CHAPTER 10

Conic Sections

CRACKING THE SUPER EGG

You can use conic sections to create your own super egg and discover the many different uses of super ellipses.

go.hrw.com
Chapter Project Online
KEYWORD: MB7 ChProj

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. vertex of a parabola
2. axis of symmetry
3. solution set of a system of equations
4. asymptote

A. a line that divides a plane figure or a graph into two congruent reflected halves

B. a line approached by the graph of a function

C. a line that is neither horizontal nor vertical

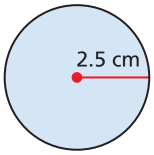
D. the turning point of a parabola

E. the set of points that make all equations in a system true

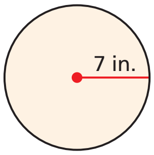
Circumference and Area of Circles

Find the circumference and area of each circle.

5.



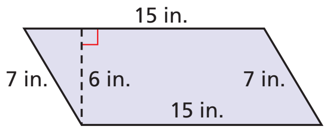
6.



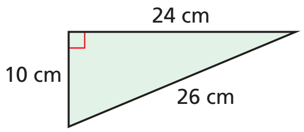
Area of Polygons

Find the area of each figure.

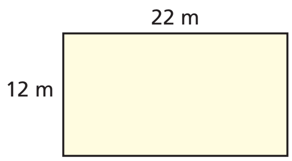
7.



8.



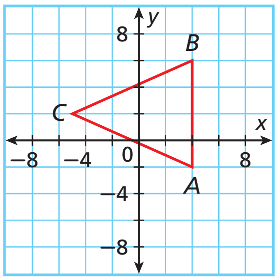
9.



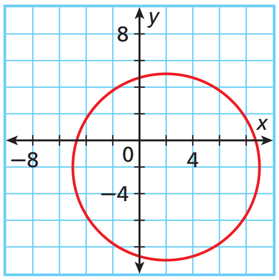
Find Areas in the Coordinate Plane

Find the area of each figure.

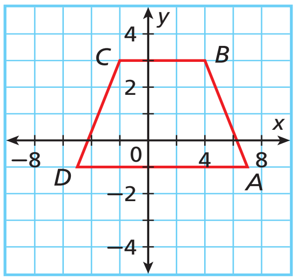
10.



11.



12.



Complete the Square

Complete the square for each expression. Write the resulting expression as a binomial squared.

13. $x^2 - 4x + \square$

14. $x^2 - x + \square$

15. $x^2 + 6x + \square$

CHAPTER 10

Study Guide: Preview

Where You've Been

Previously, you

- graphed parabolas defined by quadratic functions.
- studied graphs of piecewise functions.
- solved systems of linear equations.

In This Chapter

You will study

- graphs of parabolas and other conic sections that are not functions.
- graphs of conic sections represented by two functions together.
- methods for solving systems of nonlinear equations.

Where You're Going

You can use the skills in this chapter

- in all of your future math classes, including Calculus and Statistics.
- in other classes such as Chemistry, Physics, and Economics.
- outside of school in engineering, architecture, astronomy, photography, and communications.

Key Vocabulary/Vocabulario

circle	círculo
conic section	sección cónica
directrix	directriz
ellipse	elipse
foci of an ellipse	focos de una elipse
foci of a hyperbola	focos de una hipérbola
focus of a parabola	foco de una parábola
hyperbola	hipérbola
nonlinear system of equations	sistema no lineal de ecuaciones
tangent line	línea tangente
vertices of an ellipse	vértices de una elipse
vertices of a hyperbola	vértices de una hipérbola

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. When you use the word *focus* in most contexts, you mean a center of activity or attention. What is the focus of a painting? How can this help you understand a **focus** of a conic section?
2. In the geometry book, you saw the term *vertex* used for triangles. In this book you have already seen the term *vertex* used for parabolas. In this chapter, you will see the term **vertex** used for ellipses and hyperbolas. Why is the same term used in all of these different situations?
3. How often does a *tangent* touch a circle? Can curves other than circles also have **tangents**? What do you mean when you say that someone went "off on a tangent" during a discussion?

Reading and Writing Math

Study Strategy: Learn Vocabulary

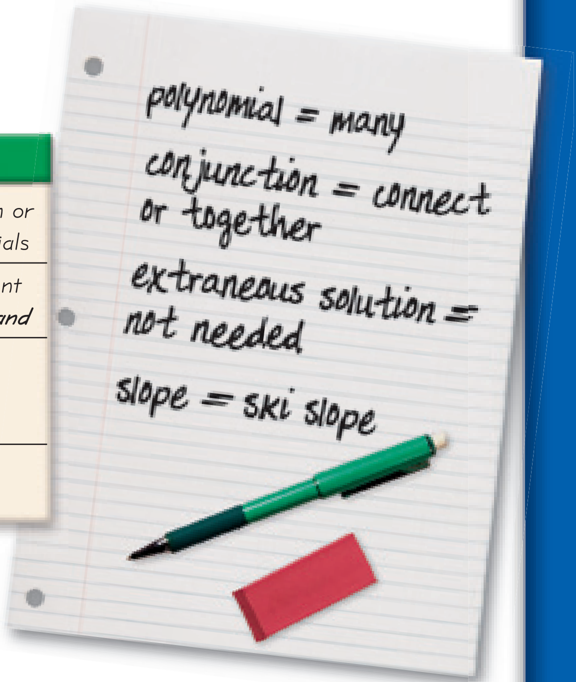
Understanding math terminology and vocabulary is important to learning and using new math concepts. You have already learned many new terms and as you progress in your studies of math, you will need to learn many more.

To learn new vocabulary:

- Look for the meaning of a new word through the context in which it is introduced.
- Use the prefix or suffix to determine the meaning of the root word.
- Relate the new term to familiar, everyday words.

Once you know what a word means, write its definition in your own words.

Vocabulary Word	Study Tips	Definition
Polynomial	Prefix *poly*-, meaning "many"	A monomial or a sum or difference of monomials
Conjunction	Prefix *con*-, meaning "connect" or "together"	A compound statement that uses the word *and*
Extraneous Solution	Relate to the word *extra*, meaning "not needed."	Extra roots that are not solutions to the original equation
Slope	Think of a *ski slope*.	The measure of the steepness of a line



Try This

Fill in the chart with information that can help you learn the vocabulary words.

	Vocabulary Word	Study Tips	Definition
1.	Trinomial	■	■
2.	Disjunction	■	■
3.	Variable	■	■
4.	Multiplicity	■	■

Use the given prefix's meaning to write the definition of the corresponding vocabulary words.

5. *dia*- through, across, between: diameter; diagonal

6. *trans*- across, beyond, through: transformation; translation

10-1 Introduction to Conic Sections

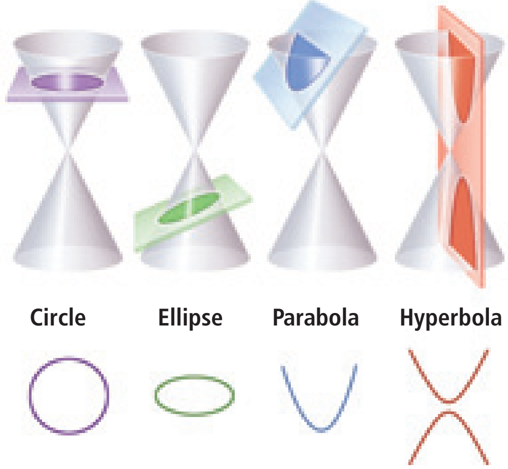
Objectives
Recognize conic sections as intersections of planes and cones.

Use the distance and midpoint formulas to solve problems.

Vocabulary
conic section

Who uses this?
Archaeologists use distance and midpoint to organize excavation sites. (See Exercise 43.)

In Chapter 5, you studied the parabola. The parabola is one of a family of curves called *conic sections*. **Conic sections** are formed by the intersection of a double right cone and a plane. There are four types of conic sections: circles, ellipses, hyperbolas, and parabolas.



Although the parabolas you studied in Chapter 5 are functions, most conic sections are not. This means that you often must use two functions to graph a conic section on a calculator.

A circle is defined by its center and its radius. An ellipse, an elongated shape similar to a circle, has two perpendicular axes of different lengths.

EXAMPLE 1 Graphing Circles and Ellipses on a Calculator

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

A $x^2 + y^2 = 25$

Step 1 Solve for y so that the expression can be used in a graphing calculator.

$y^2 = 25 - x^2$ *Subtract x^2 from both sides.*

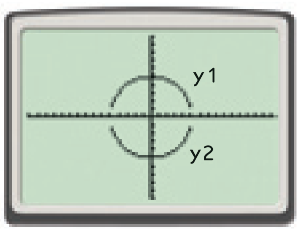
$y = \pm\sqrt{25 - x^2}$ *Take the square root of both sides.*

Step 2 Use two equations to see the complete graph.

$y_1 = \sqrt{25 - x^2}$

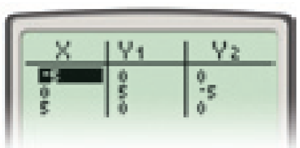
$y_2 = -\sqrt{25 - x^2}$

Use a square window on your graphing calculator for an accurate graph. The graphs meet and form a complete circle, even though it may not appear that way on your calculator.



The graph is a circle with center $(0, 0)$ and intercepts $(5, 0)$, $(-5, 0)$, $(0, 5)$, and $(0, -5)$.

Check Use a table to confirm the intercepts.



Remember!
When you take the square root of both sides of an equation, remember that you must include the positive and negative roots.

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

B $16x^2 + 9y^2 = 144$

Step 1 Solve for y so that the expression can be used in a graphing calculator.

$9y^2 = 144 - 16x^2$ *Subtract $16x^2$ from both sides.*

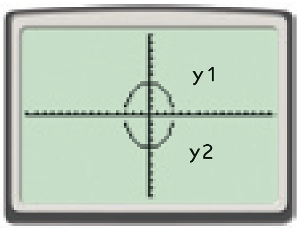
$y^2 = \frac{144 - 16x^2}{9}$ *Divide both sides by 9.*

$y = \pm\sqrt{\frac{144 - 16x^2}{9}}$ *Take the square root of both sides.*

Step 2 Use two equations to see the complete graph.

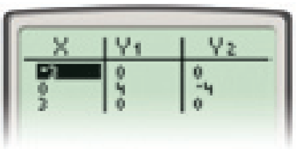
$y_1 = \sqrt{\frac{144 - 16x^2}{9}}$

$y_2 = -\sqrt{\frac{144 - 16x^2}{9}}$



Use a square window on your graphing calculator. The graphs meet and form a complete ellipse, even though it may not appear that way on your calculator.

The graph is an ellipse with center $(0, 0)$ and intercepts $(3, 0)$, $(-3, 0)$, $(0, 4)$, and $(0, -4)$.



Check Use a table to confirm the intercepts.

CHECK IT OUT! **Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.**

1a. $x^2 + y^2 = 49$ **1b.** $9x^2 + 25y^2 = 225$

A parabola is a single curve, whereas a hyperbola has two congruent branches. The equation of a parabola usually contains either an x^2 term or a y^2 term, but not both. The equations of the other conics will usually contain both x^2 and y^2 terms.

EXAMPLE 2 Graphing Parabolas and Hyperbolas on a Calculator

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

A $3y^2 = x$

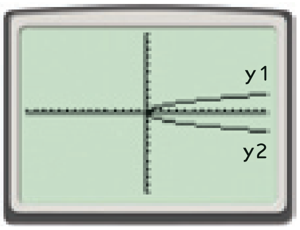
Step 1 Solve for y so that the expression can be used in a graphing calculator.

$y^2 = \frac{x}{3}$ *Divide both sides by 3.*

$y = \pm\sqrt{\frac{x}{3}}$ *Take the square root of both sides.*

Step 2 Use two equations to see the complete graph.

$y_1 = \sqrt{\frac{x}{3}}$ and $y_2 = -\sqrt{\frac{x}{3}}$



The graph is a parabola with vertex $(0, 0)$ that opens to the right.

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

B $x^2 - y^2 = 4$

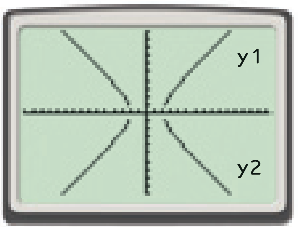
Step 1 Solve for y so that the expression can be used in a graphing calculator.

$-y^2 = 4 - x^2$	*Subtract x^2 from both sides.*
$y^2 = -(4 - x^2)$	*Multiply both sides by −1.*
$y^2 = -4 + x^2$	*Distribute.*
$y^2 = x^2 - 4$	*Rearrange.*
$y = \pm\sqrt{x^2 - 4}$	*Take the square root of both sides.*

Step 2 Use two equations to see the complete graph.

$y_1 = \sqrt{x^2 - 4}$

$y_2 = -\sqrt{x^2 - 4}$



The graph is a hyperbola that opens horizontally with vertices at $(2, 0)$ and $(-2, 0)$.

Helpful Hint

Because hyperbolas contain two curves that open in opposite directions, classify them as opening horizontally, vertically, or neither.

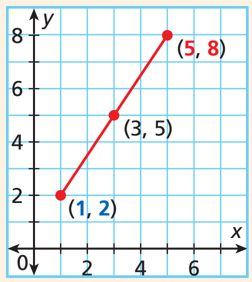
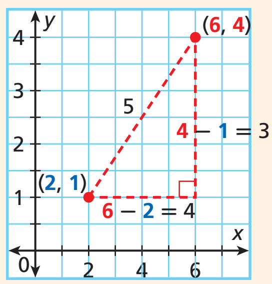
CHECK IT OUT! **Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.**

2a. $2y^2 = x$ **2b.** $x^2 - y^2 = 16$

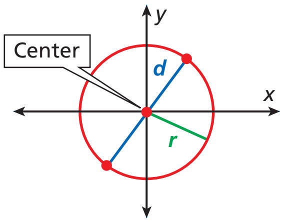
Every conic section can be defined in terms of distances. You can use the Midpoint and Distance Formulas to find the center and radius of a circle.

Know it! Note

Midpoint and Distance Formulas

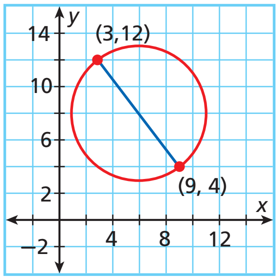
FORMULA	EXAMPLE	GRAPH
The **midpoint** (x_M, y_M) of the segment with endpoints (x_1, y_1) and (x_2, y_2) is $(x_M, y_M) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.	The midpoint of the segment with endpoints $(1, 2)$ and $(5, 8)$ is $\left(\frac{1 + 5}{2}, \frac{2 + 8}{2}\right) = (3, 5)$.	
The **distance** d between the points with coordinates (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	The distance between the points $(2, 1)$ and $(6, 4)$ is $\sqrt{(6 - 2)^2 + (4 - 1)^2} = 5$.	

Because a diameter must pass through the center of a circle, the midpoint of a diameter is the center of the circle. The radius of a circle is the distance from the center to any point on the circle and equal to half the diameter.



EXAMPLE 3 Finding the Center and Radius of a Circle

Find the center and radius of a circle that has a diameter with endpoints (3, 12) and (9, 4).



Step 1 Find the center of the circle.

Use the Midpoint Formula with the endpoints, (3, 12) and (9, 4).

$$\left(\frac{3+9}{2}, \frac{12+4}{2}\right) = (6, 8)$$

The center of the circle is (6, 8).

Step 2 Find the radius.

Use the Distance Formula with (6, 8) and (3, 12).

$$r = \sqrt{(6-3)^2 + (8-12)^2}$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= 5$$

The radius of the circle is 5.

Check Use the other endpoint (9, 4) and the center (6, 8). The radius should equal 5 for any point on the circle.

$$r = \sqrt{(9-6)^2 + (4-8)^2} = 5 \checkmark$$

The radius is the same using (9, 4).

Helpful Hint

The midpoint formula uses averages. You can think of x_M as the average of the x-values and y_M as the average of the y-values.

CHECK IT OUT! **3.** Find the center and radius of a circle that has a diameter with endpoints (2, 6) and (14, 22).

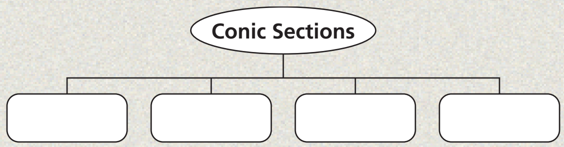
THINK AND DISCUSS

1. If you know one endpoint and the midpoint of a line segment, how could you find the other endpoint of the segment?

2. Find the domain and range of each of the graphs in Examples 1 and 2.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. List the types of conic sections, and sketch an example of each.



10-1 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-1
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** What are the four different types of *conic sections*?

SEE EXAMPLE 1
p. 722

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

2. $3x^2 + 3y^2 = 48$
3. $9x^2 + 16y^2 = 144$
4. $x^2 + y^2 = 36$

SEE EXAMPLE 2
p. 723

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

5. $5y^2 = x$
6. $x^2 = y^2 + 9$
7. $y^2 - x^2 = 25$
8. $12y = 6x^2$
9. $2x^2 - y^2 = 4$
10. $-y^2 = 4 + x$

SEE EXAMPLE 3
p. 725

Find the center and radius of a circle that has a diameter with the given endpoints.

11. $(3, 6)$ and $(13, 30)$
12. $(-4, 1)$ and $(-16, -8)$
13. $(6, -9)$ and $(-8, 39)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–22	1
23–31	2
32–34	3

Extra Practice

Skills Practice p. S22
Application Practice p. S41

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

14. $49x^2 + 36y^2 = 1764$
15. $\frac{x^2}{9} + \frac{y^2}{9} = 1$
16. $243 - 3x^2 - 3y^2 = 0$
17. $\frac{x^2}{4} = 1 - \frac{y^2}{25}$
18. $4x^2 + 81y^2 = 324$
19. $\frac{4x^2}{25} + \frac{4y^2}{225} = 1$
20. $\frac{3}{4}x^2 + \frac{3}{4}y^2 = 75$
21. $4x^2 + 4y^2 = 81$
22. $x^2 + y^2 = \frac{4}{9}$

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

23. $y = 2x^2$
24. $x^2 = y^2 + 64$
25. $x + 2y^2 = 0$
26. $x = \frac{2}{3}y^2$
27. $0 = 1 + \frac{x^2}{64} - \frac{y^2}{36}$
28. $5y^2 - 5x^2 = 180$
29. $x = 4y^2 - 3$
30. $y = 4 - \frac{x^2}{5}$
31. $9x^2 - 16y^2 = 144$

Find the center and radius of a circle that has a diameter with the given endpoints.

