. Write a linear equation in three variables to represent this situation.

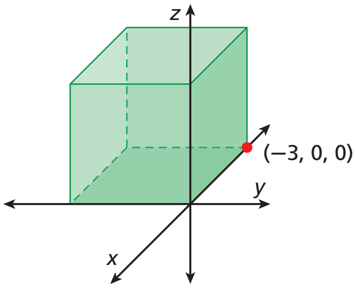
b. Complete the table for the possible numbers of sea creatures Gordon may purchase.

Hermit Crabs	Anemones	Clownfish
	2	2
10	1	
2		3
	1	5

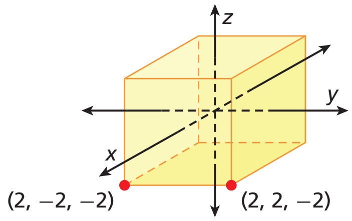
25. Sports Basketball players can score in three different ways: one-point free throws, two-point field goals, or three-point field goals. Cindy Brown of Long Beach State holds the Division 1 NCAA women's record for the most points in a single game, 60, including 20 free throws. Identify five possible combinations of two-point field goals and three-pointers that she may have had in the game.

Geometry **Identify the unlabeled vertices of each cube.**

26.



27.



28. Architecture An architect is planning the flooring for a 2000 ft^2 home. There will be three types of flooring: hardwood, tile, and carpet. The architect has budgeted \$8000 for the floors and has decided to buy 400 ft^2 of tile. Is it possible for the rest of the flooring to be half wood and half carpet? Explain.

Flooring Sale!

Laminate: \$1.50/ft^2

Carpet: \$2/ft^2

Ceramic Tile: \$4/ft^2

Hardwood Flooring: \$6/ft^2

29. Critical Thinking Does moving forward and backward along a line represent two dimensions? Explain.

30. Write About It A friend calls you on the phone and asks you how to draw a three-dimensional coordinate system. What would you tell your friend?

MULTI-STEP TEST PREP

31. This problem will prepare you for the Multi-Step Test Prep on page 228.

Engineers use three-dimensional coordinates to design construction projects. An overhead light is anchored at the point $(7, 12, 10)$ in a design where the floor of a building is represented by the xy-plane and increments on the plane are in feet.

a. Lights are spaced 4 feet apart in each direction. What are the coordinates of the anchors for two other lights?

b. The light fixture will hang 1.5 feet below the anchor in the ceiling. What are the coordinates of the fixture?

c. What would the new coordinates of the light be if the engineers want to raise the ceiling 4 feet?

32. **ERROR ANALYSIS** Below are two methods of finding the x-intercept of $-5x + 3z = 15$. Which is incorrect? Explain the error.

A

$$-5x + 3z = 15$$
$$-5x + 3(0) = 15$$
$$-5x = 15$$
$$x = -3$$

B

$$-5x + 3z = 15$$
$$-5(0) + 3z = 15$$
$$3z = 15$$
$$z = 5$$

TEST PREP

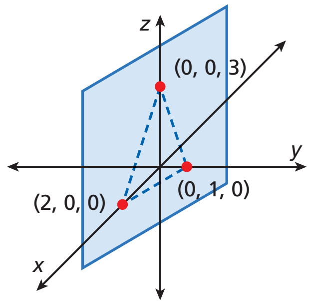
33. Which point is 5 units away from $(1, 1, 4)$?

Ⓐ $(-4, 1, 4)$ Ⓑ $(1, -4, 9)$ Ⓒ $(2, 3, 6)$ Ⓓ $(6, 6, 9)$

34. The graph of which equation is shown?

Ⓐ $x + 2y + 3z = 6$ Ⓒ $3x + 6y + 2z = 6$

Ⓑ $2x + y + 3z = 6$ Ⓓ $6x + 3y + 2z = 6$



35. Which point is located at the y-intercept of $2x - 4y + 3z = -12$?

Ⓕ $(0, -3, 0)$ Ⓗ $(0, 3, 0)$

Ⓖ $(0, 0, -3)$ Ⓙ $(3, 0, 0)$

36. **Gridded Response** Find the z-intercept of $5x - 2y - 4z = -3$.

CHALLENGE AND EXTEND

When a linear equation involves only two variables, its graph in three-dimensional space is a plane parallel to one of the axes. Graph each linear equation in three-dimensional space.

37. $x + y = 2$ **38.** $y - 2z = 4$ **39.** $x + z = 3$ **40.** $\frac{1}{2}x + \frac{1}{4}y = 1$

Write a linear equation in three dimensions with the indicated intercepts.

41. x-intercept $= 4$; y-intercept $= 2$; z-intercept $= -1$

42. x-intercept $= 25$; y-intercept $= 50$; z-intercept $= 10$

SPIRAL REVIEW

Name the three-dimensional solid with the given number of edges and vertices. *(Previous course)*

43. 5 vertices, 8 edges **44.** 6 vertices, 9 edges **45.** 0 vertices, 0 edges

46. **Fund-raising** The Wheels for Charity cycling club rode 1920 miles from Maryland to Montana to raise money for homeless shelters. Each day, the cyclists traveled about 120 miles. How many days of cycling did the trip take? *(Lesson 2-2)*

Use substitution or elimination to solve each system of equations. *(Lesson 3-2)*

47. $\begin{cases} 5y = x \\ \frac{2}{5}x + 7y = 18 \end{cases}$ **48.** $\begin{cases} 6x - y = 5 \\ 4y - 3x = 1 \end{cases}$ **49.** $\begin{cases} x + 3y = 6 \\ 2x - 3y = 9 \end{cases}$

Connecting Algebra to

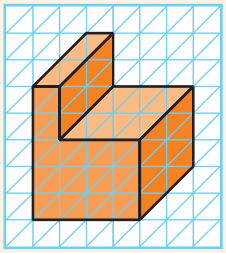
Geometry

See Skills Bank page S65

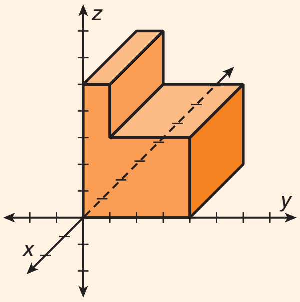
Views of Solid Figures

Here are four different ways to represent the same three-dimensional object.

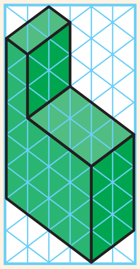
Oblique Grid



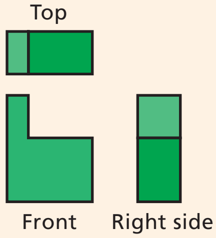
3-D Axes



Isometric Grid



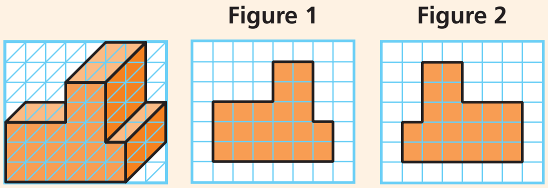
Orthographic Views



Example

Use graph paper. Show what this figure looks like from the back.

1. Draw the front of the figure. The answer is shown in figure 1.
2. The back view is the mirror image of the front view. Flip the front view to get the back view. The answer is shown in figure 2.



Try This

Use figures 3–6 for Problems 1–5.

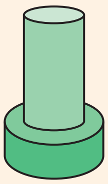


Figure 3

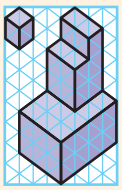


Figure 4

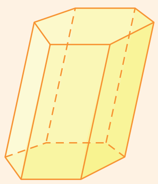


Figure 5

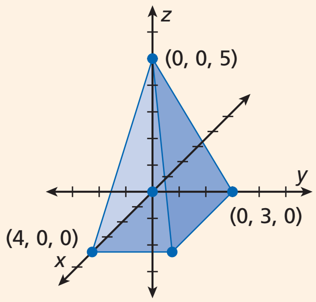


Figure 6

1. Figure 3 is made from two cylinders, one centered on top of the other. Draw front, top, and right-side views of figure 3.
2. In figure 4, the blue cube represents 1 cubic unit. How many of the blue cubes are needed to build the figure?
3. Draw front, top, and right-side views of figure 4.
4. Figure 5 shows an oblique prism. The sides do not make right angles with the bases. Describe the bases and lateral faces of this prism.
5. In figure 6, three of the vertices are labeled. What are the coordinates of the other two vertices? What is the name of this solid figure?

3-6 Solving Linear Systems in Three Variables

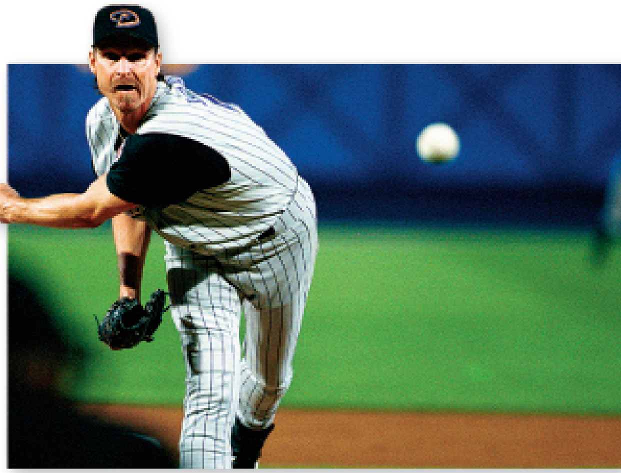
Objectives

Represent solutions to systems of equations in three dimensions graphically.

Solve systems of equations in three dimensions algebraically.

Why learn this?

You can use systems of equations in three variables to find out the scoring systems for sports awards. (See Example 2.)

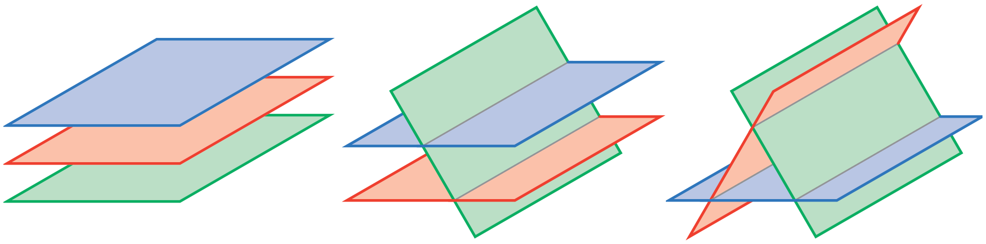


You have learned to solve systems of two equations with two variables, or 2-by-2 systems. Systems of three equations with three variables are often called 3-by-3 systems. In general, to find a single solution to *any* system of equations, you need as many equations as you have variables.

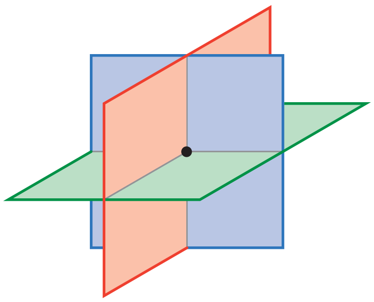
Recall from Lesson 3-5 that the graph of a linear equation in three variables is a plane. When you graph a system of three linear equations in three dimensions, the result is three planes that may or may not intersect. The solution to the system is the set of points where all three planes intersect. These systems may have one, infinitely many, or no solution.

Know it! Note

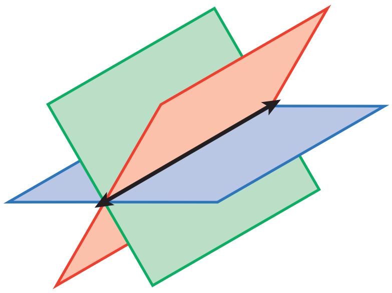
No Solutions
Inconsistent Systems



One Solution
Independent Systems



Infinitely Many Solutions
Dependent Systems



Identifying the exact solution from a graph of a 3-by-3 system can be very difficult. However, you can use the methods of elimination and substitution to reduce a 3-by-3 system to a 2-by-2 system and then use the methods that you learned in Lesson 3-2.

EXAMPLE 1 Solving a Linear System in Three Variables

Use elimination to solve the following system of equations.

$$\begin{cases} x + 2y - 3z = -2 & ❶ \\ 2x - 2y + z = 7 & ❷ \\ x + y + 2z = -4 & ❸ \end{cases}$$

Step 1 Eliminate one variable.

In this system, y is a reasonable choice to eliminate first because the coefficients of y are opposites in the equations ❶ and ❷.

$$\begin{array}{lrcl} ❶ & x + 2y - 3z &=& -2 \\ ❷ & \underline{2x - 2y + z} &\underline{=}& \underline{7} \\ & 3x - 2z &=& 5 \; ❹ \end{array}$$

Add equations ❶ and ❷.

Use equations ❶ and ❸ to create a second equation in x and z.

$$\begin{array}{lrcl} ❶ & x + 2y - 3z = -2 & & x + 2y - 3z = -2 \\ ❸ & \underline{-2(x + y + 2z = -4)} & \rightarrow & \underline{-2x - 2y - 4z = 8} \\ & & & -x - 7z = 6 \; ❺ \end{array}$$

Multiply equation ❸ by −2, and add to equation ❶.

You now have a 2-by-2 system. $\begin{cases} 3x - 2z = 5 & ❹ \\ -x - 7z = 6 & ❺ \end{cases}$

Step 2 Eliminate another variable. Then solve for the remaining variable.

You can eliminate x by using methods from Lesson 3-2.

$$\begin{array}{lrcl} ❹ & 3x - 2z = 5 & & 3x - 2z = 5 \\ ❺ & \underline{3(-x - 7z = 6)} & \rightarrow & \underline{-3x - 21z = 18} \\ & & & -23z = 23 \\ & & & z = -1 \end{array}$$

Multiply equation ❺ by 3, and add to equation ❹.

Solve for z.

Step 3 Use one of the equations in your 2-by-2 system to solve for x.

❺ $-x - 7z = 6$

$-x - 7(-1) = 6$ *Substitute −1 for z.*

$x = 1$ *Solve for x.*

Step 4 Substitute for x and z in one of the original equations to solve for y.

❸ $x + y + 2z = -4$

$(1) + y + 2(-1) = -4$ *Substitute 1 for x and −1 for z.*

$y = -3$ *Solve for y.*

The solution is $(1, -3, -1)$.

CHECK IT OUT!

1. Use elimination to solve the following system of equations.

$$\begin{cases} -x + y + 2z = 7 \\ 2x + 3y + z = 1 \\ -3x - 4y + z = 4 \end{cases}$$

You can also use substitution to solve a 3-by-3 system. Again, the first step is to reduce the 3-by-3 system to a 2-by-2 system.

EXAMPLE 2 Sports Application

In 2001, Randy Johnson of the Arizona Diamondbacks won Major League Baseball's Cy Young Award as the best pitcher in the National League. The winner is the pitcher who receives the most points, and a different number of points are given for each first-, second-, and third-place vote. The table shows the votes for the top three finishers. Find the number of points awarded for each vote.

Player	1st Place	2nd Place	3rd Place	Total Points
Randy Johnson	30	2	0	156
Curt Schilling	2	29	1	98
Matt Morris	0	1	28	31

Step 1 Let x represent the number of points for a first-place vote, y for a second-place vote, and z for a third-place vote.

Write a system of equations to represent the data in the table.

$$\begin{cases} 30x + 2y = 156 & (1) \\ 2x + 29y + z = 98 & (2) \\ y + 28z = 31 & (3) \end{cases}$$

Randy Johnson's votes
Curt Schilling's votes
Matt Morris's votes

Some variables are "missing" in the equations; however, the same solution methods apply. Substitution is a good choice because solving for y is straightforward.

Step 2 Solve for y in equation (1).

$$(1)\quad 30x + 2y = 156$$

$$y = -15x + 78 \qquad \textit{Solve for y.}$$

Step 3 Substitute for y in equations (2) and (3).

$$\begin{cases} 2x + 29(-15x + 78) + z = 98 & (2) \\ (-15x + 78) + 28z = 31 & (3) \end{cases} \qquad \textit{Substitute } -15x + 78 \textit{ for y.}$$

$$\begin{cases} -433x + z = -2164 & (4) \\ -15x + 28z = -47 & (5) \end{cases} \qquad \textit{Simplify to find a 2-by-2 system.}$$

Step 4 Solve equation (4) for z.

$$(4)\quad -433x + z = -2164$$

$$z = 433x - 2164 \qquad \textit{Solve for z.}$$

Step 5 Substitute for z in equation (5).

$$(5)\quad -15x + 28(433x - 2164) = -47 \qquad \textit{Substitute } 433x - 2164 \textit{ for z.}$$

$$12{,}109x = 60{,}545$$

$$x = 5 \qquad \textit{Solve for x.}$$

Step 6 Substitute for x to solve for z and then for y.

$$\begin{aligned} (4)\quad z &= 433x - 2164 \\ z &= 433(5) - 2164 \\ z &= 1 \end{aligned} \qquad \begin{aligned} (3)\quad y + 28z &= 31 \\ y + 28(1) &= 31 \\ y &= 3 \end{aligned}$$

The solution to the system is $(5, 3, 1)$. So, a first-place vote is worth 5 points, a second-place vote is worth 3 points, and a third-place vote is worth 1 point.

CHECK IT OUT!

2. Jada's chili won first prize at the winter fair. The table shows the results of the voting.

How many points are first-, second-, and third-place votes worth?

Winter Fair Chili Cook-off				
Name	**1st Place**	**2nd Place**	**3rd Place**	**Total Points**
Jada	3	1	4	15
Maria	2	4	0	14
Al	2	2	3	13

The systems in Examples 1 and 2 have unique solutions. However, 3-by-3 systems may have no solution or an infinite number of solutions.