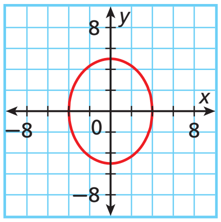
32. $(20, 21)$ and $(12, 6)$
33. $\left(\frac{9}{2}, \frac{5}{2}\right)$ and $\left(\frac{5}{2}, \frac{17}{2}\right)$
34. $(7, -5)$ and $(-1, 10)$

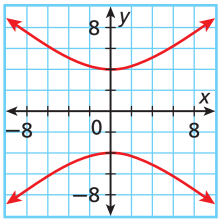
35. **Geometry** A circle has center $(-7, 10)$ and contains the point $(23, -6)$.
   a. Find the circumference and area of the circle.
   b. Find the other endpoint of the diameter with one endpoint $(23, -6)$.

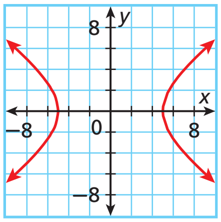
MULTI-STEP TEST PREP

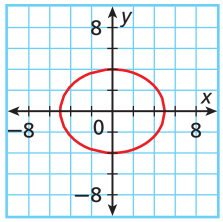
36. This problem will prepare you for the Multi-Step Test Prep on page 758.
The orbit of an asteroid can be modeled by the equation $16x^2 + 25y^2 = 400$.
   a. Graph the equation on a graphing calculator, and identify the conic section.
   b. Identify the x- and y-intercepts of the orbit.
   c. Suppose that each unit of the coordinate plane represents 50 million miles. What is the maximum width of the asteroid's orbit?

Use your graphing calculator to match each equation to one of the following graphs.

A.


B.


C.


D.


37. $16x^2 + 25y^2 = 400$

38. $16x^2 - 25y^2 = 400$

39. $25x^2 + 16y^2 = 400$

40. $25y^2 - 16x^2 = 400$

41. Geometry A quadrilateral has vertices $A(2, 3)$, $B(12, 3)$, $C(18, 11)$, and $D(8, 11)$.

a. Find the length of each side.

b. Classify the figure *ABCD*.

c. Find the area of *ABCD*.

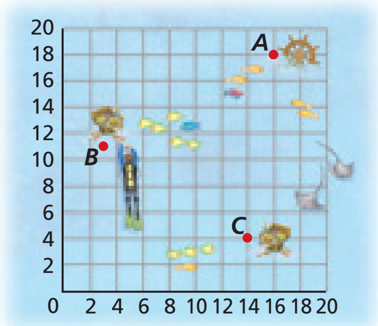
LINK

Archaeology

Chichén Itzá, in Yucatán, Mexico, was a major city of the Maya civilization. Its city center covers about 2 square miles and was used primarily for religious ceremonies. Today the ruins are the most visited archaeological site in Mexico.

42. Critical Thinking How can you tell if the graph of an equation in the form $ax^2 + by^2 = c$ is a circle or an ellipse?

43. Archaeology Archaeologists exploring an underwater site have set up a grid so that they can precisely label where any artifacts they discover were found. The archaeologists have found two treasure chests at points *B* and *C* and a ship's wheel at point *A*. Which treasure is the wheel closer to? Explain.



44. ERROR ANALYSIS Which solution is incorrect? Explain the error. Find the distance between $(0, 0)$ and $(2, 3)$.

A

$d = \sqrt{(2-0)^2 + (3-0)^2}$

$d = \sqrt{2^2 + 3^2}$

$d = \sqrt{4 + 9}$

$d = \sqrt{13}$

B

$d = \sqrt{(2-0)^2 + (3-0)^2}$

$d = \sqrt{2^2 + 3^2}$

$d = 2 + 3$

$d = 5$

45. Geometry A triangle has vertices $A(8, 2)$, $B(13, 14)$, and $C(-4, 6)$.

a. Find the length of $\overline{AB}$.

b. Find the length of the segment joining the midpoints of $\overline{BC}$ and $\overline{AC}$.

c. Find the slopes of $\overline{AB}$ and the segment joining the midpoints of the other two sides. What do the slopes tell you about the two segments?

Tell whether each statement is sometimes, always, or never true. If it is sometimes true, give examples to support your answer.

46. A circle is a function.

47. The domain of a parabola is all real numbers.

48. The distance between two points is positive.

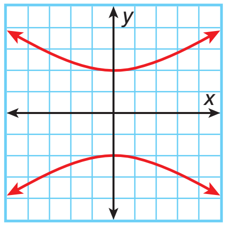
49. Write About It If a right triangle has a hypotenuse with length c and legs with lengths a and b, the Pythagorean Theorem states that $a^2 + b^2 = c^2$. Explain how the Distance Formula is related to the Pythagorean Theorem.

TEST PREP

50. Which of the following could be the equation of the graph shown?

Ⓐ $9x^2 - 4y^2 = 36$ Ⓒ $9y^2 - 4x^2 = 36$

Ⓑ $4x^2 + 9y^2 = 36$ Ⓓ $9x^2 + 4y^2 = 36$



51. One endpoint of a line segment is $(-4, -8)$, and the midpoint of the line segment is $(2, -12)$. Which of the following is the other endpoint?

Ⓕ $(-1, -10)$ Ⓖ $(3, -2)$ Ⓗ $(-8, 16)$ Ⓙ $(8, -16)$

52. Which of the following are the x-intercepts of the graph of $4x^2 + 25y^2 = 100$?

Ⓐ $(2, 0)$ and $(-2, 0)$ Ⓒ $(5, 0)$ and $(-5, 0)$

Ⓑ $(4, 0)$ and $(-4, 0)$ Ⓓ $(10, 0)$ and $(-10, 0)$

53. What is the distance between the points $(-2, 6)$ and $(5, 30)$?

Ⓕ $3\sqrt{145}$ Ⓖ 31 Ⓗ $3\sqrt{65}$ Ⓙ 25

CHALLENGE AND EXTEND

Find a so that the two points are the given distance apart.

54. $(-5, 8)$ and $(3, a)$; 17

55. $(4, -10)$ and $(a, 5)$; 39

56. **Multi-Step** A degenerate conic is formed when a plane passes through the vertex of a hollow double cone. A point, a line, and a pair of intersecting lines are all degenerate conics.

 a. The graph of $y^2 - x^2 = 0$ is a degenerate hyperbola. Graph $y^2 - x^2 = 0$.

 b. What is the graph of $x^2 + y^2 = 0$?

 c. Explain how a plane could intersect a hollow double cone to result in the graphs from parts **a** and **b.**

57. The midpoint and distance formulas can be extended to three dimensions by including an additional term in each formula for the variable z.

 a. Find the midpoint of the segment with endpoints $(6, -3, -9)$ and $(12, 7, -13)$.

 b. Write a formula to find the midpoint of a segment in three dimensions.

 c. Find the distance between the points $(1, 2, 3)$ and $(5, 8, 10)$.

 d. Write a formula to find the distance between two points in three dimensions.

SPIRAL REVIEW

58. **Construction** A construction crew is repainting the center line on a 12 mi road. If the crew has completed 2.5 mi after 45 min, about how much more time should the painting take? *(Lesson 2-2)*

Find the zeros of each function by factoring. *(Lesson 5-3)*

59. $f(x) = x^2 - 2x - 48$

60. $f(x) = x^2 + 12x + 27$

61. $f(x) = x^2 - 11x + 28$

62. $f(x) = x^2 + 10x - 24$

63. $f(x) = 2x^2 - 25x + 33$

64. $f(x) = 3x^2 + 22x + 24$

Graph each exponential function. Find the y-intercept and the asymptote. Then describe how the graph transformed from the graph of its parent function $f(x) = 5^x$. *(Lesson 7-7)*

65. $f(x) = -\frac{1}{2}(5^x) + 3$

66. $f(x) = 4(5^x)$

67. $f(x) = 6(5^x) - 1$

10-2 Circles

Objectives
Write an equation for a circle.

Graph a circle, and identify its center and radius.

Vocabulary
circle
tangent

Why learn this?
You can use circles to find locations within a given radius of an address. (See Example 3.)

A **circle** is the set of points in a plane that are a fixed distance, called the radius, from a fixed point, called the center. Because all of the points on a circle are the same distance from the center of the circle, you can use the Distance Formula to find the equation of a circle.

EXAMPLE 1 Using the Distance Formula to Write the Equation of a Circle

Write the equation of a circle with center $(2, 1)$ and radius $r = 5$.

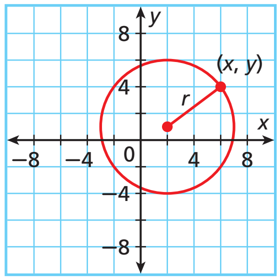
Use the Distance Formula with $(x_2, y_2) = (x, y)$, $(x_1, y_1) = (2, 1)$, and distance equal to the radius, 5.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Use the Distance Formula.*

$5 = \sqrt{(x - 2)^2 + (y - 1)^2}$ *Substitute.*

$5^2 = (x - 2)^2 + (y - 1)^2$ *Square both sides.*

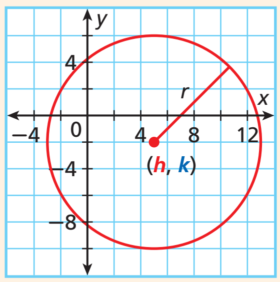
$25 = (x - 2)^2 + (y - 1)^2$



CHECK IT OUT! **1.** Write the equation of a circle with center $(4, 2)$ and radius $r = 7$.

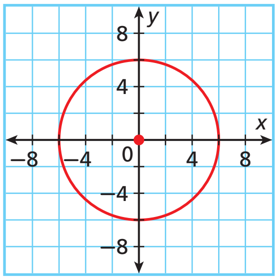
Notice that r^2 and the center are visible in the equation of a circle. This leads to a general formula for a circle with center (h, k) and radius r.

Know it! Note — **Equation of a Circle**

EQUATION	EXAMPLE	GRAPH
The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.	The equation of the circle with center $(5, -2)$ and radius $r = 8$ is $(x - 5)^2 + (y - (-2))^2 = 8^2$ or $(x - 5)^2 + (y + 2)^2 = 64$.	

EXAMPLE 2 Writing the Equation of a Circle

Write the equation of each circle.



A **the graphed circle with center $(0, 0)$ and radius $r = 6$**

$(x - h)^2 + (y - k)^2 = r^2$ *Equation of a circle*

$(x - 0)^2 + (y - 0)^2 = 6^2$ *Substitute.*

$x^2 + y^2 = 36.$

Helpful Hint

If the center of the circle is at the origin, the equation simplifies to $x^2 + y^2 = r^2$.

B **the circle with center $(2, 4)$ and containing the point $(8, 12)$**

$r = \sqrt{(8 - 2)^2 + (12 - 4)^2}$ *Use the Distance Formula to find the radius.*

$= \sqrt{6^2 + 8^2}$

$= \sqrt{100} = 10$

$(x - 2)^2 + (y - 4)^2 = 10^2$ *Substitute the values into the equation of a circle.*

$(x - 2)^2 + (y - 4)^2 = 100$

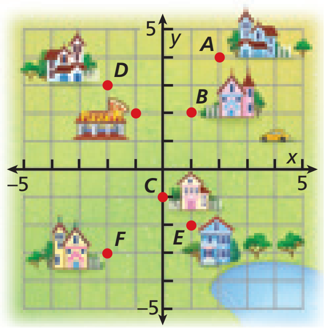
CHECK IT OUT!

2. Find the equation of the circle with center $(-3, 5)$ and containing the point $(9, 10)$.

The location of points in relation to a circle can be described by inequalities. The points inside the circle satisfy the inequality $(x - h)^2 + (y - k)^2 < r^2$. The points outside the circle satisfy the inequality $(x - h)^2 + (y - k)^2 > r^2$.

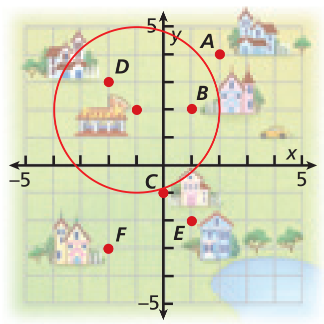
EXAMPLE 3 *Consumer Application*

Raul and his friends are having a pizza party and will decide where to have the party based on the delivery area of the pizza restaurant. Suppose that the pizza restaurant is located at the point $(-1, 2)$ and the letters represent the homes of Raul and his friends. Use the equation of a circle to find the houses that are within a 3-mile radius and will get free delivery.



The circle has center $(-1, 2)$ and radius 3. The points inside the circle will satisfy the inequality $(x + 1)^2 + (y - 2)^2 < 3^2$.

Points D and B are within a 3-mile radius.



Check Point $C(0, -1)$ is near the boundary.

$(0 + 1)^2 + (-1 - 2)^2 < 3^2$

$(1)^2 + (-3)^2 < 3^2$

$1 + 9 < 9$ ✘ *Point $C(0, -1)$ is not inside the circle.*

CHECK IT OUT!

3. **What if...?** Which homes are within a 3-mile radius of a restaurant located at $(2, -1)$?

A **tangent** is a line in the same plane as the circle that intersects the circle at exactly one point. Recall from geometry that a tangent to a circle is perpendicular to the radius at the point of tangency.

EXAMPLE 4 Writing the Equation of a Tangent

Write the equation of the line that is tangent to the circle $25 = x^2 + y^2$ at the point $(3, 4)$.

Step 1 Identify the center and radius of the circle.

From the equation $25 = x^2 + y^2$, the circle has center $(0, 0)$ and radius $r = 5$.

Step 2 Find the slope of the radius at the point of tangency and the slope of the tangent.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$m = \frac{4 - 0}{3 - 0}$ *Substitute $(3, 4)$ for (x_2, y_2) and $(0, 0)$ for (x_1, y_1).*

$m = \frac{4}{3}$ *The slope of the radius is $\frac{4}{3}$.*

Because the slopes of perpendicular lines are negative reciprocals, the slope of the tangent is $-\frac{3}{4}$.

Step 3 Find the slope-intercept equation of the tangent by using the point $(3, 4)$ and the slope $m = -\frac{3}{4}$.

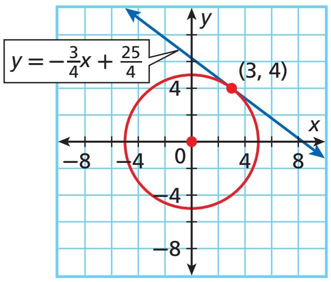
$y - y_1 = m(x - x_1)$ *Use the point-slope formula.*

$y - 4 = -\frac{3}{4}(x - 3)$ *Substitute $(3, 4)$ for (x_1, y_1) and $-\frac{3}{4}$ for m.*

$y = -\frac{3}{4}x + \frac{25}{4}$ *Rewrite in slope-intercept form.*

The equation of the line that is tangent to $25 = x^2 + y^2$ at $(3, 4)$ is $y = -\frac{3}{4}x + \frac{25}{4}$.

Check Graph the circle and the line.



Remember!
To review linear functions, see Lesson 2-4.

CHECK IT OUT!

4. Write the equation of the line that is tangent to the circle $25 = (x - 1)^2 + (y + 2)^2$ at the point $(5, -5)$.

THINK AND DISCUSS

1. Explain the transformation of $x^2 + y^2 = 1$ that is necessary to get the equation $(x - h)^2 + (y - k)^2 = 1$.

2. Explain what happens to the radius if the equation of a circle changes from $x^2 + y^2 = 4$ to $x^2 + y^2 = 16$.

3. GET ORGANIZED Copy and complete the graphic organizer. Sketch each circle, and give its equation.

Know it! Note

	$r = 1$	$r = 3$
Center (0, 0)		
Center (1, 2)		

10-2 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How can you recognize a *tangent* of a circle?

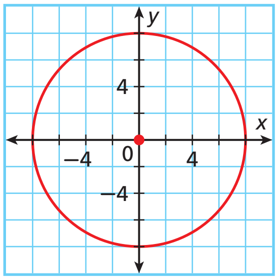
SEE EXAMPLE 1
p. 729

Write the equation of each circle.

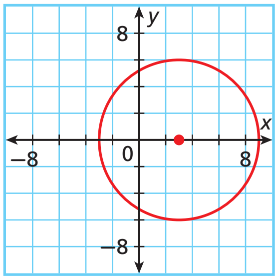
2. center $(6, -5)$ and radius $r = 4$
3. center $(-11, 3)$ and radius $r = 9$

SEE EXAMPLE 2
p. 730

4.



5.



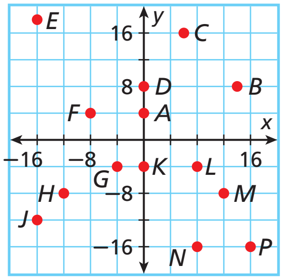
6. center $(-1, 9)$ and containing the point $(2, 5)$
7. center $(-2, -5)$ and containing the point $(-10, -20)$

SEE EXAMPLE 3
p. 730

Depending on its strength, an earthquake can be felt in locations miles away from the epicenter.

8. **Multi-Step** Suppose that the epicenter of the earthquake is located at the point $(5, -2)$ and is felt up to 10 mi away. Use the equation of a circle to find the locations that are affected.

9. **Multi-Step** Suppose that the epicenter of the earthquake is located at the point $(-5, -7)$ and is felt up to 8 mi away. Use the equation of a circle to find the locations that are affected.