EXAMPLE 3 Classifying Systems with Infinitely Many Solutions or No Solution

Classify the system as consistent or inconsistent, and determine the number of solutions.

$$\begin{cases} 4x - 2y + 4z = 8 & ❶ \\ -3x + y - z = -4 & ❷ \\ -2x + 2y - 6z = 4 & ❸ \end{cases}$$

The elimination method is convenient because the numbers you need to multiply the equations by are small. First, eliminate y.

$$\begin{array}{lrl} ❶ & 4x - 2y + 4z = 8 & \\ ❸ & \underline{-2x + 2y - 6z = 4} & \\ & 2x \qquad - 2z = 12 & ❹ \end{array}$$

Add equations ❶ and ❸.

$$\begin{array}{rcl} 4x - 2y + 4z = 8 & & 4x - 2y + 4z = 8 \\ \underline{2(-3x + y - z = -4)} & \rightarrow & \underline{-6x + 2y - 2z = -8} \\ & & -2x \qquad + 2z = 0 \quad ❺ \end{array}$$

Multiply equation ❷ by 2, and add to equation ❶.

You now have a 2-by-2 system: $\begin{cases} 2x - 2z = 12 & ❹ \\ -2x + 2z = 0 & ❺ \end{cases}$

Eliminate x.

$$\begin{array}{lr} ❹ & 2x - 2z = 12 \\ ❺ & \underline{-2x + 2z = 12} \\ & 0 = 12 \ ✘ \end{array}$$

Add equations ❹ and ❺.

Because 0 is never equal to 12, the equation is a contradiction. Therefore, the system is inconsistent and has no solution.

Remember!

Consistent means that the system of equations has at least one solution.

CHECK IT OUT!

Classify the system, and determine the number of solutions.

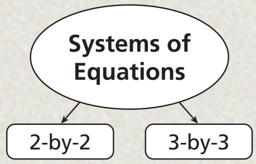
3a. $\begin{cases} 3x - y + 2z = 4 \\ 2x - y + 3z = 7 \\ -9x + 3y - 6z = -12 \end{cases}$

3b. $\begin{cases} 2x - y + 3z = 6 \\ 2x - 4y + 6z = 10 \\ y - z = -2 \end{cases}$

THINK AND DISCUSS

1. Look at the inconsistent and dependent systems shown on page 220. Describe one other arrangement of three planes that results in an inconsistent system. Describe one other arrangement that results in a dependent system.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the similarities and differences between 2-by-2 and 3-by-3 systems.



3-6 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 221

Use elimination to solve each system of equations.

1. $\begin{cases} -2x + y + 3z = 20 \\ -3x + 2y + z = 21 \\ 3x - 2y + 3z = -9 \end{cases}$

2. $\begin{cases} x + 2y + 3z = 9 \\ x + 3y + 2z = 5 \\ x + 4y - z = -5 \end{cases}$

3. $\begin{cases} x + 2y + z = 8 \\ 2x + y - z = 4 \\ x + y + 3z = 7 \end{cases}$

SEE EXAMPLE 2 p. 222

4. **Business** Mabel's Mini-Golf has different prices for seniors, adults, and children. The table shows the total revenue for three hours on a particular night. How much does each type of ticket cost?

Mabel's Mini-Golf Prices

Time	Senior	Adult	Child	Revenue
6:00 P.M.–7:00 P.M.	5	10	12	\$310
7:00 P.M.–8:00 P.M.	5	5	4	\$155
8:00 P.M.–9:00 P.M.	4	2	3	\$92

SEE EXAMPLE 3 p. 223

Classify each system as consistent or inconsistent, and determine the number of solutions.

5. $\begin{cases} 2x + 4y - 2z = 4 \\ -x - 2y + z = 4 \\ 3x + 6y - 3z = 10 \end{cases}$

6. $\begin{cases} 2x + 4y - 5z = -10 \\ -x - 2y + 8z = 16 \\ -2x + 4y + 2z = 4 \end{cases}$

7. $\begin{cases} -2x + 3y + z = 15 \\ x + 3y - z = -1 \\ -5x - 6y + 4z = -16 \end{cases}$

PRACTICE AND PROBLEM SOLVING

Use elimination to solve each system of equations.

8. $\begin{cases} 2x - y - 3z = 1 \\ 4x + 3y + 2z = -4 \\ -3x + 2y + 5z = -3 \end{cases}$

9. $\begin{cases} 5x - 6y + 2z = 21 \\ 2x + 3y - 3z = -9 \\ -3x + 9y - 4z = -24 \end{cases}$

10. $\begin{cases} 4x + 7y - z = 42 \\ -2x + 2y + 3z = -26 \\ 2x - 3y + 5z = 10 \end{cases}$

Independent Practice	
For Exercises	See Example
8–10	1
11	2
12–14	3

Extra Practice

Skills Practice p. S9
Application Practice p. S34

11. Entertainment On the *Star Quality* show, judges score contestants in three categories: Talent, Presentation, and Star Quality. Each category is worth a percent of the final score. Based on the scores in the table below, what percent of the final score is each category worth?

Star Quality Scores

Contestant	Talent	Presentation	Star Quality	Final Score
Wanda Wynn	8	9	10	9.2
Amiya Starr	9	7	8	8.1
Kenny Singh	6	10	8	7.8

Classify each system as consistent or inconsistent, and determine the number of solutions.

12. $\begin{cases} 4x - 3y + z = -9 \\ -3x + 2y - z = 6 \\ -x + 3y + 2z = 9 \end{cases}$

13. $\begin{cases} 3x + 3y + 3z = 4 \\ 2x - y - 5z = 2 \\ 5x + 2y - 2z = 8 \end{cases}$

14. $\begin{cases} -x + y + z = 8 \\ 2x - 2y - 2z = -16 \\ 2x - y + 4z = -6 \end{cases}$

15. Geometry In triangle *ABC*, the measure of angle *A* is twice the sum of the measures of angles *B* and *C*. The measure of angle *B* is three times the measure of angle *C*. What are the measures of the angles?

16. Sports Louie Dampier was the leading scorer in the history of the American Basketball Association (ABA). His 13,726 points were scored on three-point baskets, two-point baskets, and one-point free throws. In his ABA career, Dampier made 2144 more two-point baskets than free throws and 1558 more free throws than three-point baskets. How many three-point baskets, two-point baskets, and free throws did Dampier make?

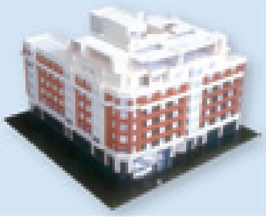
17. Critical Thinking The following system of equations has three variables and two equations. $\begin{cases} x + 2y + 4z = 4 \\ 2x + 3y + z = 12 \end{cases}$

a. Describe what happens when you attempt to solve this system.

b. Explain why a system of equations must have at least as many equations as there are variables to have a single solution.

18. Write About It The graphs of two equations in a 3-by-3 system intersect in a line. What types of solutions could the system have? Explain.

MULTI-STEP TEST PREP



19. This problem will prepare you for the Multi-Step Test Prep on page 228.

The roof lines of a building can be described by the system of equations

$\begin{cases} x + y + z = 53 \\ 3x - 2y + z = 69 \\ -x + 2y - z = -59 \end{cases}$, where the floor is represented by the *xy*-plane and measurements are in feet.

a. Find the point of intersection of the roof lines.

b. A support column will be placed under the intersection point. How tall must the column be to reach from the floor to the intersection point?

c. What are the coordinates of the center of the base of the column?

TEST PREP

20. Which point is the solution to this system of equations? $\begin{cases} 2x + y + 3z = -1 \\ 4x + 2y + 3z = 1 \\ x - y + 4z = -6 \end{cases}$

Ⓐ $(2, -2, -1)$ Ⓑ $(0, 2, -1)$ Ⓒ $(2, 1, -1)$ Ⓓ $(3, -2, 2)$

21. Ann, Betty, and Charlotte are sisters. Ann is twice as old as Betty, and Betty is 12 years younger than Charlotte. In 5 years, Charlotte will be twice as old as Betty. What are the sisters' ages?

Ⓕ Ann is 6, Betty is 3, and Charlotte is 15.

Ⓖ Ann is 34, Betty is 17, and Charlotte is 29.

Ⓗ Ann is 5, Betty is 10, and Charlotte is 22.

Ⓙ Ann is 14, Betty is 7, and Charlotte is 19.

22. **Short Response** What is the value of the x-coordinate of the solution to the following system of equations? $\begin{cases} x + 4y = 6 \\ 2x + 3z = 12 \\ 4y + z = 10 \end{cases}$

CHALLENGE AND EXTEND

23. Use any method to solve the following 4-by-4 system. $\begin{cases} w + 2x + 2y + z = -2 \\ w + 3x - 2y - z = -6 \\ -2w - x + 3y + 3z = 6 \\ w + 4x + y - 2z = -14 \end{cases}$

24. **Economics** Three investors each put \$1000 into their retirement accounts. They had three funds to choose from—fund A, fund B, and fund C. Each investor divided the money differently, as shown in the table below. The table also shows the gain for each investor for the year. Find the yield in percents for each fund.