SEE EXAMPLE 4
p. 731

Multi-Step **Write the equation of the line that is tangent to each circle at the given point.**

10. $x^2 + y^2 = 100$; $(8, 6)$
11. $(x + 6)^2 + (y + 4)^2 = 25$; $(-9, -8)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–13	1
14–17	2
18–19	3
20–21	4

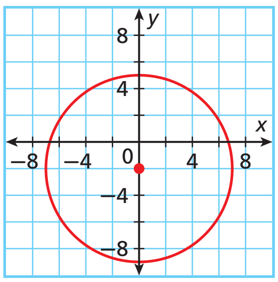
Extra Practice

Skills Practice p. S22
Application Practice p. S41

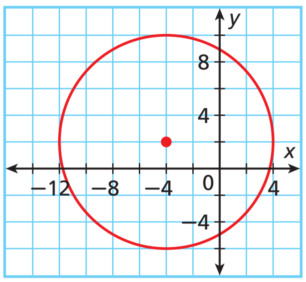
Write the equation of each circle.

12. center $(3, 2)$ and radius $r = 7$
13. center $(5, 1)$ and radius $r = 10$

14.

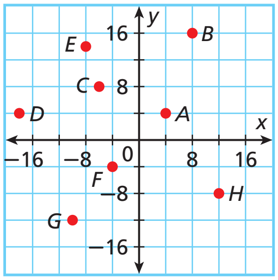


15.



16. center $(12, -3)$ and containing the point $(-12, 7)$
17. center $(-6, -4)$ and containing the point $(-2, -1)$

Aida's puppy escaped from the backyard and is lost. Aida has created a map of places that the puppy may have gone.



18. **Multi-Step** Suppose that Aida's house is located at the point $(3, 8)$. The puppy has been gone for 4 hours, and Aida estimates that the puppy cannot have traveled more than 12 miles. Use the equation of a circle to find the possible locations of the puppy.

19. **Multi-Step** Suppose that Aida's house is located at the point $(-6, 15)$. The puppy has been gone for 1 hour, and Aida estimates that the puppy cannot have traveled more than 3 miles. Use the equation of a circle to find the possible locations of the puppy.

LINK

History

Stonehenge, in southern England, is thought to have been built in three stages, from 2950-1600 B.C.E. It is not a single structure but consists of many stone, earth, and wood constructions.

Multi-Step **Write the equation of the line that is tangent to each circle at the given point.**

20. $x^2 + y^2 = 169, (-5, 12)$

21. $(x - 2)^2 + (y - 4)^2 = 289, (-15, 4)$

22. **History** The outermost ring of the ancient monument Stonehenge can be modeled by the equation $x^2 + y^2 = 27{,}225$. The Sarsen Circle, the center ring of stones usually associated with the monument, can be modeled by the equation $x^2 + y^2 = 2916$.

a. The Heel Stone is located outside of the circles, approximately at the point $(0, 300)$. Find the maximum and minimum distances, in feet, to the Heel Stone from both the outer and inner circles.

b. Graph the outer circle and the Sarsen Circle.

c. Two Station Stones surrounded by circular ditches are located within the outer circle. One stone is located at approximately $(-100, 100)$ and is surrounded by a ditch of radius 12 ft. Write an equation to model the ditch around this Station Stone.

Find the domain and range of each relation.

23. $x^2 + y^2 = 36$

24. $(x - 2)^2 + (y + 7)^2 = 81$

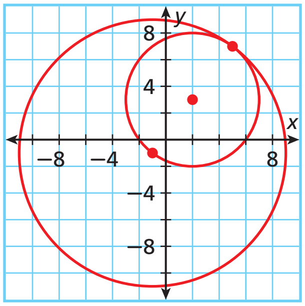
25. $(x + 2)^2 + (y)^2 = 9$

26. **Geometry** The circle with center $(2, 3)$ and the circle with center $(-1, -1)$ are tangent at the point $(5, 7)$.

a. Find an equation for the small circle.

b. Find an equation for the large circle.

c. Find the equation of the line that is tangent to both circles.



Geometry **Write the equation of each circle.**

27. center $(-4, 0)$ and circumference 16π

28. center $\left(\frac{2}{3}, \frac{5}{8}\right)$ and area 49π

MULTI-STEP TEST PREP

29. This problem will prepare you for the Multi-Step Test Prep on page 758.

The orbit of Venus is nearly circular. An astronomer develops a model for the orbit in which the Sun has coordinates $(-5, 20)$, the circular orbit of Venus passes through $(62, 20)$, and each unit of the coordinate plane represents 1 million miles.

a. Write an equation for the orbit of Venus.

b. How far is Venus from the Sun?

c. How far does Venus travel as it makes one complete orbit of the Sun?

30. **Entertainment** A radio station emits a signal that can be received by anyone within 120 miles of the station's transmitter. Write and graph an inequality for the region covered by the radio station with the transmitter located at $(0, 0)$.

31. **Critical Thinking** Is it possible to have two different lines that are tangent to the same circle at the same point? Explain.

32. **Write About It** How could you show that the line with equation $y = -\frac{5}{12}x + \frac{28}{3}$ is tangent to the circle with equation $169 = (x - 3)^2 + (y + 6)^2$ at the point $(8, 6)$?

TEST PREP

33. Which of the following lines is tangent at $(13, 9)$ to the circle with center $(5, 3)$?

Ⓐ $y = \frac{3}{4}x - \frac{3}{4}$ Ⓑ $y = -\frac{3}{4}x + \frac{75}{4}$ Ⓒ $y = -\frac{4}{3}x + \frac{79}{3}$ Ⓓ $y = \frac{4}{3}x - \frac{25}{3}$

34. Which of the following points is inside the circle with the equation $121 = (x - 5)^2 + (y + 9)^2$?

Ⓕ $(12, 2)$ Ⓖ $(-8, 6)$ Ⓗ $(2, -6)$ Ⓙ $(-9, -3)$

35. **Short Response** Give the equation of a circle with center $(-4, 8)$ and radius $r = 9$.

CHALLENGE AND EXTEND

36. Consider the circle with equation $100 = x^2 + (y - 4)^2$
   - **a.** Find the equation of the tangents of the circle at $(8, 10)$ and at $(8, -2)$.
   - **b.** Find where the equations in part **a** intersect.
   - **c.** Find the distance from the point of intersection to the tangent points.

37. The lines $y = -3x + 1$ and $y = 2x - 9$ each contain diameters of a particular circle. The point $(9, 19)$ is on the circle.
   - **a.** Find the center of the circle.
   - **b.** Write the equation of the circle.

Graph each system of inequalities.

38. $\begin{cases} x - 3y > -12 \\ (x - 2)^2 + (y - 1)^2 \le 49 \end{cases}$

39. $\begin{cases} (x - 3)^2 + (y - 2)^2 \le 36 \\ (x - 4)^2 + (y + 4)^2 \le 25 \end{cases}$

SPIRAL REVIEW

Write the equation of each line. *(Lesson 2-4)*

40. slope 2 through $(1, 4)$

41. slope $\frac{1}{2}$ through $(-2, 1)$

42. slope $\frac{4}{3}$ and y-intercept 1

43. **Travel** Patrick drives a bus. When he picks up 20 passengers or fewer, his route takes him 15 minutes plus half a minute for each passenger. When Patrick picks up more than 20 passengers, his route takes him 20 minutes plus 1 minute for every passenger. *(Lesson 9-2)*
   - **a.** Write a piecewise function for the amount of time that Patrick's route requires.
   - **b.** Graph the function.
   - **c.** How long does it take Patrick to pick up 20 passengers?

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens. *(Lesson 10-1)*

44. $\frac{y^2}{3} = x$

45. $16y^2 = -x$

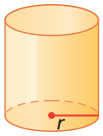
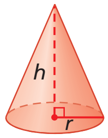
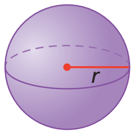
46. $4x^2 - 9y^2 = 36$

Connecting Algebra to **Geometry**

See Skills Bank page S64

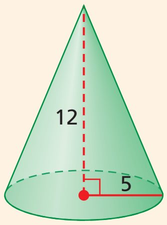
Surface Area and Volume

You can use formulas to find the surface area and volume of three-dimensional figures such as cylinders, cones, and spheres.

Solid	Cylinder with radius r and height h 	Cone with radius r and height h 	Sphere with radius r 
Volume	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$
Surface Area	$S = 2\pi r(r + h)$	$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$	$S = 4\pi r^2$

Example

Find the surface area and volume of the cone shown.



In order to use the formulas, identify the radius and height of the cone. $r = 5$ and $h = 12$. Find the surface area. Use the formula.

$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$ *Formula for surface area of a cone.*

$S = \pi \cdot 5\sqrt{(5)^2 + (12)^2} + \pi(5)^2$ *Substitute 5 for r and 12 for h.*

$S = 90\pi$ *Simplify.*

Find the volume.

$V = \frac{1}{3}\pi r^2 h$ *Formula for the volume of a cone.*

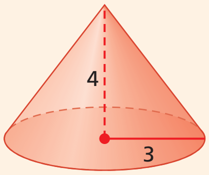
$V = \frac{1}{3}\pi(5)^2(12)$ *Substitute 5 for r and 12 for h.*

$V = 100\pi$ *Simplify.*

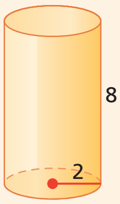
Try This

Find the surface area and volume of each figure.

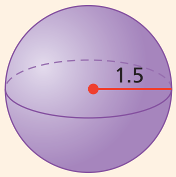
1.



2.



3.



10-3 Ellipses

Objectives
Write the standard equation for an ellipse.

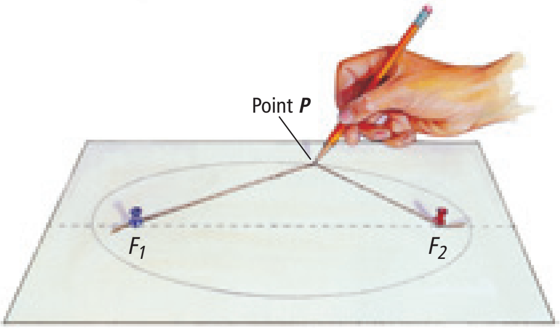
Graph an ellipse, and identify its center, vertices, co-vertices, and foci.

Vocabulary
ellipse
focus of an ellipse
major axis
vertices of an ellipse
minor axis
co-vertices of an ellipse

Who uses this?
The whispering gallery at the Chicago Museum of Science and Industry was designed by using an ellipse. (See Exercise 31.)

If you pulled the center of a circle apart into two points, it would stretch the circle into an ellipse.

An **ellipse** is the set of points $P(x, y)$ in a plane such that the sum of the distances from any point P on the ellipse to two fixed points F_1 and F_2, called the **foci** (singular: focus), is the constant sum $d = PF_1 + PF_2$. This distance d can be represented by the length of a piece of string connecting two pushpins located at the foci.



You can use the distance formula to find the constant sum of an ellipse.

EXAMPLE 1 **Using the Distance Formula to Find the Constant Sum of an Ellipse**

Find the constant sum for an ellipse with foci $F_1\,(-3, 0)$ and $F_2\,(3, 0)$ and the point on the ellipse $(0, 4)$.

$d = PF_1 + PF_2$ *Definition of the constant sum of an ellipse*

$d = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$ *Distance Formula*

$d = \sqrt{(-3 - 0)^2 + (0 - 4)^2} + \sqrt{(3 - 0)^2 + (0 - 4)^2}$ *Substitute.*

$d = \sqrt{25} + \sqrt{25}$ *Simplify.*

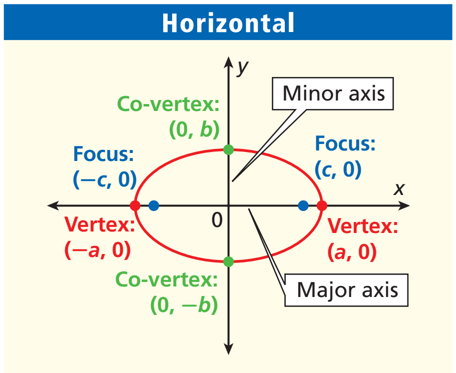
$d = 10$

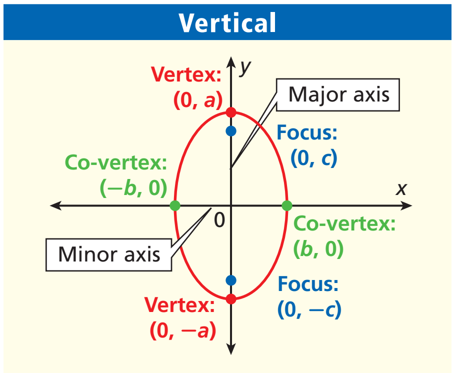
The constant sum is 10.

CHECK IT OUT! **1.** Find the constant sum for an ellipse with foci $F_1\,(0, -8)$ and $F_2\,(0, 8)$ and the point on the ellipse $(0, 10)$.

Instead of a single radius, an ellipse has two axes. The longer axis of an ellipse is the **major axis** and passes through both foci. The endpoints of the major axis are the **vertices of the ellipse**. The shorter axis of an ellipse is the **minor axis**. The endpoints of the minor axis are the **co-vertices of the ellipse**. The major axis and minor axis are perpendicular and intersect at the center of the ellipse.

The standard form of an ellipse centered at (0, 0) depends on whether the major axis is horizontal or vertical.





The values a, b, and c are related by the equation $c^2 = a^2 - b^2$. Also note that the length of the major axis is $2a$, the length of the minor axis is $2b$, and $a > b$.

Know it! Note

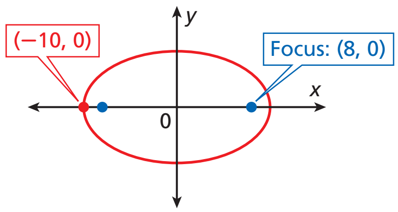
Standard Form for the Equation of an Ellipse Center at (0, 0)

MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$

EXAMPLE 2 Using Standard Form to Write an Equation for an Ellipse

Write an equation in standard form for each ellipse with center (0, 0).

A



Step 1 Choose the appropriate form of equation.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ *The horizontal axis is longer.*

Step 2 Identify the values of a and c.

$a = 10$ *The vertex $(-10, 0)$ gives the value of a.*

$c = 8$ *The focus $(8, 0)$ gives the value of c.*

Step 3 Use the relationship $c^2 = a^2 - b^2$ to find b^2.

$8^2 = 10^2 - b^2$ *Substitute 10 for a and 8 for c.*

$b^2 = 36$

Step 4 Write the equation.

$\frac{x^2}{100} + \frac{y^2}{36} = 1$ *Substitute the values into the equation of an ellipse.*

Write an equation in standard form for each ellipse with center $(0, 0)$.

B the ellipse with vertex $(0, 8)$ and co-vertex $(3, 0)$

Step 1 Choose the appropriate form of equation.

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$ *The vertex is on the y-axis.*

Step 2 Identify the values of a and b.

$a = 8$ *The vertex $(0, 8)$ gives the value of a.*

$b = 3$ *The co-vertex $(0, 3)$ gives the value of b.*

Step 3 Write the equation.

$$\frac{y^2}{64} + \frac{x^2}{9} = 1$$ *Substitute the values into the equation of an ellipse.*

CHECK IT OUT! Write an equation in standard form for each ellipse with center $(0, 0)$.

2a. Vertex $(9, 0)$ and co-vertex $(0, 5)$

2b. Co-vertex $(4, 0)$ focus $(0, 3)$

Ellipses may also be translated so that the center is not the origin.

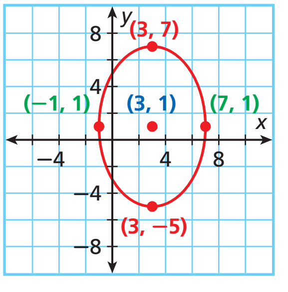
Know it! Note

Standard Form for the Equation of an Ellipse **Center at (h, k)**

MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h + a, k), (h - a, k)$	$(h, k + a), (h, k - a)$
Foci	$(h + c, k), (h - c, k)$	$(h, k + c), (h, k - c)$
Co-vertices	$(h, k + b), (h, k - b)$	$(h + b, k), (h - b, k)$

EXAMPLE 3 Graphing Ellipses

Graph the ellipse $\frac{(x-3)^2}{16} + \frac{(y-1)^2}{36} = 1$.



Step 1 Rewrite the equation as

$$\frac{(x-3)^2}{4^2} + \frac{(y-1)^2}{6^2} = 1.$$

Step 2 Identify the values of h, k, a, and b.

$h = 3$ and $k = 1$, so the center is $(3, 1)$.

$a = 6$ and $b = 4$; Because $6 > 4$, the major axis is vertical.

Step 3 The vertices are $(3, 1 \pm 6)$, or $(3, 7)$ and $(3, -5)$, and the co-vertices are $(3 \pm 4, 1)$, or $(7, 1)$ and $(-1, 1)$.

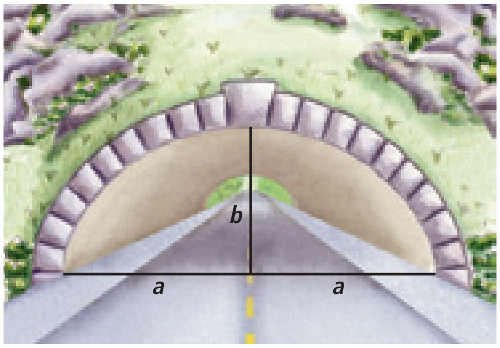
CHECK IT OUT! Graph each ellipse.

3a. $\frac{x^2}{64} + \frac{y^2}{25} = 1$

3b. $\frac{(x-2)^2}{25} + \frac{(y-4)^2}{9} = 1$

EXAMPLE 4 *Engineering Application*

A road passes through a tunnel in the form of a semi-ellipse. In order to widen the road to accommodate more traffic, engineers must design a larger tunnel that is twice as wide and 1.5 times as tall as the original tunnel. The design for the original tunnel can be modeled by the equation $\frac{x^2}{100} + \frac{y^2}{64} = 1$, measured in feet.



a. Find the dimensions of the larger tunnel.

Step 1 Find the dimensions of the original tunnel.