Investor	Fund A	Fund B	Fund C	Gain
M. Nguyen	\$300	\$300	\$400	\$56
A. O'Sullivan	\$600	\$200	\$200	\$76
T. Lane	\$100	\$300	\$600	\$30

SPIRAL REVIEW

Perform the given translation of the point $(-3, 2)$, and give the coordinates of the translated point. *(Lesson 1-8)*

25. 6 units right and 1 unit up

26. 4 units left and 2 units down

27. **Construction** The blueprint for a house showed the kitchen as 11 cm by 8 cm. If the blueprint is drawn to a 1 cm : 0.65 m scale, what are the dimensions of the kitchen in the house? *(Lesson 2-2)*

Write each equation in slope-intercept form, and then graph. *(Lesson 2-3)*

28. $4x - 3y = -6$

29. $3y - 2x = -12$

30. $2x + 5y = 15$

3-6 Technology LAB

Explore Parametric Equations

Use with Lesson 3-6

A set of *parametric equations* is a system of two equations with the same independent variable, usually t. The independent variable t is called the *parameter*.

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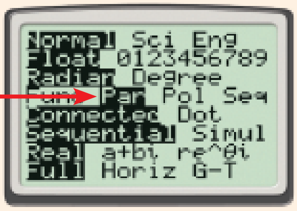
KEYWORD: MB7 Lab3

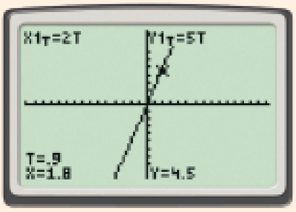
Activity

Follow the steps to graph the following parametric equations. $\begin{cases} x = 2t \\ y = 5t \end{cases}$

Press MODE, and change the graphing mode to **PAR** (for parametric). Press Y=. Enter $2t$ for the first equation (X_{1T}) and $5t$ for the second equation (Y_{1T}). To enter the variable t, use the X,T,θ,n key. Graph the equations.



You can use the TRACE key to view values of both equations and t. You can change the look of the graph and the values by adjusting the WINDOW settings.



Try This

1. Consider $\begin{cases} x = 2t \\ y = \frac{1}{2}t \end{cases}$.

a. Graph the relation and estimate the slope of the line created by the graph. Use the following window settings: **Tmin = −10**, **Tmax = 10**, and **Tstep = 0.1**, and use the square viewing window.

b. **Critical Thinking** How does the slope of the line relate to the slope of the two parametric equations?

2. An airplane is traveling at a constant horizontal speed of 500 ft/s and is ascending at a constant rate of 50 ft/s. A model of the path of the airplane is given parametrically by $\begin{cases} x = 500t \\ y = 50t \end{cases}$ for t s.

a. Graph the path of the airplane for $t = 20$ to $t = 180$ s and identify the location of the plane after 50 s.

b. When will the plane reach an altitude of 5000 ft? (*Hint:* You may have to adjust your viewing window.)

3. **Critical Thinking** Create a graph of the line $y = x$ by using parametric equations.

4. **Extension** Consider $\begin{cases} x = t^2 \\ y = t \end{cases}$.

a. Graph the relation. Use the following window settings: **Tmin = −10**, **Tmax = 10**, and **Tstep = 0.1**, and use the square viewing window.

b. Describe the relation and write the formula in terms of x and y only. Is the relation a function?

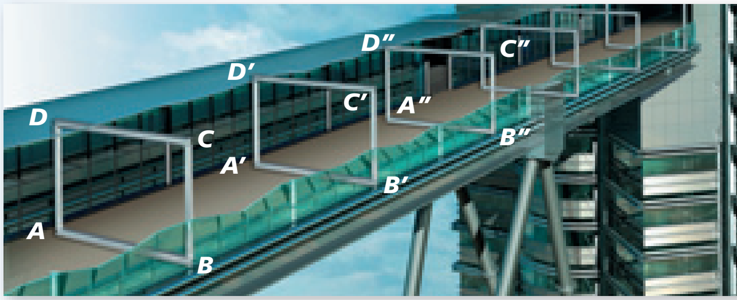
CHAPTER 3

SECTION 3B

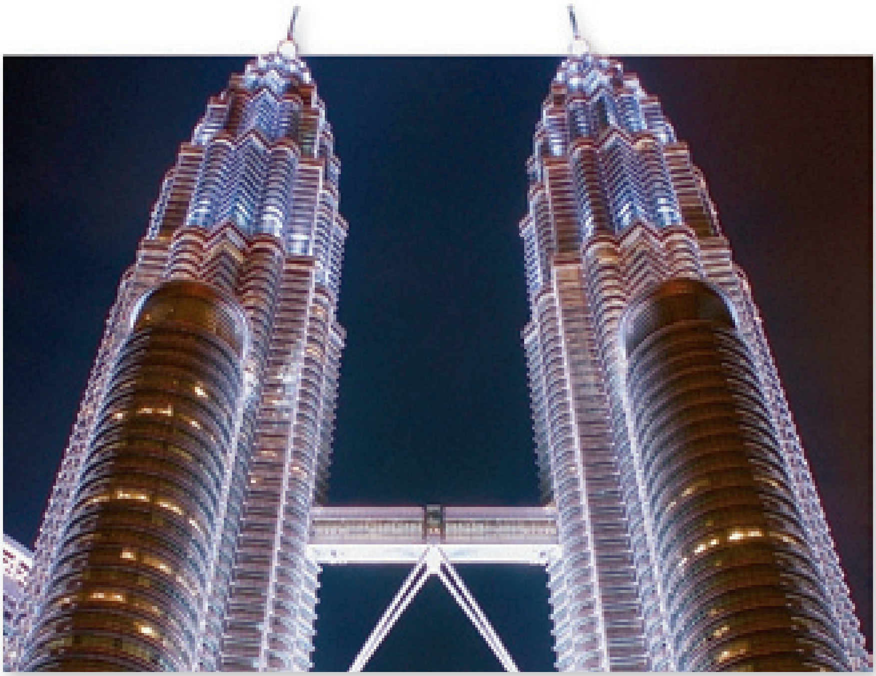
MULTI-STEP TEST PREP

Linear Equations in Three Dimensions

Building Bridges A bridge is to be constructed between two buildings. The cross sections of the bridge are 12 steel rectangles 16 feet wide by 11 feet high. The bridge will be represented by a three-dimensional model.



1. The coordinates of the vertices defining the first cross section are $A(0, 0, 0)$, $B(0, 16, 0)$, $C(0, 16, 11)$, and $D(0, 0, 11)$. What are the coordinates of the points A', B', C', and D' if the rectangular cross sections are 30 feet away from each other?
2. What are the coordinates of the points A'', B'', C'', and D''?
3. A supervisor changes the sketch so that point A has coordinates $(150, 0, -16)$. What are the new coordinates for points B, C, and D? What are the new coordinates for points A', B' C', and D'? What about A'', B'', C'', and D''?
4. The steel cross sections will be rigged with sliding metal fire doors that will close in case of emergency. What is the area covered by one of these doors?
5. Due to some budget restrictions, the bridge has to be reduced to a width of 14 feet and a height of 9 feet. What will be the coordinates of points A, B, and D if point C is defined as the origin $(0, 0, 0)$?



READY TO GO ON?

CHAPTER 3

SECTION 3B

Quiz for Lessons 3-5 Through 3-6

3-5 Linear Equations in Three Dimensions

Graph each point in three-dimensional space.

1. $(-3, 2, 1)$ **2.** $(2, -3, -2)$ **3.** $(3, 1, -3)$

Graph each linear equation in three-dimensional space.

4. $2x - 2y + 4z = 8$ **5.** $2x + y - 2z = -4$ **6.** $x + 5y + 3z = 15$

Finance Use the following information and the table for Problems 7 and 8. A dental office charges \$50 for teeth cleanings, \$100 for performing a one-surface filling, and \$75 for an initial visit with X rays. The dental office's total income was exactly \$3500 for each of the days shown in the table.

Day	Cleaning	Filling	Initial Visit
Monday	20	22	■
Tuesday	■	18	6
Wednesday	16	■	4
Thursday	25	21	■

7. Write a linear equation in three variables to represent this situation.

8. Complete the table for the possible numbers of appointments each day.

3-6 Solving Linear Systems in Three Variables

Use elimination to solve each system of equations.

9. $\begin{cases} x + y + z = 0 \\ 2x + y - 2z = -8 \\ -x + 4z = 10 \end{cases}$ **10.** $\begin{cases} x + 2y + z = 7 \\ x - 2y - 4z = 0 \\ 2x - y + 4z = -3 \end{cases}$ **11.** $\begin{cases} 2x + 2y + z = 10 \\ x - 2y + 3z = 13 \\ x - y + 3z = 12 \end{cases}$

Business Use the following information and the table for Problems 12 and 13. JoJo's Pretzel Stand has three different types of pretzels, indicated by type A, type B, and type C. The table shows the total revenue for three hours on a particular afternoon.

Time	Type A	Type B	Type C	Revenue
2:00 P.M.–3:00 P.M.	6	8	14	\$65
3:00 P.M.–4:00 P.M.	10	10	15	\$80
4:00 P.M.–5:00 P.M.	12	6	9	\$60

12. Write a system in three variables to represent the data in the table.

13. How much does each type of pretzel cost?

Classify each system as consistent or inconsistent, and determine the number of solutions.