Because $100 > 64$, the major axis of the tunnel is horizontal.

$a^2 = 100$, so $a = 10$ and the width of the tunnel is $2a = 20$ ft.

$b^2 = 64$, so $b = 8$ and the height of the tunnel is 8 ft.

Step 2 Find the dimensions of the larger tunnel.

The width of the larger tunnel is $2(20) = 40$ ft.

The height is $1.5(8) = 12$ ft.

b. Write an equation for the design of the larger tunnel.

Step 1 Use the dimensions of the larger tunnel to find the values of a and b.

For the larger tunnel, $a = 20$ and $b = 12$.

Step 2 Write the equation.

The equation in standard form for the larger tunnel is $\frac{x^2}{20^2} + \frac{y^2}{12^2} = 1$, or $\frac{x^2}{400} + \frac{y^2}{144} = 1$.

CHECK IT OUT!

Engineers have designed a tunnel with the equation $\frac{x^2}{64} + \frac{y^2}{36} = 1$, measured in feet. A design for a larger tunnel needs to be twice as wide and 3 times as tall.

4a. Find the dimensions for the larger tunnel.

4b. Write an equation for the design of the larger tunnel.

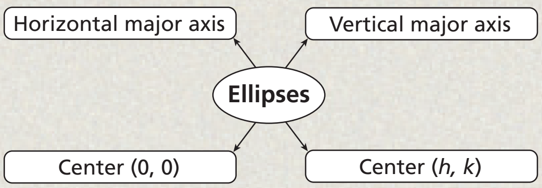
THINK AND DISCUSS

1. Explain where the foci are located in relation to the vertices.

2. Compare circles and ellipses by using lines of symmetry.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Give an equation for each type of ellipse.



10-3 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-3
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How can you tell the difference between the *major axis* and the *minor axis* of an ellipse?

SEE EXAMPLE 1 p. 736

Find the constant sum of an ellipse with the given foci and point on the ellipse.

2. $F_1(-5, 0), F_2(5, 0), P(0, -12)$
3. $F_1(0, -12), F_2(0, 12), P(9, 0)$

SEE EXAMPLE 2 p. 737

Multi-Step **Write an equation in standard form for each ellipse with center $(0, 0)$.**

4. vertex $(-9, 0)$, co-vertex $(0, 7)$
5. vertex $(0, 25)$, focus $(0, -20)$
6. co-vertex $(10, 0)$, focus $(0, 24)$
7. vertex $(-7, 0)$, focus $(\sqrt{13}, 0)$

SEE EXAMPLE 3 p. 738

Graph each ellipse.

8. $\frac{x^2}{36} + \frac{y^2}{81} = 1$
9. $\frac{x^2}{121} + \frac{y^2}{49} = 1$
10. $\frac{(x-5)^2}{16} + \frac{(y+2)^2}{36} = 1$
11. $\frac{(x+1)^2}{64} + \frac{(y-6)^2}{9} = 1$

SEE EXAMPLE 4 p. 739

12. **Engineering** Engineers are building semi-elliptical bridges across two rivers. The larger river is 4 times as wide as the smaller river and must accommodate boats that are 3 times as tall. The equation for the bridge over the smaller river is $\frac{x^2}{225} + \frac{y^2}{144} = 1$, measured in feet.
    a. Find the dimensions of the larger bridge.
    b. Write an equation for the design of the larger bridge.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–14	1
15–18	2
19–22	3
23	4

Extra Practice

Skills Practice p. S22
Application Practice p. S41

Find the constant sum of an ellipse with the given foci and point on the ellipse.

13. $F_1(-20, 0), F_2(20, 0), P(-21, 0)$
14. $F_1(0, -8), F_2(0, 8), P(9, 13.6)$

Multi-Step **Write an equation in standard form for each ellipse with center $(0, 0)$.**

15. vertex $(5, 0)$, co-vertex $(0, -2)$
16. co-vertex $(0, -8)$, focus $(6, 0)$
17. co-vertex $(4, 0)$, focus $(0, -3)$
18. vertex $(0, -9)$, focus $(0, 3\sqrt{5})$

Graph each ellipse.

19. $\frac{(x+2)^2}{169} + \frac{(y-7)^2}{25} = 1$
20. $\frac{(x-6)^2}{36} + \frac{(y-4)^2}{100} = 1$
21. $\frac{x^2}{256} + \frac{y^2}{196} = 1$
22. $\frac{x^2}{225} + \frac{y^2}{289} = 1$

23. **National Parks** South of the White House in Washington, D.C., is the President's Park South, or the Ellipse, which hosts events such as the White House Garden Tours. The Ellipse is 880 ft from north to south and 1057 ft from east to west. Write an equation for the Ellipse, centered at the origin.

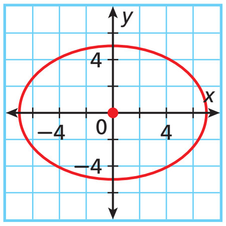
Write an equation in standard form for each ellipse.

24. tangent to the x-axis at $(9, 0)$ and tangent to the y-axis at $(0, -6)$
25. center $(-4, 7)$, vertex $(-4, -3)$, focus $(-4, 0)$

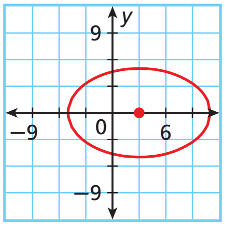
26. Estimation An ellipse has a vertex at the point $(2.4, -6.1)$, focus $(0.35, -6.1)$, and center $(-4.5, -6.1)$. Estimate the coordinates of the co-vertices.

Write an equation for each graph, and give the domain and range. (*Hint:* The domain and range depend on the center and the lengths of the major and minor axes.)

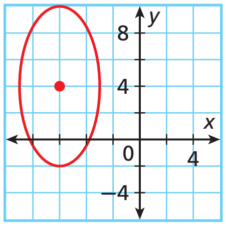
27.



28.



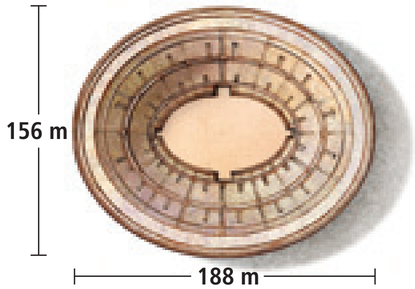
29.



LINK History

Located in Rome, Italy, the Colosseum was designed to hold as many as 50,000 spectators, who would arrive and leave through 80 entrances.

30. History The Roman Colosseum is shaped like a large ellipse, with an external width of 188 m and a length of 156 m. Write an equation that can be used to model the shape of the Colosseum.



31. Architecture As a result of their unique elliptical shapes, whispering galleries enable the smallest sound generated at one focus to be carried across the room to the other focus. The whispering gallery at the Chicago Museum of Science and Industry is 47 ft 4 in. long and 13 ft 6 in. wide.

a. Supposing that the center of the floor of the whispering gallery is located at the origin, write an equation for the gallery floor.

b. Find the coordinates of the foci. How far apart are they?

Find the center, vertices, co-vertices, foci, domain, and range of each ellipse.

32. $\frac{(x-1)^2}{225} + \frac{(y+5)^2}{324} = 1$

33. $9(x+9)^2 + 81(y+4)^2 = 729$

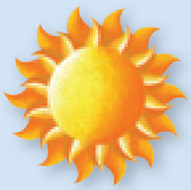
34. Critical Thinking An ellipse is defined by the distance $PF_1 + PF_2 = d$. Could the distance between the foci be less than $PF_1 + PF_2$? Explain.

35. Geometry The area of an ellipse in standard form is given by $A = \pi ab$.

a. Critical Thinking How is the formula for the area of an ellipse related to the formula for the area of a circle?

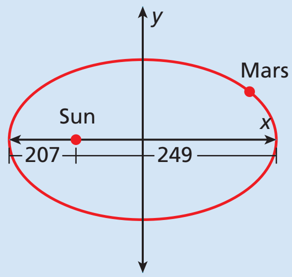
b. Find the area of $\frac{(x+2)^2}{169} + \frac{(y-7)^2}{25} = 1$.

MULTI-STEP TEST PREP



36. This problem will prepare you for the Multi-Step Test Prep on page 758.

The figure shows the elliptical orbit of Mars, where each unit of the coordinate plane represents 1 million kilometers. As shown, the planet's maximum distance from the Sun is 249 million kilometers and its minimum distance from the Sun is 207 million kilometers.



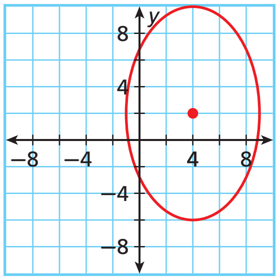
a. The Sun is at one focus of the ellipse. What are the coordinates of the Sun?

b. What is the length of the minor axis of the ellipse?

c. Write an equation that models the orbit of Mars.

37. **Write About It** How is the distance $PF_1 + PF_2$ related to the length of the ellipse's major axis?

TEST PREP



38. Which of the following is the equation for the graph?

Ⓐ $\frac{(x-4)^2}{25} - \frac{(y-2)^2}{64} = 1$

Ⓑ $25(x-4)^2 + 64(y-2)^2 = 1600$

Ⓒ $\frac{(x-4)^2}{64} + \frac{(y-2)^2}{25} = 1$

Ⓓ $64(x-4)^2 + 25(y-2)^2 = 1600$

39. The graph of which equation has the greatest distance between foci?

Ⓕ $\frac{(x-12)^2}{49} + \frac{(y+23)^2}{25} = 1$

Ⓗ $\frac{(x-1)^2}{20} + \frac{(y-1)^2}{150} = 1$

Ⓖ $\frac{x^2}{625} + \frac{y^2}{576} = 1$

Ⓙ $\frac{x^2}{175} + \frac{y^2}{225} = 1$

40. **Short Response** Give an equation for the ellipse with center $(2, -3)$, focus $(26, -3)$, and major axis length 50.

CHALLENGE AND EXTEND

41. The eccentricity of an ellipse is defined as $e = \frac{c}{a}$. Recall that $c^2 = a^2 - b^2$ for an ellipse in standard form.
   - **a.** Find the eccentricity of the ellipse with equation $\frac{x^2}{841} + \frac{y^2}{400} = 1$.
   - **b.** Find the equation of the ellipse with vertices $(13, 0)$ and $(-13, 0)$ and $e = \frac{5}{13}$.
   - **c.** What are the possible values for the eccentricity of an ellipse?
   - **d.** Describe the relationship between eccentricity and the shape of an ellipse.

42. **Astronomy** The path that the Moon travels around Earth is an ellipse with Earth at one focus. The length of the major axis is about 477,700 mi, and the length of the minor axis is about 476,980 mi.
   - **a.** Write an equation for the Moon's orbit.
   - **b.** Find the minimum and maximum distances from Earth to the Moon.

43. Write an equation for an ellipse with foci $F_1\ (-3, 0)$ and $F_2\ (3, 0)$ and a constant sum of 10. (*Hint:* Use $d = PF_1 + PF_2$ and the point (x, y).)

SPIRAL REVIEW

44. **Recreation** Rhonda exercises no more than 60 minutes a day. She runs and lifts weights. *(Lesson 2-5)*
   - **a.** Write and graph an inequality for the number of minutes that Rhonda can run and lift weights each day.
   - **b.** How long does Rhonda lift weights if she runs for 25 minutes?

Given $f(x) = 2x^2 + 6$ and $g(x) = -\frac{1}{2}x + 4$, find each value. *(Lesson 9-4)*

45. $f(g(2))$ 46. $g(f(2))$ 47. $f(g(-2))$ 48. $g(f(-2))$

Write the equation of each circle. *(Lesson 10-2)*

49. center $(0, -1)$, containing the point $(6, 7)$ 50. center $(-5, 9)$, radius $r = 6$

10-3 Algebra LAB

Use with Lesson 10-3

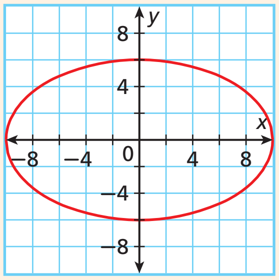
Locate the Foci of an Ellipse

You have seen how an ellipse is defined by its foci and how to draw an ellipse given the foci. You can find the foci of a given ellipse by using a compass.

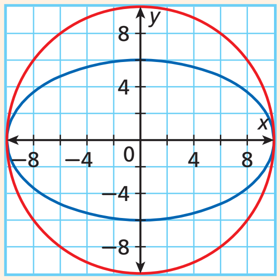
Activity

Find the foci of the ellipse with major axis length 20 and minor axis length 12.

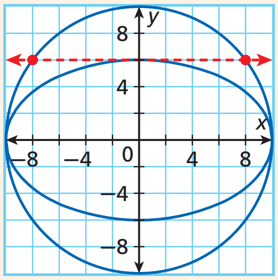
1 Graph the ellipse so that the center is at $(0, 0)$. Mark the endpoints of the major axis: $(-10, 0)$ and $(10, 0)$. Mark the endpoints of the minor axis at $(0, -6)$ and $(0, 6)$. Draw the ellipse.



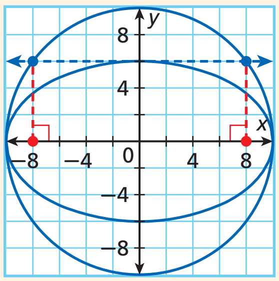
2 Use a compass to draw a circle with radius 10 units centered at $(0, 0)$.



3 Draw the line with equation $y = 6$ on the graph. Mark the points where the line intersects the circle.



4 Draw lines from the points where $y = 6$ intersects the circle perpendicular to the x-axis. The foci of the ellipse are the points where the perpendicular lines intersect the x-axis. Where are the foci of your ellipse? Check by using the formula $c^2 = a^2 - b^2$.



Try This

Use a compass to find the foci of each ellipse with a horizontal major axis.

1. major axis length 26, minor axis length 10

2. major axis length 34, minor axis length 16

Use a compass to find the foci of each ellipse with a vertical major axis.

3. major axis length 25, minor axis length 24

4. major axis length 20, minor axis length 12

5. **Critical Thinking** In Step 3 above, what other line could you have drawn to get the same foci?

6. **Critical Thinking** Why does this method of locating the foci of an ellipse work?

10-4 Hyperbolas

Objectives

Write the standard equation for a hyperbola.

Graph a hyperbola, and identify its vertices, co-vertices, center, foci, and asymptotes.

Vocabulary

hyperbola
focus of a hyperbola
branch of a hyperbola
transverse axis
vertices of a hyperbola
conjugate axis
co-vertices of a hyperbola

Who uses this?

Biologists use hyperbolas to locate and track whales based on the sounds that the whales make. (See Exercise 33.)

What would happen if you pulled the two foci of an ellipse so far apart that they moved outside the ellipse? The result would be a *hyperbola*, another conic section.

A **hyperbola** is the set of points $P(x, y)$ in a plane such that the difference of the distances from P to fixed points F_1 and F_2, the **foci**, is constant. For a hyperbola, $d = |PF_1 - PF_2|$, where d is the constant difference. You can use the distance formula to find the equation of a hyperbola.

EXAMPLE 1 Using the Distance Formula to Find the Constant Difference of a Hyperbola

Find the constant difference for a hyperbola with foci $F_1\,(-5, 0)$ and $F_2\,(5, 0)$ and the point on the hyperbola $(4, 0)$.

$d = |PF_1 - PF_2|$ *Definition of the constant difference of a hyperbola*

$= \left|\sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} - \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}\right|$ *Distance Formula*

$= \left|\sqrt{(-5 - 4)^2 + (0 - 0)^2} - \sqrt{(5 - 4)^2 + (0 - 0)^2}\right|$ *Substitute.*

$= \left|\sqrt{81} - \sqrt{1}\right|$ *Simplify.*

$= 8$

The constant difference is 8.

CHECK IT OUT!

1. Find the constant difference for a hyperbola with foci at $F_1\,(0, -10)$ and $F_2\,(0, 10)$ and the point on the hyperbola $(6, 7.5)$.

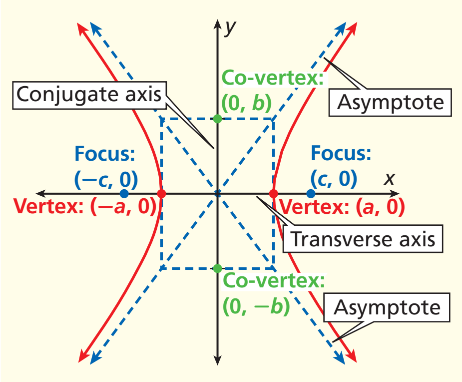
As the graphs in the following table show, a hyperbola contains two symmetrical parts called **branches**.

A hyperbola also has two axes of symmetry. The **transverse axis** of symmetry contains the vertices and, if it were extended, the foci of the hyperbola. The **vertices of a hyperbola** are the endpoints of the transverse axis.

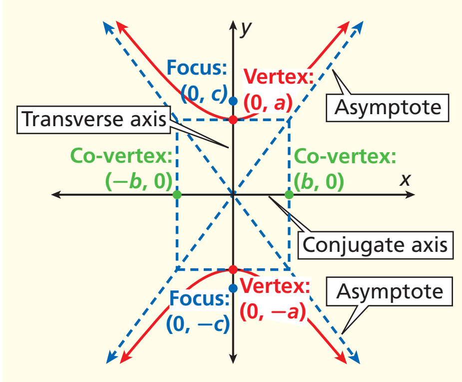
The **conjugate axis** of symmetry separates the two branches of the hyperbola. The **co-vertices of a hyperbola** are the endpoints of the conjugate axis. The transverse axis is not always longer than the conjugate axis.

The standard form of the equation of a hyperbola depends on whether the hyperbola's transverse axis is horizontal or vertical.

Horizontal



Vertical



The values a, b, and c are related by the equation $c^2 = a^2 + b^2$. Also note that the length of the transverse axis is $2a$ and the length of the conjugate axis is $2b$.

Know it! Note

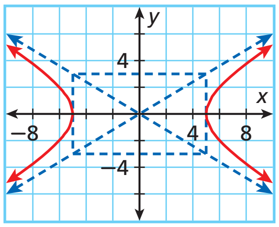
Standard Form for the Equation of a Hyperbola Center at (0, 0)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

EXAMPLE 2 Writing Equations of Hyperbolas

Write an equation in standard form for each hyperbola.

A



Step 1 Identify the form of the equation.

The graph opens horizontally, so the equation will be in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Step 2 Identify the center and vertices.

The center of the graph is $(0, 0)$, the vertices are $(-5, 0)$ and $(5, 0)$, and the co-vertices are $(0, -3)$ and $(0, 3)$. So $a = 5$ and $b = 3$.

Step 3 Write the equation.

Because $a = 5$ and $b = 3$, the equation of the graph is $\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1$, or $\frac{x^2}{25} - \frac{y^2}{9} = 1$.

Write an equation in standard form for each hyperbola.

B **the hyperbola with center (0, 0), vertex (0, 12), and focus (0, 20)**

Step 1 Because the vertex and the focus are on the vertical axis, the transverse axis is vertical and the equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Step 2 $a = 12$ and $c = 20$; Use $c^2 = a^2 + b^2$ to solve for b^2.

$20^2 = 12^2 + b^2$ *Substitute 12 for a and 20 for c.*

$256 = b^2$

Step 3 The equation of the hyperbola is $\frac{y^2}{144} - \frac{x^2}{256} = 1$

CHECK IT OUT!

Write an equation in standard form for each hyperbola.

2a. Vertex (0, 9), co-vertex (7, 0)

2b. Vertex (8, 0), focus (10, 0)

As with circles and ellipses, hyperbolas do not have to be centered at the origin.

Know it! Note

Standard Form for the Equation of a Hyperbola Center at (h, k)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$
Asymptotes	$y-k = \pm\frac{b}{a}(x-h)$	$y-k = \pm\frac{a}{b}(x-h)$

EXAMPLE 3 Graphing a Hyperbola

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

A $\frac{y^2}{25} - \frac{x^2}{36} = 1$

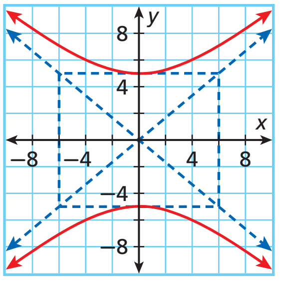
Step 1 The equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, so the transverse axis is vertical with center (0, 0).

Step 2 Because $a = 5$ and $b = 6$, the vertices are (0, 5) and (0, −5) and the co-vertices are (6, 0) and (−6, 0).

Step 3 The equations of the asymptotes are $y = \frac{5}{6}x$ and $y = -\frac{5}{6}x$.

Step 4 Draw a box by using the vertices and co-vertices. Draw the asymptotes through the corners of the box.

Step 5 Draw the hyperbola by using the vertices and the asymptotes.

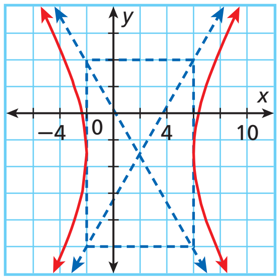


Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

B $\dfrac{(x-2)^2}{16} - \dfrac{(y+3)^2}{49} = 1$

Step 1 The equation is in the form $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ so the transverse axis is horizontal with center $(2, -3)$.

Step 2 Because $a = 4$ and $b = 7$, the vertices are $(6, -3)$ and $(-2, -3)$ and the co-vertices are $(2, 4)$ and $(2, -10)$.



Step 3 The equations of the asymptotes are $y + 3 = \frac{7}{4}(x - 2)$ and $y + 3 = -\frac{7}{4}(x - 2)$.

Step 4 Draw a box by using the vertices and co-vertices. Draw the asymptotes through the corners of the box.

Step 5 Draw the hyperbola by using the vertices and the asymptotes.

Caution!

The graph of the hyperbola must pass through the vertices and approach both of the asymptotes.

CHECK IT OUT! Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

3a. $\dfrac{x^2}{16} - \dfrac{y^2}{36} = 1$ **3b.** $\dfrac{(y+5)^2}{9} - \dfrac{(x-1)^2}{1} = 1$

Notice that as the parameters change, the graph of the hyperbola is transformed.

Parameter	Transformation
h	Translates the graph left for $h > 0$ and right for $h < 0$
k	Translates the graph up for $k > 0$ and down for $k < 0$
a	Stretches the graph in the direction of the transverse axis; as *a* increases, the vertices move farther apart.
b	Stretches the graph in the direction of the conjugate axis; as *b* increases, the co-vertices move farther apart.

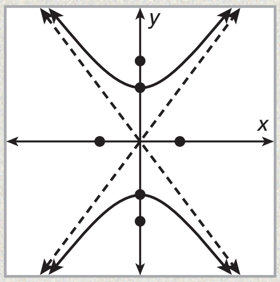
THINK AND DISCUSS

1. When is the transverse axis of a hyperbola shorter than its conjugate axis?

2. How do you tell when a hyperbola has a horizontal transverse axis?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Label all of the parts of the hyperbola.



10-4 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The vertices of a hyperbola lie on the __?__ (*transverse axis* or *conjugate axis*).

SEE EXAMPLE 1
p. 744

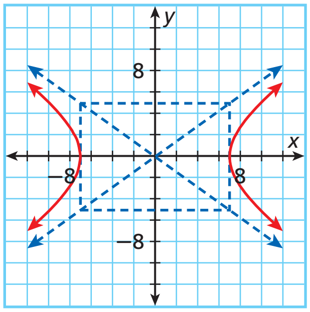
Find the constant difference for a hyperbola with the given foci and point on the hyperbola.

2. $F_1(-13, 0)$, $F_2(13, 0)$, $P(5, 0)$
3. $F_1(0, -17)$, $F_2(0, 17)$, $P(0, -15)$

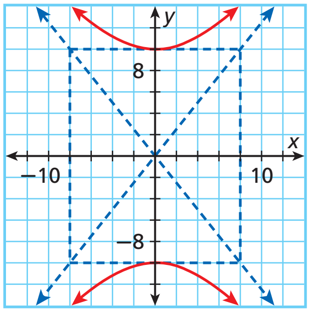
SEE EXAMPLE 2
p. 745

Write an equation in standard form for each hyperbola.

4. center $(0, 0)$, vertex $(0, 5)$, and focus $(0, 13)$
5. center $(0, 0)$, vertex $(9, 0)$, and co-vertex $(0, 7)$
6.



7.



SEE EXAMPLE 3
p. 746

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

8. $\frac{x^2}{49} - \frac{y^2}{36} = 1$
9. $\frac{x^2}{25} - \frac{y^2}{64} = 1$
10. $\frac{y^2}{25} - \frac{x^2}{36} = 1$
11. $\frac{y^2}{100} - \frac{x^2}{81} = 1$
12. $\frac{(x-4)^2}{9} - \frac{(y-3)^2}{64} = 1$
13. $\frac{(x-4)^2}{16} - \frac{(y+6)^2}{49} = 1$
14. $\frac{(y+8)^2}{36} - \frac{(x+3)^2}{25} = 1$
15. $\frac{(y+7)^2}{4} - \frac{x^2}{25} = 1$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–17	1
18–21	2
22–29	3

Extra Practice

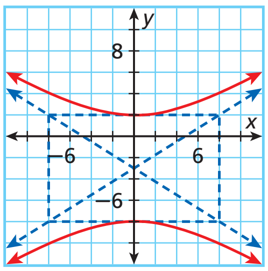
Skills Practice p. S22
Application Practice p. S41

Find the constant difference for a hyperbola with the given foci and point on the hyperbola.

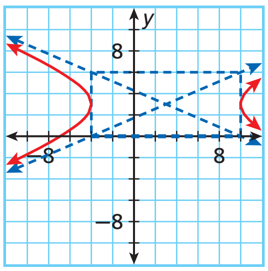
16. $F_1(0, -10)$, $F_2(0, 10)$, $P(0, 6)$
17. $F_1(-29, 0)$, $F_2(29, 0)$, $P(21, 0)$

Write an equation in standard form for each hyperbola.

18. center $(0, 0)$, vertex $(15, 0)$, co-vertex $(0, -13)$
19. center $(0, 0)$, vertex $(-8, 0)$, focus $(17, 0)$
20.



21.



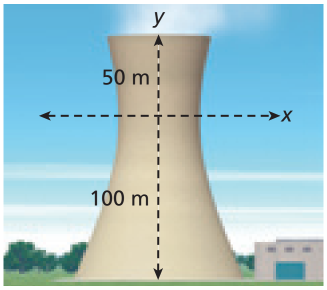
Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

22. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ **23.** $\frac{y^2}{25} - \frac{x^2}{81} = 1$ **24.** $\frac{y^2}{81} - \frac{x^2}{16} = 1$ **25.** $\frac{x^2}{4} - \frac{y^2}{121} = 1$

26. $\frac{(y-1)^2}{64} - \frac{(x+2)^2}{36} = 1$ **27.** $\frac{(x+5)^2}{25} - \frac{(y-3)^2}{16} = 1$

28. $\frac{(y-8)^2}{25} - \frac{(x+6)^2}{36} = 1$ **29.** $\frac{(x-6)^2}{9} - \frac{(y-2)^2}{16} = 1$

30. Architecture If the x-axis is placed at a height of 100 meters, the outer edge of a cooling tower can be modeled by the hyperbola $\frac{x^2}{900} - \frac{y^2}{1600} = 1$, measured in meters. If the tower is 150 meters tall, find the width of the cooling tower at the top.

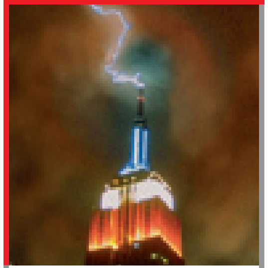


31. Critical Thinking What happens to the graph of $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ as the values of a increase? What happens to the graph of $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ as the values of b increase?

32. Physics Two people standing 10,000 feet apart see lightning strike. One person hears the thunder 5 seconds after the other person. Because sound travels at 1100 feet per second, one person is 5500 feet farther from the lightning strike than the other. The possible locations of the strike then form a hyperbola with the two people at the foci. Place the origin midway between the two people, and write an equation that could be used to represent the possible locations of the lightning strike.

LINK

Physics



The saying "lightning never strikes twice in the same place" is often disproven. The Empire State Building is struck by lightning about 100 times each year and serves as a lightning rod for the surrounding area.

33. Biology Two underwater listening devices 12,000 feet apart detect a whale call. One device detects the call 2 seconds before the other. The possible locations of the whale form a hyperbola with the two devices at the foci.

a. If the speed of sound in water is 5000 feet per second, write an equation for the possible locations of the whale. (*Hint:* Place the origin midway between the devices.)

b. What if...? Could the location of the whale be more precisely located if there were a third listening device? Explain.

34. Critical Thinking How could you identify the domain and range of a hyperbola? Explain.

35. Critical Thinking Consider a hyperbola with equation $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Which parameter—$a$, b, or c—has the greatest value? Which has the least value? Explain.

36. Write About It Suppose you have two hyperbolas that are the same except that the transverse axis and conjugate axis are switched. How does switching the axes affect the equations of the asymptotes for the two hyperbolas? Why?

MULTI-STEP TEST PREP

37. This problem will prepare you for the Multi-Step Test Prep on page 758.

A comet's path as it approaches the Sun is modeled by one branch of the hyperbola $\frac{y^2}{900} - \frac{x^2}{44{,}896} = 1$, where the Sun is at the corresponding focus. Each unit of the coordinate plane represents 1 million miles.

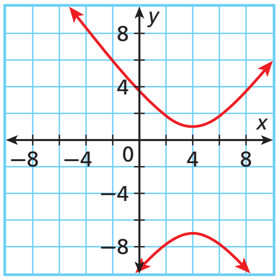
a. Find the coordinates of the Sun, assuming that it is at the focus with nonnegative coordinates.

b. How close does the comet come to the Sun?

c. When the comet is far from the Sun, the comet's path can be modeled by the hyperbola's asymptotes. Write the equations of the asymptotes.

TEST PREP

38. Which of the following is the equation of the graph shown?



Ⓐ $\dfrac{(x-3)^2}{16} - \dfrac{(y+4)^2}{9} = 1$ Ⓒ $\dfrac{(y-3)^2}{16} - \dfrac{(x+4)^2}{9} = 1$

Ⓑ $\dfrac{(x+3)^2}{16} - \dfrac{(y-4)^2}{9} = 1$ Ⓓ $\dfrac{(y+3)^2}{16} - \dfrac{(x-4)^2}{9} = 1$

39. Which of the following is an asymptote of the graph of $1 = \dfrac{x^2}{4} - \dfrac{y^2}{9}$?

Ⓕ $y = -\dfrac{2}{3}x$ Ⓖ $y = \dfrac{3}{2}x$ Ⓗ $y = -\dfrac{9}{4}x$ Ⓙ $y = \dfrac{4}{9}x$

40. The graph of which of the following equations will have the greatest distance between foci?

Ⓐ $\dfrac{(x-6)^2}{36} - \dfrac{(y+2)^2}{81} = 1$ Ⓒ $\dfrac{(y+115)^2}{49} - \dfrac{(x-225)^2}{100} = 1$

Ⓑ $\dfrac{(x+22)^2}{45} - \dfrac{(y-36)^2}{125} = 1$ Ⓓ $\dfrac{(y-59)^2}{90} - \dfrac{(x+76)^2}{95} = 1$

41. What is the length of the conjugate axis of the hyperbola with equation $\dfrac{x^2}{49} - \dfrac{y^2}{121} = 1$?

Ⓕ 7   Ⓖ 11   Ⓗ 14   Ⓙ 22

CHALLENGE AND EXTEND

Write an equation in standard form for each hyperbola.

42. co-vertex $(-12, 0)$, asymptote $y = -\dfrac{4}{3}x$

43. vertex $(27, -9)$, asymptote $y + 9 = -\dfrac{3}{5}(x - 7)$

44. The eccentricity of a hyperbola is defined as $e = \frac{c}{a}$. Recall that $c^2 = a^2 + b^2$ for a hyperbola in standard form.

a. Find the eccentricity of $\frac{(x-4)^2}{144} - \frac{(y+2)^2}{1225} = 1$.

b. Find the equation of a hyperbola with vertices $(0, 6)$ and $(0, -6)$, and eccentricity $e = \frac{4}{3}$.

c. What are the possible values for the eccentricity of a hyperbola?

d. Describe the relationship between eccentricity and the shape of a hyperbola.

45. Use the distance formula to write the equation of a hyperbola with foci at $F_1(-5, 0)$ and $F_2(5, 0)$ and $d = 8$. (*Hint:* Use $d = PF_1 - PF_2$ and the point (x, y).)

SPIRAL REVIEW

Graph each function by using a table. *(Lesson 5-1)*

46. $f(x) = 2x^2 + 3x - 6$ **47.** $f(x) = -x^2 + 2x + 5$ **48.** $f(x) = x^2 - 5x + 4$

49. Finance Carlton's starting salary was \$30,000. Every year, he received a raise of \$3000. Let x represent years and y represent Carlton's salary. *(Lesson 9-1)*

a. Write and graph an equation to represent this situation.

b. After how many years will Carlton earn \$60,000?

Write an equation in standard form for each ellipse with center $(0, 0)$. *(Lesson 10-3)*

50. vertex $(5, 0)$, co-vertex $(0, 4)$ **51.** vertex $(0, -2)$, focus $(0, \sqrt{2})$

10-5 Parabolas

Objectives

Write the standard equation of a parabola and its axis of symmetry.

Graph a parabola, and identify its focus, directrix, and axis of symmetry.

Vocabulary

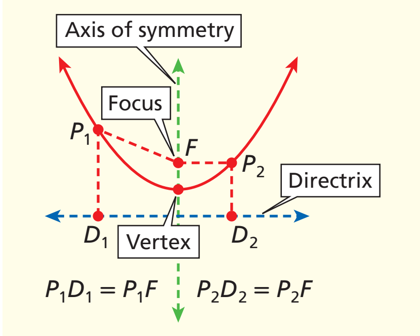
focus of a parabola
directrix

Why learn this?

Parabolas are used with microphones to pick up sounds from sports events. (See Example 4.)

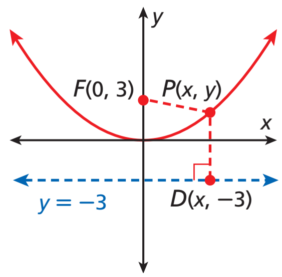
In Chapter 5, you learned that the graph of a quadratic function is a parabola. Because a parabola is a conic section, it can also be defined in terms of distance.

A parabola is the set of all points $P(x, y)$ in a plane that are an equal distance from both a fixed point, the **focus**, and a fixed line, the **directrix**. A parabola has an axis of symmetry perpendicular to its directrix and that passes through its vertex. The vertex of a parabola is the midpoint of the segment connecting the focus and the directrix.



EXAMPLE 1 Using the Distance Formula to Write the Equation of a Parabola

Use the Distance Formula to find the equation of a parabola with focus $F(0, 3)$ and directrix $y = -3$.



$PF = PD$	*Definition of a parabola*
$\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$	*Distance Formula*
$\sqrt{(x - 0)^2 + (y - 3)^2} = \sqrt{(x - x)^2 + (y + 3)^2}$	*Substitute (0, 3) for (x_1, y_1) and (x, −3) for (x_2, y_2).*
$\sqrt{x^2 + (y - 3)^2} = \sqrt{(y + 3)^2}$	*Simplify.*
$x^2 + (y - 3)^2 = (y + 3)^2$	*Square both sides.*
$x^2 + y^2 - 6y + 9 = y^2 + 6y + 9$	*Expand.*
$x^2 - 6y = 6y$	*Subtract y^2 and 9 from both sides.*
$x^2 = 12y$	*Add 6y to both sides.*
$y = \frac{1}{12}x^2$	*Solve for y.*

Remember!

The distance from a point to a line is defined as the length of the line segment from the point perpendicular to the line.

CHECK IT OUT!