14. $\begin{cases} 2x - 2y + 3z = -2 \\ 4y + 6z = 1 \\ 4x - 4y + 6z = 5 \end{cases}$ **15.** $\begin{cases} 4x - y + z = 5 \\ 3x + y + 2z = 5 \\ 2x - 5z = -8 \end{cases}$ **16.** $\begin{cases} 2x + y - 3z = 4 \\ x - 3y + z = -8 \\ -x + 3y - z = 8 \end{cases}$

EXTENSION Parametric Equations

Objectives
Graph parametric equations, and use them to model real-world applications.

Write the function represented by a pair of parametric equations.

Vocabulary
parameter
parametric equations

As an airplane ascends after takeoff, its altitude increases at a rate of 45 ft/s while its distance on the ground from the airport increases at 210 ft/s.

Both of these rates can be expressed in terms of time. When two variables, such as x and y, are expressed in terms of a third variable, such as t, the third variable is called a **parameter**. The equations that define this relationship are **parametric equations**.

EXAMPLE 1 Writing and Graphing Parametric Equations

As an airplane ascends after takeoff, its altitude increases at a rate of 45 ft/s while its distance on the ground from the airport increases at 210 ft/s.

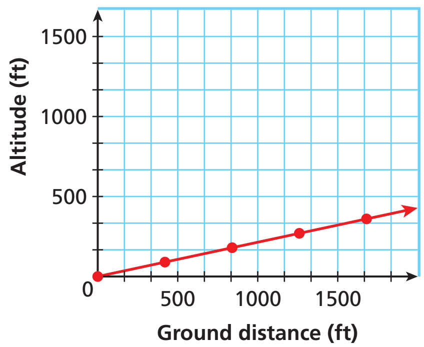
A **Write parametric equations to model the location of the plane described above. Then graph the equations on a coordinate grid.**

Using the horizontal and vertical speeds given above, write equations for the ground distance x and altitude y in terms of t.

$$\begin{cases} x = 210t \\ y = 45t \end{cases}$$ *Use the distance formula $d = rt$*

Make a table to help you draw the graph. Use different t-values to find x- and y-values. The x and y rows give the points to plot.

t	0	2	4	6	8
x	0	420	840	1260	1680
y	0	90	180	270	360



Plot and connect $(0, 0)$, $(420, 90)$, $(840, 180)$, $(1260, 270)$, and $(1680, 360)$.
The graph is shown at right.

B **Find the location of the airplane 15 seconds after takeoff.**

$x = 210t = 210(15) = 3150$
$y = 45t = 45(15) = 675$ *Substitute $t = 15$*

At $t = 15$, the airplane has a ground distance of 3150 feet from the airport and an altitude of 675 feet.

CHECK IT OUT! **A helicopter takes off with a horizontal speed of 5 ft/s and a vertical speed of 20 ft/s**

1a. Write equations for and draw a graph of the motion of the helicopter.

1b. Describe the location of the helicopter at $t = 10$ seconds.

You can use parametric equations to write a function that relates the two variables by using the substitution method.

EXAMPLE 2 Writing Functions Based on Parametric Equations

Use the data from Example 1 to write an equation for the airplane's altitude *y* in terms of ground distance *x*.

Solve one of the two parametric equations for *t*. Then substitute to get one equation whose variables are *x* and *y*.

$x = 210t$, so $\frac{x}{210} = t$ — *Solve for t in the first equation.*

$y = 45t$ — *Second equation*

$y = 45\left(\frac{x}{210}\right) = \frac{3}{14}x$ — *Substitute and simplify.*

$y = \frac{3}{14}x$

The equation for the airplane's altitude in terms of ground distance is $y = \frac{3}{14}x$.

CHECK IT OUT! **Recall that the helicopter in Check It Out Problem 1 takes off with a horizontal speed of 5 ft/s and a vertical speed of 20 ft/s.**

2. Write an equation for the helicopter's motion in terms of only *x* and *y*.

EXTENSION Exercises

Draw a graph to represent each set of parametric equations.

1. $\begin{cases} x = 4t \\ y = 2t \end{cases}$ **2.** $\begin{cases} x = t - 2 \\ y = 4t \end{cases}$ **3.** $\begin{cases} x = \frac{t}{4} \\ y = -3t \end{cases}$ **4.** $\begin{cases} x = 20t \\ y = 10t + 10 \end{cases}$

Write one equation for each set of parametric equations in terms of only *x* and *y*.

5. $\begin{cases} x = 3t \\ y = 2t \end{cases}$ **6.** $\begin{cases} x = 2t + 4 \\ y = 5t \end{cases}$ **7.** $\begin{cases} x = \frac{3}{5}t \\ y = 6t \end{cases}$ **8.** $\begin{cases} x = 7.5t \\ y = 20t + 2 \end{cases}$

9. **Oceanography** Suppose a research submarine descends from the surface with a horizontal speed of 1.8 m/s and a vertical speed of 0.9 m/s.

a. Write equations for and draw a graph of the motion of the submarine.

b. Find the depth of the submarine after 50 s.

c. Find the submarine's depth after 1 day. Does this answer make sense? Explain.

10. **Hiking** From her starting point, a hiker walks along a straight path. Her north-south speed is 3 mi/h (to the north), and her east-west speed is 0.4 mi/h (to the east). Let *x* represent how far east of her starting point the hiker is, and let *y* represent how far north she is. Write an equation for her motion in terms of only *x* and *y*. Find the location of the hiker when $x = 2$.

CHAPTER 3

Study Guide: Review

Vocabulary

consistent system 183
constraint 205
dependent system 184
elimination 191
feasible region 205
inconsistent system 183
independent system 184
linear programming 205
linear system 182
objective function 206
ordered triple 214
parameter 230
parametric equations 230
substitution 190
system of equations 182
system of linear inequalities 199
three-dimensional coordinate system 214
z-axis 214

Complete the sentences below with vocabulary words from the list above.

1. A consistent and ___?___ system has infinitely many solutions.

2. ___?___ involves adding or subtracting equations to get rid of one of the variables in a system.

3. In a linear programming problem, the solution to the ___?___ can be graphed as a(n) ___?___.

4. Each point in a(n) ___?___ can be represented by a(n) ___?___.

5. A(n) ___?___ system is a set of equations or inequalities that has at least one solution.

3-1 Using Graphs and Tables to Solve Linear Systems *(pp. 182–189)*

EXAMPLES

■ **Solve $\begin{cases} x + y = 3 \\ 3x - 6y = -9 \end{cases}$ by using a graph and a table.**

Solve each equation for y.

$$\begin{cases} y = -x + 3 \\ y = \frac{1}{2}x + \frac{3}{2} \end{cases}$$

Make a table of values.

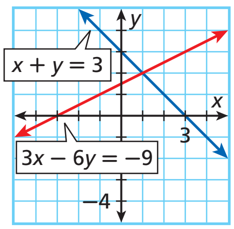
$y = -x + 3$

x	y
0	3
1	2
4	1

$y = \frac{1}{2}x + \frac{3}{2}$

x	y
0	1.5
1	2
4	2.5

Graph the lines.



The solution is (1, 2).

EXERCISES

Solve each system by using a graph and a table.

6. $\begin{cases} y = 2x \\ 3x - y = 5 \end{cases}$

7. $\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$

8. $\begin{cases} x - 6y = 2 \\ 2x - 5y = -3 \end{cases}$

9. $\begin{cases} x - 3y = 6 \\ 3x - y = 2 \end{cases}$

Classify each system and determine the number of solutions.

10. $\begin{cases} y = x - 7 \\ x + 9y = 16 \end{cases}$

11. $\begin{cases} \frac{1}{2}x + 2y = 3 \\ x + 4y = 6 \end{cases}$

12. $\begin{cases} 5x - 10y = 8 \\ x - 2y = 4 \end{cases}$

13. $\begin{cases} 4x - 3y = 21 \\ 2x - 2y = 10 \end{cases}$

14. **Security** A locksmith charges \$25 to make a house call and \$15 for each lock that is re-keyed. Another locksmith charges \$10 to make a house call and \$20 for each lock that is re-keyed. For how many locks will the total costs be the same?

3-2 Using Algebraic Methods to Solve Linear Systems *(pp. 190–197)*

EXAMPLES

■ **Use substitution to solve** $\begin{cases} y = x + 6 \\ 4x - 5y = -18 \end{cases}$.

$4x - 5(x + 6) = -18$ *Substitute for y.*

$4x - 5x - 30 = -18 \rightarrow x = -12$

Substitute the x-value into either equation.

$y = x + 6 \rightarrow y = (-12) + 6 \rightarrow y = -6$

The solution to the system is $(-12, -6)$.

■ **Use elimination to solve** $\begin{cases} 7x - 2y = 2 \\ 3x + 4y = 30 \end{cases}$.

Multiply the first equation by 2 to eliminate y.

$$\begin{cases} 7x - 2y = 2 \\ 3x + 4y = 30 \end{cases} \rightarrow \begin{array}{c} 2(7x - 2y = 2) \\ 3x + 4y = 30 \end{array} \rightarrow \begin{array}{r} 14x - 4y = 4 \\ 3x + 4y = 30 \\ \hline 17x \quad = 34 \\ x \quad = 2 \end{array}$$

Add the equations.
First part of the solution

Substitute the x-value into either equation.

$3x + 4y = 30 \rightarrow 3(2) + 4y = 30$

$\rightarrow y = 6$ *Second part of the solution*

The solution to the system is $(2, 6)$.

EXERCISES

Use substitution to solve each system of equations.

15. $\begin{cases} y = 3x \\ 2x - 3y = -7 \end{cases}$

16. $\begin{cases} y = x - 1 \\ 4x - y = 19 \end{cases}$

17. $\begin{cases} 4x - y = 0 \\ 6x - 3y = 12 \end{cases}$

18. $\begin{cases} 5x = -10y \\ 8x - 4y = 40 \end{cases}$

Use elimination to solve each system of equations.

19. $\begin{cases} 4x + 5y = 41 \\ 7x + 5y = 53 \end{cases}$

20. $\begin{cases} -4x - y = -16 \\ -4x - 5y = -32 \end{cases}$

21. $\begin{cases} 2x - y = 8 \\ x + 2y = 9 \end{cases}$

22. $\begin{cases} 9x - 5y = 13 \\ 4x - 6y = 2 \end{cases}$

23. Mixtures A popular mixture of potpourri includes pine needles and lavender. If pine needles cost \$1.50 per ounce and lavender costs \$4.00 per ounce, how much of each ingredient should be mixed to make 80 oz of the potpourri that is worth \$200?

3-3 Solving Systems of Linear Inequalities *(pp. 199–204)*

EXAMPLE

■ **The combined annual sales for a company's two divisions was almost \$12 million. One of the divisions accounted for at least 75% of the total sales. Write and graph a system of inequalities that can be used to determine the possible combinations of sales for both divisions of the company.**

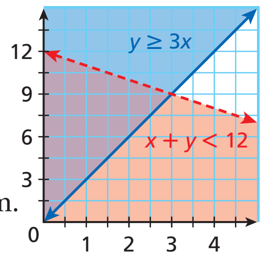
Let x be one division, and let y be the other division with 75% of the sales.

Write the system of inequalities.

$\begin{cases} x + y < 12 \\ y \ge 0.75(x + y) \end{cases} \rightarrow \begin{cases} x + y < 12 \\ y \ge 3x \end{cases}$ *dashed line* / *solid line*

Graph the boundary lines, and shade accordingly. Notice also that $x > 0$ and $y > 0$.

The overlapping region is the solution for the system.



EXERCISES

Graph each system of inequalities.

24. $\begin{cases} y + 1 > 4x \\ y \le x + 1 \end{cases}$

25. $\begin{cases} y - 3x < 3 \\ 3y \ge x + 3 \end{cases}$

Graph the system of inequalities and classify the figure created by the solution region.

26. $\begin{cases} y \le -x + 2 \\ x > -1 \\ y > -1 \end{cases}$

27. $\begin{cases} y \ge 2x \\ y < 4 \\ y > 2 \\ y \le \frac{1}{2}x + 4 \end{cases}$

28. Business A coffee shop wants to make a maximum of 120 lb of a coffee mixture that costs less than \$10/lb. The shop will mix coffee that is sold at \$8/lb with coffee sold at \$11.50/lb. Write and graph a system of inequalities that shows the possible mixtures of the two coffee types.

3-4 Linear Programming *(pp. 205–211)*

EXAMPLE

■ **A café sells cold sandwiches and hot entrées. The range of items sold is shown in the table. The café has never sold more than a total of 125 sandwiches and entrées in one day. If the café makes a profit of \$0.75 on each sandwich and \$1 on each hot entrée, how many of each item would maximize the café profit?**

Menu Item	Minimum Sold	Maximum Sold
Cold sandwiches	60	80
Hot entrées	40	60

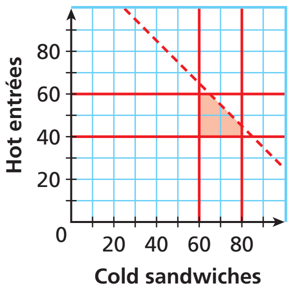
Let x be the number of cold sandwiches, and let y be the number of hot entrées.

Write the constraints.

$$\begin{cases} 60 \le x \le 80 & \textit{Number of sandwiches} \\ 40 \le y \le 60 & \textit{Number of hot entrées} \\ x + y < 125 & \textit{Number of items sold} \end{cases}$$

Graph the feasible region and identify vertices.

The feasible region has five vertices at $(60, 40)$, $(60, 60)$, $(65, 60)$, $(80, 45)$, and $(80, 40)$.



Write the objective function.

The objective function is $P = 0.75x + y$.
$P(0, 0) = 18(0) + 25(0) = 0$

Evaluate the objective function at each vertex.

$P(60, 40) = 0.75(60) + 40 = 85$
$P(60, 60) = 0.75(60) + 60 = 105$
$P(65, 60) = 0.75(65) + 60 = 108.75$
$P(80, 45) = 0.75(80) + 45 = 105$
$P(80, 40) = 0.75(80) + 40 = 100$

The objective function is maximized at $(65, 60)$. The maximum profit of \$108.75 is obtained when 65 cold sandwiches and 60 hot entrées are sold.

EXERCISES

Graph each feasible region.

29. $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 3x + 1 \\ y \le -\frac{3}{4}x + 6 \end{cases}$

30. $\begin{cases} x < 3 \\ y \ge 0 \\ y < 2x + 1 \\ y \le -x + 4 \end{cases}$

31. $\begin{cases} x > 0 \\ y < 0 \\ y > \frac{1}{2}x - 6 \end{cases}$

32. $\begin{cases} x \le 2 \\ y \ge -1 \\ x \ge -1 \\ y \le -x + 3 \end{cases}$

Maximize or minimize each objective function.

33. Maximize $P = 6x + 10y$ for the constraints from Exercise 29.

34. Minimize $P = 14x + 9y$ for the constraints from Exercise 30.

Manufacturing A shoe insole company produces two models of insoles: an extra thick insole for sports shoes and a thinner insole for dress shoes. The thick insole requires 6 min of manufacturing time and generates a profit of \$8. The thin insole requires 4 min of manufacturing time and generates a profit of \$9. The manufacturing line runs at most 12 h a day, or 720 min. Because of demand, the company manufactures at least twice as many thick insoles as thin insoles.

35. Write the constraints, and graph the feasible region.

36. Write the objective function for the company's profit.

37. What is the maximum profit that can be generated in one day?

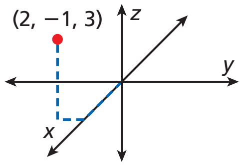
38. **Sales** Each day, a cell phone stand sells between 10 and 25 cell phones with new service contracts, and between 5 and 10 cell phones without contracts. The stand never sells more than 30 new cell phones per day. The cell phone stand makes a commission of \$35 for each phone with a contract and \$5 for each phone without a contract. How many of each option would maximize the stand's profit?

3-5 Linear Equations in Three Dimensions *(pp. 214–218)*

EXAMPLES

- **Graph (2, −1, 3) in three-dimensional space.**

From the origin, move 2 units forward along the x-axis, 1 unit left, and 3 units up.



- **Graph the linear equation $3x + 6y - z = -6$ in three-dimensional space.**

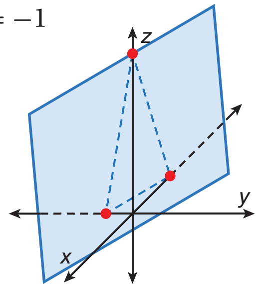
Find the intercepts.

x-intercept: $3x = -6 \rightarrow x = -2$

y-intercept: $6y = -6 \rightarrow y = -1$

z-intercept: $-z = -6 \rightarrow z = 6$

Plot the points $(-2, 0, 0)$, $(0, -1, 0)$, and $(0, 0, 6)$. Sketch a plane through the three points.



EXERCISES

Graph each point in three-dimensional space.

39. $(-1, 0, 3)$
40. $(2, -2, 1)$
41. $(0, -1, 1)$
42. $(3, 1, 0)$

Graph each linear equation in three-dimensional space.

43. $x - 3y + 2z = 6$
44. $2x - 4y - 2z = 4$
45. $-x + y - 5z = 5$
46. $3x + 2y + z = -6$

47. Consumer Economics Lee has \$35 to purchase a combination of drinks, pizza, and ice cream for a party. Each drink costs \$2, each pizza costs \$9, and each quart of ice cream costs \$4. Write a linear equation in three variables to represent this situation.

3-6 Solving Linear Systems in Three Variables *(pp. 220–226)*

EXAMPLES

- **Use elimination to solve** $\begin{cases} 3x + 2y - z = -1 \\ x + 3y - z = -10 \\ 2x - y - 3z = -3 \end{cases}$.