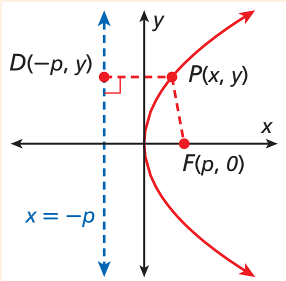
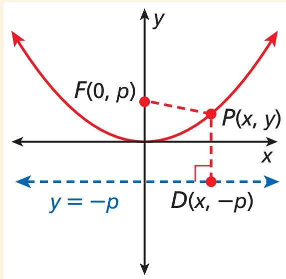
1. Use the Distance Formula to find the equation of a parabola with focus $F(0, 4)$ and directrix $y = -4$.

Previously, you have graphed parabolas with vertical axes of symmetry that open upward or downward. Parabolas may also have horizontal axes of symmetry and may open to the left or right.

The equations of parabolas use the parameter p. The $|p|$ gives the distance from the vertex to both the focus and the directrix.

Know it! Note

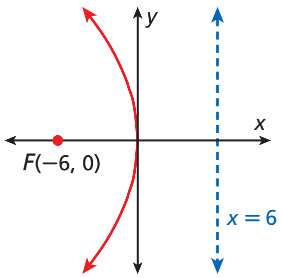
Standard Form for the Equation of a Parabola — **Vertex at (0, 0)**

AXIS OF SYMMETRY	HORIZONTAL $y = 0$	VERTICAL $x = 0$
Equation	$x = \frac{1}{4p}y^2$	$y = \frac{1}{4p}x^2$
Direction	Opens right if $p > 0$ Opens left if $p < 0$	Opens upward if $p > 0$ Opens downward if $p < 0$
Focus	$(p, 0)$	$(0, p)$
Directrix	$x = -p$	$y = -p$
Graph		

EXAMPLE 2 Writing Equations of Parabolas

Write the equation in standard form for each parabola.

A

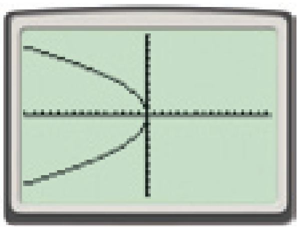


Step 1 Because the axis of symmetry is horizontal and the parabola opens to the left, the equation is in the form $x = \frac{1}{4p}y^2$ with $p < 0$.

Step 2 The distance from the focus $(-6, 0)$ to the vertex $(0, 0)$ is 6, so $p = -6$ and $4p = -24$.

Step 3 The equation of the parabola is $x = -\frac{1}{24}y^2$.

Check Use your graphing calculator. The graph of the equation appears to match.



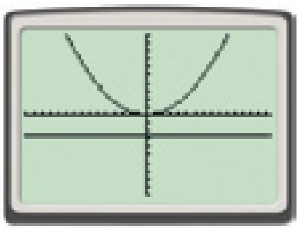
Write the equation in standard form for each parabola.

B the parabola with vertex $(0, 0)$ and directrix $y = -2.5$.

Step 1 Because the directrix is a horizontal line, the equation is in the form $y = \frac{1}{4p}x^2$. The vertex is above the directrix, so the graph will open upward.

Step 2 Because the directrix is $y = -2.5$, $p = 2.5$ and $4p = 10$.

Step 3 The equation of the parabola is $y = \frac{1}{10}x^2$.



Check Use your graphing calculator.

CHECK IT OUT! **Write the equation in standard form for each parabola.**

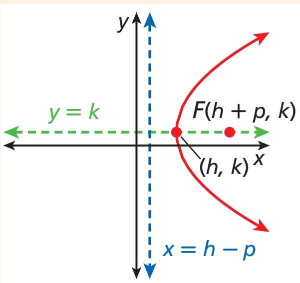
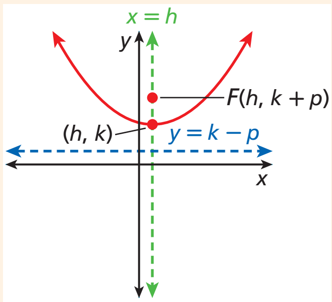
2a. vertex $(0, 0)$, directrix $x = 1.25$

2b. vertex $(0, 0)$, focus $(0, -7)$

The vertex of a parabola may not always be the origin. Adding or subtracting a value from x or y translates the graph of a parabola. Also notice that the values of p stretch or compress the graph.

Know it! Note

Standard Form for the Equation of a Parabola — Vertex at (h, k)

AXIS OF SYMMETRY	HORIZONTAL $y = k$	VERTICAL $x = h$
Equation	$x - h = \frac{1}{4p}(y - k)^2$	$y - k = \frac{1}{4p}(x - h)^2$
Direction	Opens right if $p > 0$ Opens left if $p < 0$	Opens upward if $p > 0$ Opens downward if $p < 0$
Focus	$(h + p, k)$	$(h, k + p)$
Directrix	$x = h - p$	$y = k - p$
Graph		

EXAMPLE 3 Graphing Parabolas

Find the vertex, value of p, axis of symmetry, focus, and directrix of the parabola $x - 2 = -\frac{1}{16}(y + 5)^2$. Then graph.

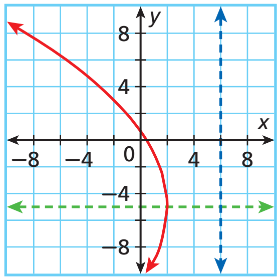
Step 1 The vertex is $(2, -5)$.

Step 2 $\frac{1}{4p} = -\frac{1}{16}$, so $4p = -16$ and $p = -4$.

Step 3 The graph has a horizontal axis of symmetry, with equation $y = -5$, and opens left.

Step 4 The focus is $(2 + (-4), -5)$, or $(-2, -5)$.

Step 5 The directrix is a vertical line $x = 2 - (-4)$, or $x = 6$.



CHECK IT OUT! **Find the vertex, value of p, axis of symmetry, focus, and directrix of each parabola. Then graph.**

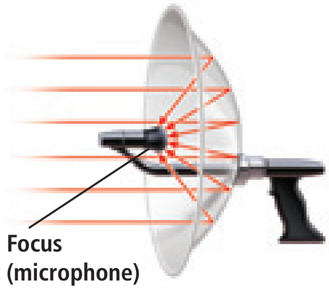
3a. $x - 1 = \frac{1}{12}(y - 3)^2$

3b. $y - 4 = -\frac{1}{2}(x - 8)^2$

Light or sound waves collected by a parabola will be reflected by the curve through the focus of the parabola, as shown in the figure. Waves emitted from the focus will be reflected out parallel to the axis of symmetry of a parabola. This property is used in communications technology.

EXAMPLE 4 Using the Equation of a Parabola

Engineers are constructing a parabolic microphone for use at sporting events. The surface of the parabolic microphone will reflect sounds to the focus of the microphone at the end of a part called a feedhorn. The equation for the cross section of the parabolic microphone dish is $x = \frac{1}{32}y^2$, measured in inches. How long should the engineers make the feedhorn?



The equation for the cross section is in the form $x = \frac{1}{4p}y^2$, so $4p = 32$ and $p = 8$. The focus should be 8 inches from the vertex of the cross section. Therefore, the feedhorn should be 8 inches long.

CHECK IT OUT! **4.** Find the length of the feedhorn for a microphone with a cross section equation $x = \frac{1}{44}y^2$.

THINK AND DISCUSS

1. By using the standard form of a parabola's equation, how can you tell which direction a parabola opens?

2. How does knowing the value of p help you in finding the focus and the directrix of a parabola?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Sketch an example and give an equation for each type of parabola.

Opens upward	Opens right
Parabola	
Opens downward	Opens left

10-5 Exercises

GUIDED PRACTICE

1. **Vocabulary** Describe the relationship between a parabola and its *directrix*.

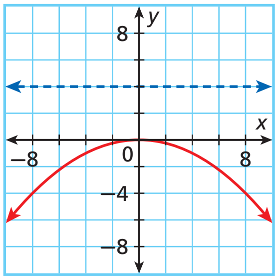
SEE EXAMPLE 1
p. 751

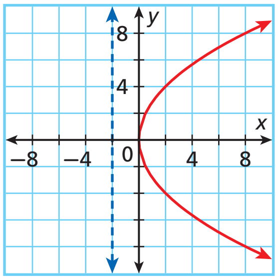
Use the distance formula to find the equation of a parabola with the given focus and directrix.

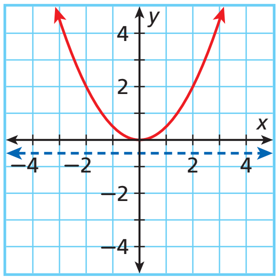
2. $F(0, -5)$, $y = 5$
3. $F(7, 0)$, $x = -7$
4. $F(-3, 0)$, $x = 6$

SEE EXAMPLE 2
p. 752

Write the equation in standard form for each parabola.

5.


6.


7.


8. vertex $(0, 0)$, focus $(0, 1)$
9. vertex $(0, 0)$, focus $(-8, 0)$

SEE EXAMPLE 3
p. 753

Find the vertex, value of p, axis of symmetry, focus, and directrix of each parabola, and then graph.

10. $y = \frac{1}{32}(x + 2)^2$
11. $x = \frac{1}{24}(y - 4)^2$
12. $y + 1 = \frac{1}{16}(x - 2)^2$

SEE EXAMPLE 4
p. 754

13. **Communications** The equation for the cross section of a parabolic satellite TV dish is $y = \frac{1}{38}x^2$, measured in inches. How far is the focus from the vertex of the cross section?

PRACTICE AND PROBLEM SOLVING

Independent Practice

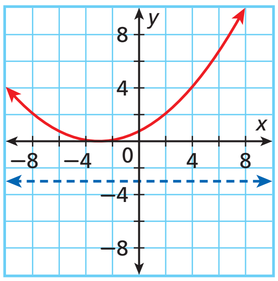
For Exercises	See Example
14–16	1
17–21	2
22–24	3
25	4

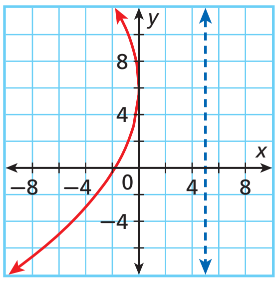
Extra Practice
Skills Practice p. S23
Application Practice p. S41

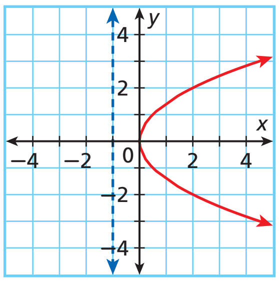
Use the distance formula to find the equation of a parabola with the given focus and directrix.

14. $F(0, 3)$, $y = -5$
15. $F(-2, 0)$, $x = 8$
16. $F(7, 0)$, $x = -1$

Write the equation in standard form for each parabola.

17.


18.


19.


20. vertex $(0, 0)$, focus $\left(\frac{1}{2}, 0\right)$
21. vertex $(0, 0)$, focus $(0, -6)$

Find the vertex, value of p, axis of symmetry, focus, and directrix of each parabola, and then graph.

22. $y = \frac{1}{8}(x - 1)^2$
23. $x = 2y^2 + 1$
24. $x - 2 = \frac{1}{2}(y + 1)^2$

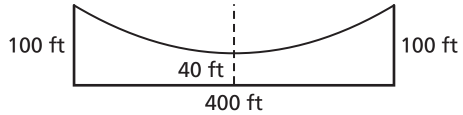
25. **Communications** Find an equation for a cross section of a parabolic microphone whose feedhorn is 9 inches long if the end of the feedhorn is placed at the origin.

LINK

Engineering

The Akashi-Kaikyo Bridge is the longest suspension bridge in the world with a main span of 1991 m. Also known as the Pearl Bridge, it connects the Kobe region of Japan to Awaji Island.

26. **Engineering** The main cables of a suspension bridge are ideally parabolic. The cables over a bridge that is 400 feet long are attached to towers that are 100 feet tall. The lowest point of the cable is 40 feet above the bridge.

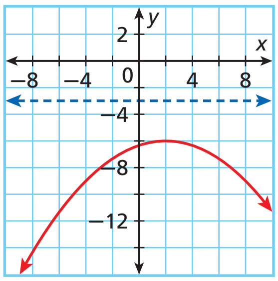


a. Find the coordinates of the vertex and the tops of the towers if the bridge represents the x-axis and the axis of symmetry is the y-axis.

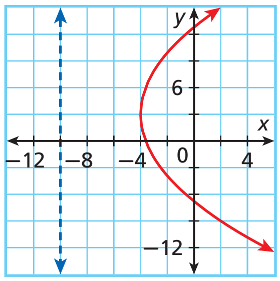
b. Find an equation that can be used to model the cables.

Write the equation in standard form for each parabola, and give the domain and range. (*Hint:* Find the domain and range by using the vertex and the direction that the parabola opens.)

27.



28.



29. vertex $(-7, -3)$, focus $(2, -3)$

30. vertex $(5, -2)$, focus $(5, -8)$

31. focus $(0, 0)$, directrix $y = 10$

32. focus $(2, 6)$, directrix $y = -8$

33. focus $(4, -5)$, directrix $x = 12$

34. focus $(-3, 1)$, directrix $x = -15$

35. **Engineering** A spotlight has parabolic cross sections.

a. Write an equation for a cross section of the spotlight if the bulb is 5 inches from the vertex and the vertex is placed at the origin.

b. Write an equation for a cross section of the spotlight if the bulb is 4 inches from the vertex and the bulb is placed at the origin.

c. If the spotlight has a diameter of 24 inches at its opening, find the depth of the spotlight if the bulb is 5 inches from the vertex.

36. **Sports** When a football is kicked, the path that the ball travels can be modeled by a parabola.

a. A placekicker kicks a football, which reaches a maximum height of 8 yards and lands 50 yards away. Assuming that the football was at the origin when it was kicked, write an equation for the height of the football.

b. **What if...?** If the placekicker was trying to kick the ball over a 10-foot-high goalpost 40 yards away, was the football high enough to go over the goalpost? Explain.

MULTI-STEP TEST PREP

37. This problem will prepare you for the Multi-Step Test Prep on page 758.

The path of a comet is modeled by the parabola $y = -\frac{1}{532}(x + 96)^2 + 174$, where each unit of the coordinate plane represents 1 million kilometers.

a. The Sun is at the focus of the parabolic path. Find the coordinates of the Sun.

b. How close does the comet come to the Sun?

c. What are the coordinates of the comet when it is at its closest point to the Sun?

Graph each equation. Identify the vertex, value of p, axis of symmetry, focus, and directrix for each equation.

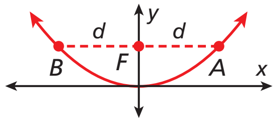
38. $20(y-2) = (x+6)^2$

39. $y = -2(x+4)^2 + 5$

40. $(y+7)^2 = \frac{x}{16}$

41. $x + 3 = \frac{1}{8}(y-2)^2$

42. Critical Thinking Find the distance d from the focus to the points on the parabola that are on the line perpendicular to the axis of symmetry and through the focus. Explain your answer.



43. Write About It Explain how changing the value of p will affect the vertex, focus, and directrix of the parabola $y - k = \frac{1}{4p}(x-h)^2$.

TEST PREP

44. The graph of which of the following parabolas opens to the left?

(A) $16y - 4x^2 = 12$ (B) $16y + 4x^2 = 12$ (C) $16x - 4y^2 = 12$ (D) $16x + 4y^2 = 12$

45. Which of the following is the axis of symmetry for the graph of $x - 4 = \frac{1}{8}(y+2)^2$?

(F) $x = 0$ (G) $y = -2$ (H) $x = 4$ (J) $y = 8$

46. Which of the following graphs has the directrix $y = 4$?

(A) $y + 3 = \frac{1}{4}(x-1)^2$

(B) $y - 5 = \frac{1}{4}(x+2)^2$

(C) $x - 5 = \frac{1}{4}(y+4)^2$

(D) $x + 3 = \frac{1}{4}(y-2)^2$

47. Short Response What are the coordinates of the focus for the graph of $x - 3 = \frac{1}{16}y^2$?

CHALLENGE AND EXTEND

Write the equation in standard form for each parabola.

48. vertex $(6, 8)$, contains the point $(4, -2)$, axis of symmetry $x = 6$

49. focus $(6, 5)$, axis of symmetry $x = 6$, contains the point $(10, 5)$

Multi-Step **The latus rectum of a parabola is the line segment perpendicular to the axis of symmetry through the focus, with endpoints on the parabola. Find the length of the latus rectum of each parabola.**

50. $y = \frac{1}{8}x^2$

51. $y - k = \frac{1}{4p}(x-h)^2$

SPIRAL REVIEW

52. Write and graph a system of linear inequalities whose solution region is the triangle given by the vertices $(0, 2)$, $(1, 4)$, and $(2, 1)$. *(Lesson 3-3)*

Find the inverse of each function. Tell whether the inverse is a function, and state its domain and range. *(Lesson 9-5)*

53. $f(x) = 4x + 22$

54. $f(x) = 3x^2 + 1$

55. $f(x) = \frac{x-2}{3}$

56. $f(x) = \frac{1}{x-1}$

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph. *(Lesson 10-4)*

57. $\frac{x^2}{81} - \frac{y^2}{25} = 1$

58. $\frac{y^2}{9} - \frac{x^2}{16} = 1$

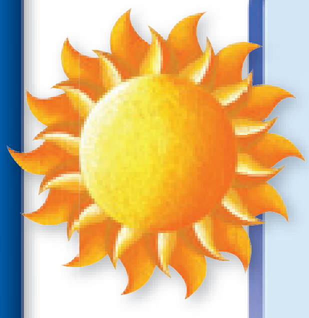
59. $\frac{y^2}{64} - \frac{x^2}{4} = 1$

60. $\frac{x^2}{49} - \frac{y^2}{36} = 1$

CHAPTER 10

SECTION 10A

MULTI-STEP TEST PREP

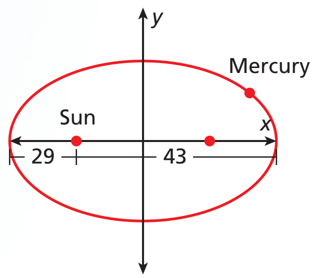


Understanding Conic Sections

The Solar System Johannes Kepler (1571–1630) is generally credited as the first astronomer to recognize the role of the conic sections in describing our solar system. Kepler's first law of planetary motion states that the path of every planet is an ellipse with the Sun at one focus.