First, eliminate z to obtain a 2-by-2 system.

$$\begin{array}{r} 3x + 2y - z = -1 \\ x + 3y - z = -10 \\ \hline 2x - y \quad = 9 \end{array} \qquad \begin{array}{r} 3(x + 3y - z = -10) \\ 2x - y - 3z = -3 \\ \hline x + 10y \quad = -27 \end{array}$$

The resulting 2-by-2 system is $\begin{cases} 2x - y = 9 \\ x + 10y = -27 \end{cases}$.

Eliminate x.

$$\begin{array}{r} 2x - \quad y = 9 \\ -2(x + 10y = -27) \\ \hline -21y = 63 \rightarrow y = -3 \end{array}$$

Substitute to solve for x and then z.

$2x - y = 9 \rightarrow 2x - (-3) = 9 \rightarrow x = 3$

$3x + 2y - z = -1 \rightarrow 3(3) + 2(-3) - z = -1 \rightarrow z = 4$

The solution to the system is $(3, -3, 4)$.

EXERCISES

Use elimination to solve each system of equations.

48. $\begin{cases} x + 3y + 2z = 13 \\ 2x + 2y - z = 3 \\ x - 2y + 3z = 6 \end{cases}$

49. $\begin{cases} x + y + z = 2 \\ 3x + 2y - z = -1 \\ 3x - y = 4 \end{cases}$

Classify each system as consistent or inconsistent, and determine the number of solutions.

50. $\begin{cases} x + y + z = -2 \\ -x + 2y - 5z = 4 \\ 3x + 3y + 3z = 5 \end{cases}$

51. $\begin{cases} -x - y + 2z = -3 \\ 4x + 4y - 8z = 12 \\ 2x + y - 3z = -2 \end{cases}$

CHAPTER 3

CHAPTER TEST

Solve each system by using a graph and a table.

1. $\begin{cases} x - y = -4 \\ 3x - 6y = -12 \end{cases}$

2. $\begin{cases} y = x - 1 \\ x + 4y = 6 \end{cases}$

3. $\begin{cases} x - y = 3 \\ 2x + 3y = 6 \end{cases}$

Classify each system and determine the number of solutions.

4. $\begin{cases} 6y = 9x \\ 8x + 4y = 20 \end{cases}$

5. $\begin{cases} 12x + 3y = -9 \\ -y - 4x = 3 \end{cases}$

6. $\begin{cases} 3x - 9y = 21 \\ 6 = x - 3y \end{cases}$

Use substitution or elimination to solve each system of equations.

7. $\begin{cases} y = x - 2 \\ x + 5y = 20 \end{cases}$

8. $\begin{cases} 5x - y = 33 \\ 7x + y = 51 \end{cases}$

9. $\begin{cases} x + y = 5 \\ 2x + 5y = 16 \end{cases}$

Graph each system of inequalities.

10. $\begin{cases} 2y - 4x \geq 4 \\ y - x \geq 1 \end{cases}$

11. $\begin{cases} x + y \geq 3 \\ y - 4 \leq 0 \end{cases}$

12. **Chemistry** A chemist wants to mix a new solution with at least 18% pure salt. The chemist has two solutions with 9% pure salt and 24% pure salt and wants to make at most 250 mL of the new solution. Write and graph a system of inequalities that can be used to find the amounts of each salt solution needed.

13. Minimize the objective function $P = 5x + 9y$ under the following constraints. $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 2x + 1 \\ y \leq -3x + 6 \end{cases}$

Graph each point in three-dimensional space.

14. $(2, -1, 3)$

15. $(0, -1, 3)$

16. $(-2, 1, -1)$

Business **Use the following information and the table for Problems 17 and 18.**
A plumber charges \$50 for repairing a leaking faucet, \$150 for installing a sink, and \$200 for an emergency situation. The plumber's total income was exactly \$1000 for each day shown in the table.

Day	Repair Faucet	Install Sink	Emergency
Monday	2	2	■
Tuesday	■	3	2
Wednesday	1	■	4
Thursday	4	4	■

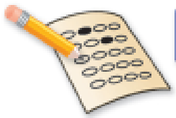
17. Write a linear equation in three variables to represent this situation.

18. Complete the table for the possible numbers of tasks each day.

Solve each system of equations using elimination, or state that the system is inconsistent or dependent.

19. $\begin{cases} x - y + z = -2 \\ 4x - y + 2z = -3 \\ 2x - 3y + 2z = -7 \end{cases}$

20. $\begin{cases} 3x - y - z = -1 \\ x + y + 2z = 8 \\ 6x - 2y - 2z = 5 \end{cases}$



College Entrance Exam Practice

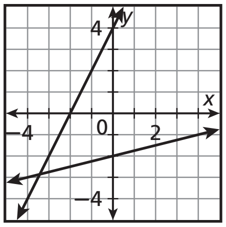
FOCUS ON SAT MATHEMATICS SUBJECT TESTS

In addition to the SAT, the SAT Mathematics Subject Tests are required by some colleges for admission. Colleges that don't require the SAT Mathematics Subject Tests may still use the scores to learn about your academic background and possibly place you in the appropriate college math class.

HOT TIP! Take the SAT Mathematics Subject Tests while the subject matter is fresh in your mind. You are not expected to be familiar with all the content covered on the tests, but you should have completed at least three years of college-prep math.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. Which of the following systems of equations is represented by the graph?



(A) $\begin{cases} y = -2x + 4 \\ y = \frac{1}{4}x + 2 \end{cases}$

(B) $\begin{cases} y = 2x - 4 \\ y = -\frac{1}{4}x - 2 \end{cases}$

(C) $\begin{cases} y = 2x + 4 \\ y = \frac{1}{4}x - 2 \end{cases}$

(D) $\begin{cases} y = \frac{1}{2}x + 4 \\ y = 4x - 2 \end{cases}$

(E) $\begin{cases} y = \frac{1}{2}x - 4 \\ y = 4x + 2 \end{cases}$

2. If $x - 2y = 1$ and $2x - y = -4$, then $x + y = ?$

(A) -9

(B) -7

(C) -5

(D) -3

(E) -1

3. In a fruit salad, there are two more bananas than apples and eight times as many cherries as apples. If a total of 22 pieces of fruit are used, how many of each type are in the salad?

(A) 2 apples, 4 bananas, 18 cherries

(B) 2 apples, 4 bananas, 16 cherries

(C) 2 apples, 0 bananas, 20 cherries

(D) 4 apples, 2 bananas, 12 cherries

(E) 4 apples, 8 bananas, 32 cherries

4. Which of the following inequalities is NOT graphed in the figure?

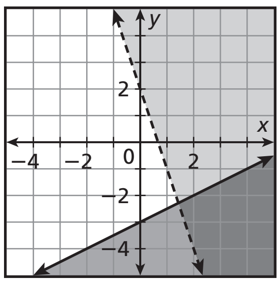
(A) $y > -3x + 2$

(B) $2y \leq x - 6$

(C) $0.5x \geq y + 3$

(D) $3x + y \geq 2$

(E) $6x + 2y > 4$



5. If $r = 3s + 1$ and $t = s - 4$, then what is r in terms of t?

(A) $3t + 13$

(B) $t + 4$

(C) -2.5

(D) $3t + 5$

(E) $3t + 1$

CHAPTER 3

TEST TACKLER

Standardized Test Strategies

Short Response: Write Short Responses

Short-response test items are designed to measure understanding and reasoning skills. Typically, you must show how you solved the problem and explain your answer. Short-response questions are scored using a scoring rubric.

EXAMPLE 1

Short Response Kathie earns $20 per day plus $0.15 for each newspaper she delivers. Kevin earns $25 per day plus $0.10 for each newspaper he delivers. After how many deliveries do they each earn the same amount of money for that day? Write and solve a system of linear equations that models this situation.

Here are examples of how three different responses were scored using the scoring rubric shown.

2-point response:

Let x= the number of newspapers delivered.
Let y= the total amount of money earned.

$$\begin{cases} y = 20 + 0.15x \\ y = 25 + 0.10x \end{cases}$$ Set up a system of equations.

$20 + 0.15x = 25 + 0.10x$ Use substitution to solve.
$0.05x = 5$
$x = 100$

Check:

$$\begin{cases} y = 20 + 0.15x = 20 + 0.15(100) = 35 \checkmark \\ y = 25 + 0.10x = 25 + 0.10(100) = 35 \end{cases}$$

Kathie and Kevin will make the same amount of money, $35, if they both deliver exactly 100 newspapers on any given day.

1-point response:

$$\begin{cases} y = 20 + 0.15x \\ y = 25 + 0.10x \end{cases}$$ Set up a system of equations.

I solved the system using my graphing calculator and determined that x = 100 newspapers.

Notice that the variables are not defined, and there is no sketch of the graph. Although the answer is correct, no explanation is provided.

0-point response:

Kathie and Kevin will never make the same amount of money on the same day.

Notice that the student provided an incorrect response without showing any work or explanation.

Scoring Rubric

2 points: The student writes and correctly solves a system of equations, showing all work. The student defines the variables, answers the question in a complete sentence, and provides an explanation.

1 point: The student writes and correctly solves a system of equations but does not show all work, does not define the variables, or does not provide an explanation.

1 point: The student writes and solves a system of equations but gives an incorrect answer. The student shows all work and provides an explanation for the answer.