1. Although the orbit of Earth around the Sun is elliptical, it very closely resembles a circle. The orbit can be modeled by $x^2 + y^2 = 8649$, where the Sun is at the origin and each unit of the coordinate plane represents 1 million miles. How far does Earth travel in 1 year as it makes one complete orbit?

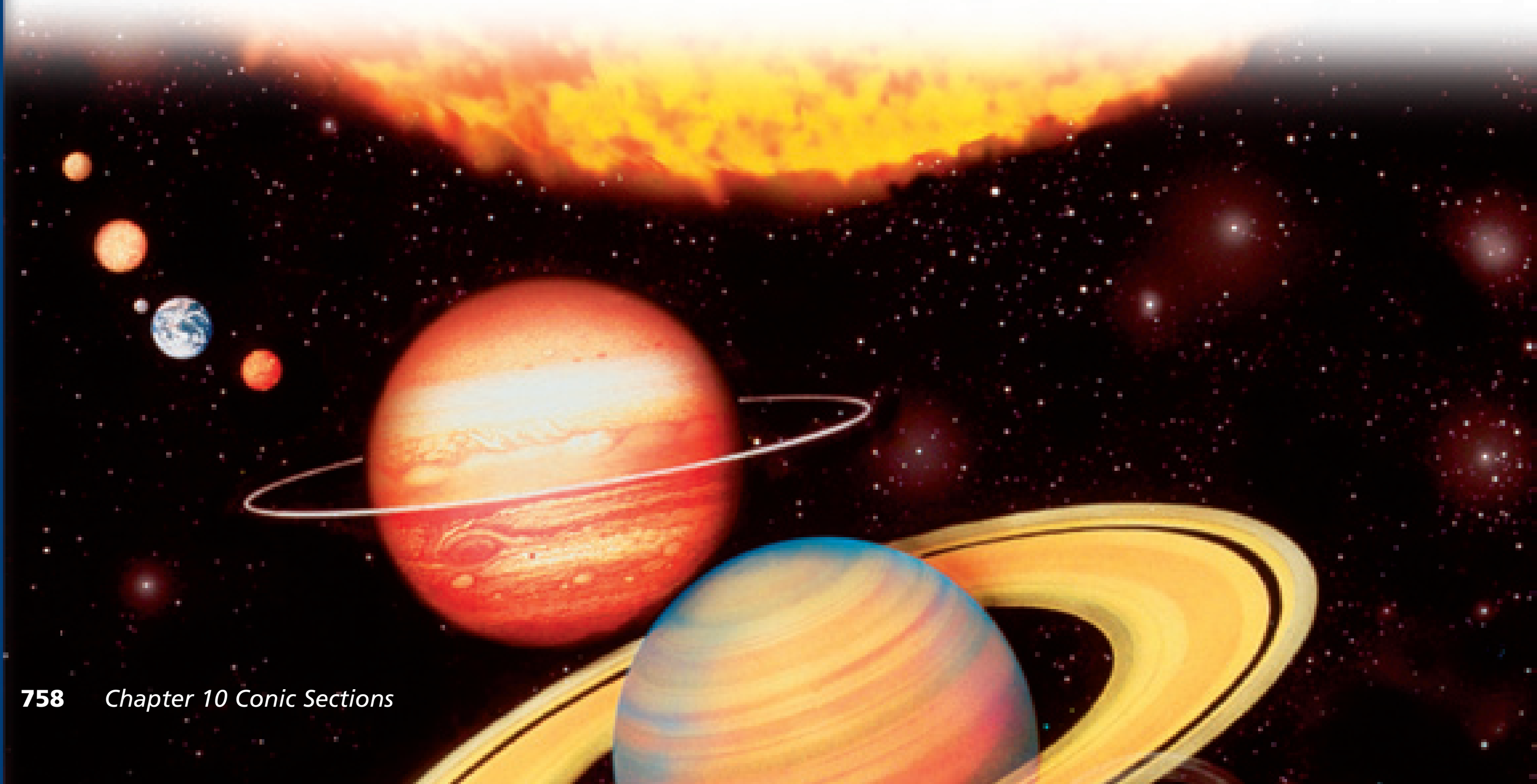
2. The figure shows the elliptical orbit of Mercury, whose minimum distance to the Sun is 29 million miles and whose maximum distance to the Sun is 43 million miles. According to Kepler's laws, the average distance of a planet to the Sun is equal to half the length of the orbit's major axis. What is the average distance of Mercury to the Sun?



3. Write an equation that models the orbit of Mercury.

4. A comet that passes through the solar system just once has a path that is modeled by a hyperbola or a parabola. Astronomers discover a comet whose path is modeled by $\frac{x^2}{2500} - \frac{y^2}{37{,}500} = 1$, with the Sun at one focus. How close will the comet come to the Sun?

5. The path of another comet is modeled by $336(x - 89) = (y - 62)^2$, with the Sun at the focus. In this model, what are the coordinates of the Sun? How close will this comet come to the Sun?



READY TO GO ON?

Quiz for Lessons 10-1 Through 10-5

10-1 Introduction to Conic Sections

1. The delivery area of a furniture store extends to the locations $(-7, 12)$ and $(5, -4)$. Write an equation for the delivery area of the store if a line between the locations represents a diameter of the delivery area.

Identify and describe each conic section.

2. $\frac{(x+2)^2}{64} + \frac{(y-8)^2}{64} = 1$
3. $25x^2 + 36y^2 = 900$
4. $x = \frac{y^2}{3} + 2$
5. $\frac{y^2}{25} - \frac{x^2}{25} = 1$

10-2 Circles

Write the equation of each circle.

6. center $(-3, 7)$ and radius $r = 12$
7. center $(4, -2)$ and containing the point $(-4, 13)$
8. Write the equation of the line that is tangent to $x^2 + y^2 = 225$ at $(9, -12)$.

10-3 Ellipses

Find the center, vertices, co-vertices, and foci of each ellipse. Then graph.

9. $\frac{x^2}{81} + \frac{y^2}{100} = 1$
10. $4(x-2)^2 + 16(y+3)^2 = 64$
11. Write the equation of the ellipse with center $(3, 5)$, vertex $(-10, 5)$, and focus $(8, 5)$.
12. A semi-elliptical bridge over a stream that is 30 feet wide must be 12 feet high at its highest point to accommodate boat traffic. Write an equation for a cross section of the bridge.

10-4 Hyperbolas

Find the center, vertices, co-vertices, foci, and asymptotes for each hyperbola. Then graph.

13. $\frac{y^2}{49} - \frac{x^2}{25} = 1$
14. $\frac{(x-5)^2}{36} - \frac{(y+3)^2}{9} = 1$
15. Write the equation of the hyperbola with vertices $(2, 3)$ and $(2, 9)$ and co-vertex $(7, 6)$.

10-5 Parabolas

Find the vertex, value of *p*, axis of symmetry, focus, and directrix for each parabola. Then graph.

16. $x = -\frac{1}{12}y^2$
17. $y = 2(x+3)^2 + 4$
18. Write the equation of the parabola with focus $(5, 2)$ and directrix $x = 1$.
19. A cross section of a parabolic microphone has the equation $35x = y^2$, where x and y are measured in inches. How far from the vertex of the microphone should the feedhorn be placed?

10-6 Identifying Conic Sections

Objectives
Identify and transform conic sections.

Use the method of completing the square to identify and graph conic sections.

Why learn this?
The path of an airplane in a dive can be modeled by a branch of a hyperbola or a parabola. (See Example 4.)

In Lessons 10-2 through 10-5, you learned about the four conic sections. Recall the equations of conic sections in standard form. In these forms, the characteristics of the conic sections can be identified.

Know it! Note

Standard Forms for the Conic Sections with Center (*h*, *k*)

Circle	$(x-h)^2+(y-k)^2=r^2$	
	HORIZONTAL AXIS	**VERTICAL AXIS**
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Parabola	$x-h=\frac{1}{4p}(y-k)^2$	$y-k=\frac{1}{4p}(x-h)^2$

EXAMPLE 1 Identifying Conic Sections in Standard Form

Identify the conic section that each equation represents.

A $\frac{(x-7)^2}{5^2}-\frac{(y+2)^2}{2^2}=1$

This equation is of the same form as a hyperbola with a horizontal transverse axis.

B $y-3=\frac{1}{12}(x-4)^2$

This equation is of the same form as a parabola with a vertical axis of symmetry.

C $\frac{(x-1)^2}{8^2}+\frac{(y-1)^2}{10^2}=1$

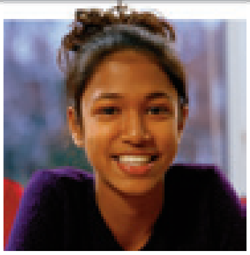
This equation is of the same form as an ellipse with a vertical major axis.

CHECK IT OUT! **Identify the conic section that each equation represents.**

1a. $x^2+(y+14)^2=11^2$ **1b.** $\frac{(y-6)^2}{2^2}-\frac{(x-1)^2}{21^2}=1$

Student to Student

Classifying Conic Sections



Mercedes Raya
Central High School

I can classify an equation in standard form just by looking. This is a good way for me to check my work.

Only one squared term → *it's a parabola.*

A squared term minus a squared term → *it's a hyperbola.*

A squared term plus a squared term → *it's a circle or an ellipse.*

All conic sections can be written in the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The conic section represented by an equation in general form can be determined by the coefficients.

Know it! Note

Classifying Conic Sections

For an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ (A, B, and C do not all equal 0.)

CONIC SECTION	COEFFICIENTS
Circle	$B^2 - 4AC < 0$, $B = 0$, and $A = C$
Ellipse	$B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$
Hyperbola	$B^2 - 4AC > 0$
Parabola	$B^2 - 4AC = 0$

EXAMPLE 2

Identifying Conic Sections in General Form

Identify the conic section that each equation represents.

A $\mathbf{6x^2 + 9y^2 + 12x - 15y - 25 = 0}$

$A = 6, B = 0, C = 9$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

$0^2 - 4(6)(9)$ — *Substitute into $B^2 - 4AC$.*

-216 — *Simplify. The conic is either a circle or an ellipse.*

$A \neq C$ — *The conic is not a circle.*

Because $B^2 - 4AC < 0$ and $A \neq C$, the equation represents an ellipse.

B $\mathbf{4x^2 + 4xy + y^2 - 12x + 8y + 36 = 0}$

$A = 4, B = 4, C = 1$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

$4^2 - 4(4)(1)$ — *Substitute into $B^2 - 4AC$.*

0 — *Simplify.*

Because $B^2 - 4AC = 0$, the equation represents a parabola.

CHECK IT OUT!

Identify the conic section that each equation represents.

2a. $9x^2 + 9y^2 - 18x - 12y - 50 = 0$

2b. $12x^2 + 24xy + 12y^2 + 25y = 0$

If you are given the equation of a conic in standard form, you can write the equation in general form by expanding the binomials.

If you are given the general form of a conic section, you can use the method of completing the square from Lesson 5-4 to write the equation in standard form.

EXAMPLE 3 Finding the Standard Form of the Equation for a Conic Section

Find the standard form of each equation by completing the square. Then identify and graph each conic.

A $x^2 - 12x - 16y + 36 = 0$

$$x^2 - 12x + \square = 16y - 36 + \square$$
Prepare to complete the square in x.

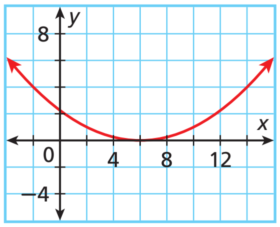
$$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = 16y - 36 + \left(\frac{-12}{2}\right)^2$$
Add $\left(-\frac{12}{2}\right)^2$, or 36, to both sides to complete the square.

$$(x-6)^2 = 16y$$
Factor and simplify.

$$\frac{1}{16}(x-6)^2 = y$$
Divide both sides by 16.

$$y = \frac{1}{16}(x-6)^2$$
Rewrite in standard form.

Because the conic is of the form $y - k = \frac{1}{4p}(x-h)^2$, it is a parabola with vertex $(6, 0)$ and $p = 4$, and it opens upward. The focus is $(6, 4)$ and the directrix is $y = -4$.



Remember!

You must factor out the leading coefficient of x^2 and y^2 before completing the square.

B $x^2 + 4y^2 + 4x - 24y + 36 = 0$

$$x^2 + 4x + \square + 4y^2 - 24y + \square = -36 + \square + \square$$
Rearrange to prepare for completing the square in x and y.

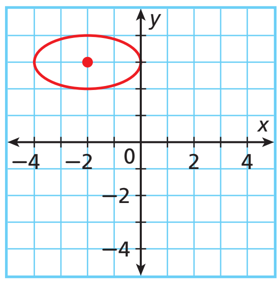
$$x^2 + 4x + \square + 4(y^2 - 6y + \square) = -36 + \square + \square$$
Factor 4 from the y terms.

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 + 4\left[y^2 - 6y + \left(-\frac{6}{2}\right)^2\right] = -36 + \left(\frac{4}{2}\right)^2 + 4\left(-\frac{6}{2}\right)^2$$
Complete both squares.

$$(x+2)^2 + 4(y-3)^2 = 4$$
Factor and simplify.

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{1} = 1$$
Divide both sides by 4.

Because the conic is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, it is an ellipse with center $(-2, 3)$, horizontal major axis length 4, and minor axis length 2. The co-vertices are $(-2, 4)$ and $(-2, 2)$, and the vertices are $(-4, 3)$ and $(0, 3)$.



CHECK IT OUT! **Find the standard form of each equation by completing the square. Then identify and graph each conic.**

3a. $y^2 - 9x + 16y + 64 = 0$

3b. $16x^2 + 9y^2 - 128x + 108y + 436 = 0$

EXAMPLE 4 *Aviation Application*

At an air show, an airplane makes a dive that can be modeled by the equation $-4x^2 + 16y^2 - 16x + 32y - 64 = 0$, measured in hundreds of feet, with the ground represented by the x-axis. How close to the ground does the airplane pass?

The graph of $-4x^2 + 16y^2 - 16x + 32y - 64 = 0$ is a conic section. Write the equation in standard form.

$$-4x^2 - 16x + \square + 16y^2 + 32y + \square = 64 + \square + \square$$

Rearrange to prepare for completing the square in x and y.

$$-4\left(x^2 + 4x + \square\right) + 16\left(y^2 + 2y + \square\right) = 64 + \square + \square$$

Factor −4 from the x terms and 16 from the y terms.

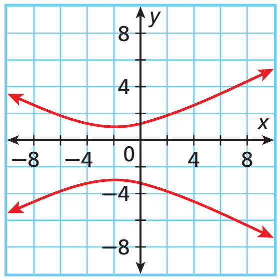
$$-4\left[x^2 + 4x + \left(\frac{4}{2}\right)^2\right] + 16\left[y^2 + 2y + \left(\frac{2}{2}\right)^2\right] = 64 - 4\left(\frac{4}{2}\right)^2 + 16\left(\frac{2}{2}\right)^2$$

Complete both squares.

$$16(y + 1)^2 - 4(x + 2)^2 = 64 \qquad \textit{Simplify.}$$

$$\frac{(y + 1)^2}{4} - \frac{(x + 2)^2}{16} = 1 \qquad \textit{Divide both sides by 64.}$$

Because the conic is of the form $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$, it is a hyperbola with vertical transverse axis length 4 and center $(-2, -1)$. The vertices are then $(-2, 1)$ and $(-2, -3)$. Because distance above ground is always positive, the airplane will be on the upper branch of the hyperbola. The relevant vertex is $(-2, 1)$ with y-coordinate 1.



The minimum height of the plane is 100 feet.

CHECK IT OUT!

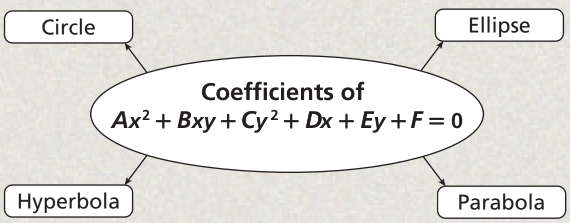
4. An airplane makes a dive that can be modeled by the equation $-16x^2 + 9y^2 + 96x + 36y - 252 = 0$, measured in hundreds of feet. How close to the ground does the airplane pass?

THINK AND DISCUSS

1. In the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, if $B = 0$, what must be true about either A or C for the equation to represent a parabola?

2. When solving by completing the square, what must be added to both sides of the equation if one side has $5x^2 - 30x$? Explain.

3. GET ORGANIZED Copy and complete the graphic organizer. Give an example of coefficients for each conic section in general form.



10-6 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-6
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

Identify the conic section that each equation represents.

SEE EXAMPLE 1 p. 760

1. $\frac{(x+4)^2}{2^2} + \frac{(y-3)^2}{3^2} = 1$
2. $\frac{(x-8)^2}{5^2} - \frac{y^2}{5^2} = 1$
3. $y + 9 = 4(x-1)^2$
4. $(x-2)^2 + (y-6)^2 = 13^2$

SEE EXAMPLE 2 p. 761

5. $12x^2 + 18y^2 - 8x + 9y - 10 = 0$
6. $-4y^2 + 15x + 12y - 8 = 0$
7. $10x^2 + 15xy + 10y^2 + 15x + 25y + 9 = 0$
8. $6x^2 = 14x + 12y^2 - 16y + 20$

SEE EXAMPLE 3 p. 762

Find the standard form of each equation by completing the square. Then identify and graph each conic.