0 points: The student gives no response or provides a solution without showing any work or explanation.

HOT TIP! Never leave a short response test item blank. Showing your work and providing a reasonable explanation will result in at least partial credit.

Read each test item, and answer the questions that follow using this scoring rubric.

Scoring Rubric

- **2 points:** The student demonstrates a thorough understanding of the concept, correctly answers the question, and provides a complete explanation.
- **1 point:** The student shows all work and provides an explanation but answers the question incorrectly.
- **1 point:** The student correctly answers the question but does not show all work or does not provide an explanation.
- **0 points:** The student gives a response showing no work or explanation or gives no response.

Item A

Write a real-world situation that can be modeled by this system of equations.

$$\begin{cases} 12x + 15y = 69 \\ 40x + 30y = 170 \end{cases}$$

Solve for x, and make sure that its value makes sense to your situation.

Let x equal the cost of one bag of soil, and let y equal the cost of one potted plant. A landscaper purchased 12 bags of soil and 15 potted plants for $69. He returned to the same store later that week and purchased 40 bags of soil and 30 potted plants for $170. Each bag of soil cost $2, and each potted plant cost $3.

1. How would you score the student's response? Explain.
2. Rewrite the response so that it receives full credit.

Item B

Describe the graph of an independent linear system. Give an example of this type of system, and list the number of solutions it has.

The graph of an independent system is hard to describe because the graph of each equation is independent of the other, and therefore has many solutions.

3. Score the response, and provide your reasoning for the score.
4. Give a response that would receive full credit.

Item C

Explain how to use the intercepts to graph this linear equation in three dimensions. Then graph the equation.

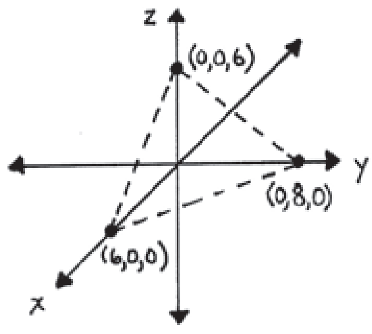
$4x + 3y + 4z = 24$

By finding the x-, y-, and z-intercepts of the linear equation, you can plot three solutions. The plane defined by these points represents the solution set.

x-intercept: $4x + 3(0) + 4(0) = 24$; $x = 6$

y-intercept: $4(0) + 3y + 4(0) = 24$; $y = 8$

z-intercept: $4(0) + 3(0) + 4z = 24$; $z = 6$



5. Should this response receive full credit? Explain your reasoning.

CHAPTER 3

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–3

Multiple Choice

1. What are the intercepts of the linear equation $2x + y - 5z = 20$?

Ⓐ $x = 0, y = 0, z = 0$

Ⓑ $x = 2, y = 1, z = -5$

Ⓒ $x = 10, y = 20, z = -4$

Ⓓ $x = 10, y = 20, z = 4$

2. Sam attends college 440 miles from home. He figures he can make the trip home in about 8 hours driving an average highway speed of 60 miles per hour. Which function represents how many miles Sam is from home after he has been driving for x hours at 60 miles per hour?

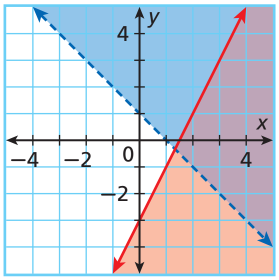
Ⓕ $f(x) = 440 - 60x$

Ⓖ $f(x) = 440 + 60x$

Ⓗ $f(x) = 440 - 8x$

Ⓙ $f(x) = 60x$

3. Which system of inequalities corresponds to the graph?



Ⓐ $\begin{cases} y \le 2x - 3 \\ y \ge -x + 1 \end{cases}$

Ⓑ $\begin{cases} y \le 2x - 3 \\ y > -x + 1 \end{cases}$

Ⓒ $\begin{cases} y < 2x - 3 \\ y > -x + 1 \end{cases}$

Ⓓ $\begin{cases} y \ge 2x - 3 \\ y < -x + 1 \end{cases}$

4. Kylie read the first 87 pages of a book in 3 hours 40 minutes. At this pace, how long will it take her to finish the book if it has a total of 214 pages?

Ⓕ 1 hour 25 minutes

Ⓕ 5 hours 5 minutes

Ⓗ 9 hours 1 minutes

Ⓙ 12 hours 41 minutes

5. What is the equation of a line with a slope of $-\frac{2}{5}$ passing through (1, 4)?

Ⓐ $y = -\frac{2}{5}x + 4\frac{2}{5}$

Ⓑ $y = -\frac{2}{5}x + 2\frac{3}{5}$

Ⓒ $y = -\frac{1}{4}x - \frac{1}{10}$

Ⓓ $y = \frac{2}{5}x + 3\frac{3}{5}$

6. Which system of equations is an independent system?

Ⓕ $\begin{cases} 2y + 3x = -8 \\ 9x = -24 - 6y \end{cases}$

Ⓖ $\begin{cases} y = -x + 4 \\ 3y + 3x = -21 \end{cases}$

Ⓗ $\begin{cases} 2y + 7x = 24 \\ 5y - 6 = -4x \end{cases}$

Ⓙ $\begin{cases} 2y = 3x - 6 \\ 8y - 12x = 80 \end{cases}$

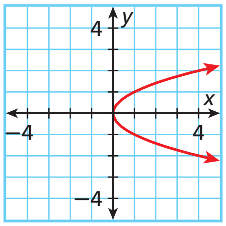
7. Which relation is a function?

Ⓐ $\{(1, 4), (4, 1), (1, 0), (0, 4)\}$

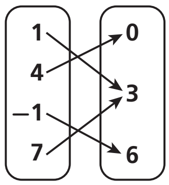
Ⓑ

x	3	5	8	8	12
y	5	6	7	8	9

Ⓒ



Ⓓ



8. A feasible region has vertices (0, 0), (−2, 6), (3, −1), (−1, 1), and (−5, −5). What is the maximum value of the objective function $P = 4x - y$ over this region?

Ⓕ 0

Ⓖ 7

Ⓗ 13

Ⓙ 25

9. Mark has two bowls of cookie dough. One has 30% raisins, and the other has 5% raisins. How much of each dough should he mix to get 18 ounces of cookie dough that has 15% raisins?

Ⓐ 14.4 ounces of the dough with 30% raisins and 3.6 ounces of the dough with 5% raisins

Ⓑ 5.4 ounces of the dough with 30% raisins and 0.9 ounces of the dough with 5% raisins

Ⓒ 8.8 ounces of the dough with 30% raisins and 9.2 ounces of the dough with 5% raisins

Ⓓ 7.2 ounces of the dough with 30% raisins and 10.8 ounces of the dough with 5% raisins

HOT TIP! In Exercise 10, to solve for the final exam percent, it is not necessary to find the percent for homework or quizzes first.

10. A final grade is based on a student's performance on homework, quizzes, and the final exam. All homework, quizzes, and the final exam are worth 100 points. Each category is worth a different percentage of the final grade. Given the scores of three students in the table below, what percent of the final grade is the final exam worth?

	Homework	Quizzes	Final Exam	Final Grade
Andy	100	82	73	82
Mia	66	94	88	82
Nick	82	46	98	88

Ⓕ 50%
Ⓖ 60%
Ⓗ 75%
Ⓙ 86%

Gridded Response

11. Find the lowest positive whole number that is a solution of $\frac{|438 - 3x|}{3} > 816$.

12. Peter eats 8 wings and 3 pieces of pizza and consumes a total of 975 Calories. K.J. eats 6 wings and 4 pieces of pizza and consumes a total of 950 Calories. How many Calories are in one piece of pizza?

Short Response

13. One group of people going to the zoo bought 5 child tickets and 4 adult tickets for a total of \$68. Another group bought 17 child tickets and 12 adult tickets for a total of \$216.

  a. Write a system of equations that models this problem.

  b. Solve the system using a graph.

  c. Solve the system using another method. Explain why the method you used may be better than using the graphing method. What is the price of each kind of ticket?

14. Point A has coordinates (3, 4). Point B is a reflection of point A across the x-axis.

  a. Give the coordinates of point B.

  b. Point C is a translation of point B 3 units left and 2 units down. Give the coordinates of point C.

15. The function $g(x)$ is a vertical translation of $f(x) = 4x - 3$ down 5 units.

  a. Write the rule for $g(x)$.

  b. The function $h(x)$ is a reflection of $g(x)$ across the x-axis. Write the rule for $h(x)$.

Extended Response

16. A curtain manufacturer has 820 lots of cotton fiber and 1250 lots of synthetic fiber. A discount curtain uses 18 lots of cotton fiber and 32 lots of synthetic fiber. A premium curtain uses 36 lots of cotton fiber and 28 lots of synthetic fiber.

  a. Write the constraints.

  b. Graph the feasible region. Give the vertices of the polygon that defines the feasible region.

  c. The manufacturer makes a profit of \$170 on each discount curtain sold and a profit of \$190 on each premium curtain sold. Write the objective function.

  d. How many of each kind of curtain should be manufactured to maximize profit?