9. $x^2 + y^2 - 16x + 10y + 53 = 0$
10. $x^2 + 14x - 12y + 97 = 0$
11. $25x^2 + 9y^2 + 72y - 81 = 0$
12. $16x^2 + 36y^2 + 160x - 432y + 1120 = 0$

SEE EXAMPLE 4 p. 763

13. **Multi-Step** A moth is circling an outdoor light in a path that can be modeled by the equation $4x^2 + 9y^2 - 108y = -288$, measured in inches. How close does the moth pass to a lizard located at the origin?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–17	1
18–21	2
22–31	3
32	4

Extra Practice
Skills Practice p. S23
Application Practice p. S41

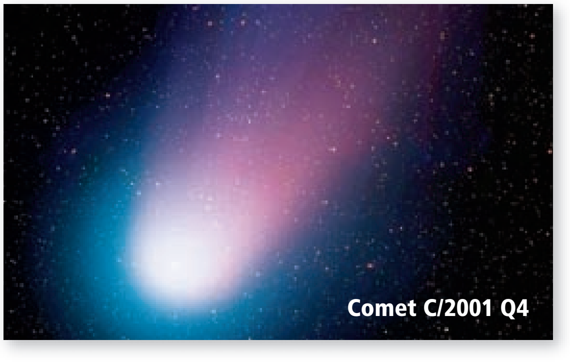
Identify the conic section that each equation represents.

14. $\frac{(y-11)^2}{2^2} - \frac{(x+15)^2}{9^2} = 1$
15. $x - 4 = \frac{1}{16}(y-3)^2$
16. $(x+2)^2 + (y-4)^2 = 3^2$
17. $\frac{(x+2)^2}{6^2} + \frac{(y-7)^2}{8^2} = 1$
18. $12x^2 - 18y^2 - 18x - 12y + 12 = 0$
19. $7x^2 + 28x - 29y - 16 = 0$
20. $-12x^2 - 3y^2 + 7x + 9y - 5 = 0$
21. $12x^2 + 9y^2 - 2xy + 9 = 8y - 3y^2$

Find the standard form of each equation by completing the square. Then identify and graph each conic.

22. $x^2 + 20x - 4y + 100 = 0$
23. $x^2 + y^2 - 8y - 33 = 0$
24. $9x^2 + 36y^2 - 72x - 180 = 0$
25. $25x^2 - 4y^2 - 72y - 424 = 0$
26. $x^2 - 2x - 20y - 79 = 0$
27. $x^2 + y^2 + 10x + 4y + 9 = 0$
28. $64x^2 + 49y^2 + 256x - 196y - 2684 = 0$
29. $9x^2 - 4y^2 + 18x + 56y - 223 = 0$
30. $y^2 + 6x + 12y - 6 = 0$
31. $x^2 + y^2 - 5x + 9y + 10.5 = 0$

32. **Astronomy** Scientists find that the path of a comet as it travels around the Sun can be modeled by the function $225x^2 + 64y^2 + 7650x + 50{,}625 = 0$, with the Sun as one focus.
    a. Write the equation in standard form.
    b. If measurements are in millions of miles, about how close will the comet come to the sun?


Comet C/2001 Q4

MULTI-STEP TEST PREP

33. This problem will prepare you for the Multi-Step Test Prep on page 776.

A water-skier is towed along a path that can be modeled by $25x^2 + 4y^2 + 300x - 24y + 836 = 0$. Each unit of the coordinate plane represents 10 m.

a. What is the shape of the water-skier's path?

b. The edge of a dock is represented by the y-axis. How close does the water-skier come to the dock?

c. A second water-skier is towed along the same path. What is the maximum possible distance between the two water-skiers?

Write each equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

34. $(x - 7)^2 + (y + 12)^2 = 81$ **35.** $\frac{(x-5)^2}{25} + \frac{(y+8)^2}{36} = 1$ **36.** $\frac{(x+10)^2}{49} - \frac{(y-6)^2}{81} = 1$

Determine whether the origin lies inside, outside, or on the graph of each equation.

37. $36x^2 + 4y^2 - 432x + 1152 = 0$

38. $4x^2 + 36y^2 - 48x = 0$

39. $16x^2 + 64y^2 - 192x + 16y - 447 = 0$

40. $3x^2 + 3y^2 = 147$

LINK

Math History

In 1604, German astronomer Johannes Kepler introduced a new way of thinking about the conic sections—as a family of related curves. For example, the parabola could be considered simply a hyperbola with one focus at infinity.

41. Multi-Step A model of the solar system includes a satellite orbiting the Moon on a path that can be modeled by the equation $6x^2 + 6y^2 = 24$, measured in centimeters (1 cm:10,000 km). If the Moon is located at the point $(0, 38.4)$, how close will the satellite pass to the Moon in the model?

42. Critical Thinking What does the graph of $x^2 - xy = 0$ look like? Explain.

43. Agriculture A farmer is planning to fence in part of the farm. Placing the farmhouse at the origin, the farmer finds that the path for the fence can be modeled by the equation $x^2 + y^2 - 80x - 60y - 37{,}500 = 0$, measured in feet.

a. Write the equation in standard form.

b. Find the area enclosed by the fence.

c. Is the farmhouse inside or outside of the fence?

44. ERROR ANALYSIS In which case below was the conic section $4y^2 + 3x - 12y = 2x^2 + 18$ identified incorrectly? Explain the error.

A

$4y^2 + 3x - 12y = 2x^2 + 18$
$-2x^2 + 4y^2 + 3x - 12y - 18 = 0$
$A = -2, B = 0, C = 4$
$B^2 - 4AC = 0 - 4(-2)(4)$
$B^2 - 4AC = 32$
The equation represents a hyperbola.

B

$4y^2 + 3x - 12y = 2x^2 + 18$
$A = 2, B = 0, C = 4$
$B^2 - 4AC = 0 - 4(2)(4)$
$B^2 - 4AC = 32$
$A \neq C$
The equation represents an ellipse.

45. Sports The path followed by a baseball after it is hit can be modeled by the equation $2x^2 - 800x + 1000y - 4000 = 0$, measured in feet.

a. Write the equation in standard form.

b. What is the maximum height of the ball?

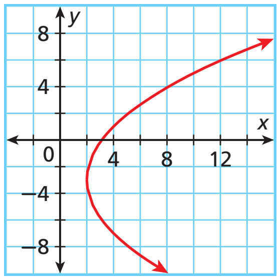
c. What was the height of the ball when it was hit?

d. What if...? How would changing the 4000 in the equation to 5000 change your answers to parts **b** and **c?**

46. Write About It Compare the equations and graphs of parabolas and hyperbolas.

Test Prep

47. Which of the following is the equation for the graph shown?



Ⓐ $3y^2 - 24x + 18y + 75 = 0$

Ⓑ $5x^2 + 30x - 40y + 125 = 0$

Ⓒ $2x^2 - 3y^2 + 18x - 24y + 75 = 0$

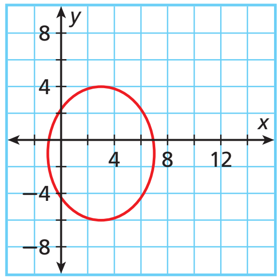
Ⓓ $3x^2 + 2y^2 - 24x + 18y + 125 = 0$

48. The graph of $9x^2 + 15x - 9y^2 - 15y + 25 = 0$ is which of the following?

Ⓕ Circle   Ⓖ Ellipse

Ⓗ Hyperbola   Ⓙ Parabola

49. Which of the following is the equation for the graph shown?



Ⓐ $25x^2 + 25y^2 - 150x + 32y - 159 = 0$

Ⓑ $25x^2 - 150x + 32y = 159$

Ⓒ $25x^2 - 150x = 16y^2 - 32y + 159$

Ⓓ $16y^2 + 32y - 159 = 150x - 25x^2$

50. **Short Response** Write the equation $x^2 + y^2 + 8x - 6y + 16 = 0$ in standard form, and identify the conic section that it represents. What are the coordinates of the center?

CHALLENGE AND EXTEND

In order to graph the general form of conic sections, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, use the quadratic formula,

$$y = \frac{-(Bx + E) \pm \sqrt{(Bx + E)^2 - 4C(Ax^2 + Dx + F)}}{2C},$$ **and a graphing calculator.**

51. Graph $4x^2 + 8xy - 9y^2 - 36 = 0$.

52. Graph $9x^2 - 12xy + 16y^2 - 144 = 0$.

53. What effect does the term Bxy have on the graph?

54. **What if...?** What happens to the formula if $C = 0$?

SPIRAL REVIEW

Use substitution to determine if the given point is a solution to the system of equations. *(Lesson 3-1)*

55. $(1, 2)\begin{cases} 8y - 3x = 13 \\ 5x + 6y = 18 \end{cases}$

56. $(10, 5)\begin{cases} x + y = 15 \\ x - y = 5 \end{cases}$

57. $(-2, 4)\begin{cases} x = 8 - y \\ 2x - 7y = -32 \end{cases}$

Use elimination to solve each system of equations. *(Lesson 3-2)*

58. $\begin{cases} 7x - 2y = 20 \\ -7x + 10y = 12 \end{cases}$

59. $\begin{cases} 3x + 4y = 16 \\ 2x - 4y = 4 \end{cases}$

60. $\begin{cases} x + 5y = -13 \\ -2x - 7y = 14 \end{cases}$

61. **Business** In 1980, a baseball card was valued at $1.65. The value of the baseball card increased at a rate of 5% per year. *(Lesson 7-1)*

a. Write an equation to model the value of the baseball card where t is the number of years since 1980.

b. What was the value of the baseball card in 2004?

10-6 Technology LAB

Conic-Section Art

You can use graphs of conic sections to design and create pictures on the coordinate grid.

Use with Lesson 10-6

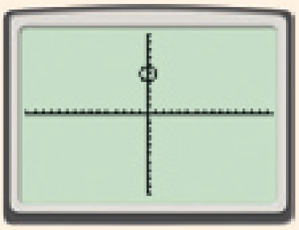
go.hrw.com
Lab Resources Online
KEYWORD: MB7 LAB10

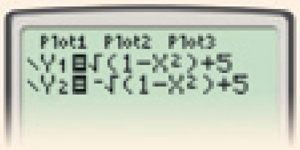
Activity

Create a picture of a dragonfly by using one circle and six ellipses.

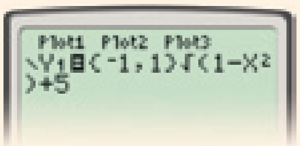
1 Graph the head by using $x^2 + (y - 5)^2 = 1$.

Solve for y, $y = \pm\sqrt{1 - x^2} + 5$, and graph. There are two ways to enter the two halves of the circle into the calculator.

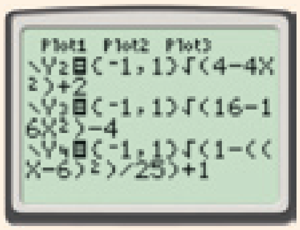




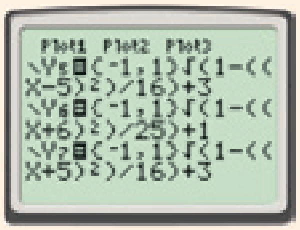
2 The part of the equation $\{-1, 1\}$ represents $\pm$ and can be used to graph both halves of a conic section at one time.



3 Graph the body parts and one right wing by using $x^2 + \frac{(y-2)^2}{4} = 1$, $x^2 + \frac{(y+4)^2}{16} = 1$, *and* $\frac{(x-6)^2}{25} + (y-1)^2 = 1$.

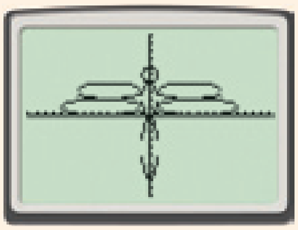


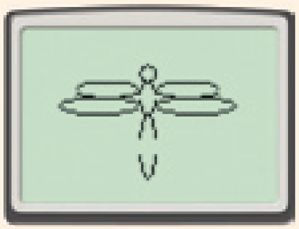
4 Graph the other right wing and left wings by using $\frac{(x-5)^2}{16} + (y-3)^2 = 1$, $\frac{(x+6)^2}{25} + (y-1)^2 = 1$, *and* $\frac{(x+5)^2}{16} + (y-3)^2 = 1$.



5 The dragonfly is now complete.

Turn off the axes by using the **Format** function and setting **AxesOff**.





Try This

1. Create your own picture by using the graphs of conic sections. Use at least four conic sections. You may also use lines if necessary.
2. Trade equations with a classmate, and attempt to re-create his or her picture by using only the equations.

10-7 Solving Nonlinear Systems

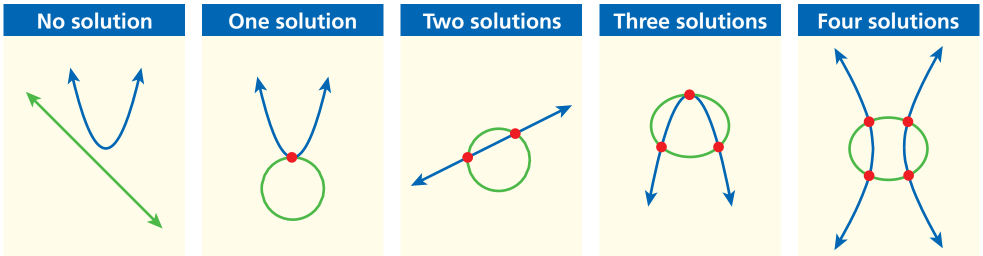
Objective
Solve systems of equations in two variables that contain at least one second-degree equation.

Vocabulary
nonlinear system of equations

Who uses this?
Harbormasters can solve nonlinear systems to ensure that ships traveling in a variety of patterns do not collide. (See Example 4.)

A **nonlinear system of equations** is a system in which at least one of the equations is not linear. You have been studying one class of nonlinear equations, the conic sections.

The solution set of a system of equations is the set of points that make all of the equations in the system true, or where the graphs intersect. For systems of nonlinear equations, you must be aware of the number of possible solutions.



You can use your graphing calculator to find solutions to systems of nonlinear equations and to check algebraic solutions.

EXAMPLE 1 Solving a Nonlinear System by Graphing

Solve $\begin{cases} 2x - y = 1 \\ y + 7 = 2(x+1)^2 \end{cases}$ **by graphing.**

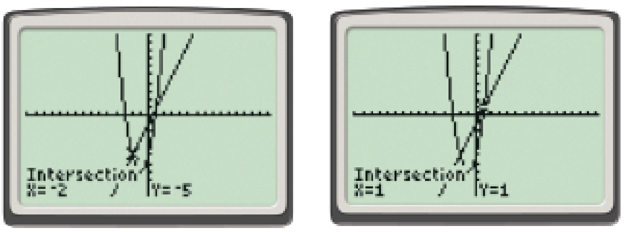
The graph of the first equation is a line, and the graph of the second equation is a parabola, so there may be as many as two points of intersection.

Step 1 Solve each equation for y.

$y = 2x - 1$ *Solve the first equation for y.*

$y = 2(x+1)^2 - 7$ *Solve the second equation for y.*

Step 2 Graph the system on your calculator, and use the intersect feature to find the solution set.



The points of intersection are $(-2, -5)$ and $(1, 1)$.

Check Substitute the points into each equation.

Check $(-2, -5)$. Check $(1, 1)$.

y	$2x - 1$	y	$2(x+1)^2 - 7$	y	$2x - 1$	y	$2(x+1)^2 - 7$
-5	$2(-2) - 1$	-5	$2(-2+1)^2 - 7$	1	$2(1) - 1$	1	$2(1+1)^2 - 7$
-5	-5 ✔	-5	-5 ✔	1	1 ✔	1	1 ✔

The solution set of the system is $\{(-2, -5), (1, 1)\}$.

CHECK IT OUT! **1.** Solve $\begin{cases} 3x + y = 4.5 \\ y = \frac{1}{2}(x-3)^2 \end{cases}$ by graphing.

The substitution method for solving linear systems can also be used to solve nonlinear systems algebraically.

EXAMPLE 2 Solving a Nonlinear System by Substitution

Solve $\begin{cases} x^2 + y^2 = 25 \\ y + 5 = \frac{1}{2}x^2 \end{cases}$ by using the substitution method.

The graph of the first equation is a circle, and the graph of the second equation is a parabola. There may be as many as four points of intersection.

Step 1 It is simplest to solve for x^2 because both equations have x^2 terms.

$x^2 = 2y + 10$ *Solve for x^2 in the second equation.*

Step 2 Use substitution.

$(2y + 10) + y^2 = 25$ *Substitute this value into the first equation.*

$y^2 + 2y - 15 = 0$ *Simplify, and set equal to 0.*

$(y - 3)(y + 5) = 0$ *Factor.*

$y = 3$ or $y = -5$

Step 3 Substitute 3 and -5 into $x^2 = 2y + 10$ to find values for x.

$x^2 = 2(3) + 10$

$x^2 = 16$

$x = \pm 4$

$(4, 3)$ and $(-4, 3)$ are solutions.

$x^2 = 2(-5) + 10$

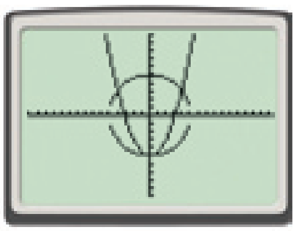
$x^2 = 0$

$x = 0$

$(0, -5)$ is a solution.

The solution set of the system is $\{(4, 3), (-4, 3), (0, -5)\}$.

Check Use a graphing calculator. The graph supports that there are three points of intersection.



CHECK IT OUT! **Solve each system of equations by using the substitution method.**

2a. $\begin{cases} x + y = -1 \\ x^2 + y^2 = 25 \end{cases}$

2b. $\begin{cases} x^2 + y^2 = 25 \\ y - 5 = -x^2 \end{cases}$

The elimination method can also be used to solve systems of nonlinear equations.

EXAMPLE 3 Solving a Nonlinear System by Elimination

Solve $\begin{cases} 25x^2 + 9y^2 = 225 \\ 16x^2 - 9y^2 = 144 \end{cases}$ **by using the elimination method.**

The graph of the first equation is an ellipse, and the graph of the second equation is a hyperbola. There may be as many as four points of intersection.

Remember!

In Example 3, you can check your work on a graphing calculator.

Step 1 Eliminate y.

$$\begin{aligned} 25x^2 + 9y^2 &= 225 \\ +\ 16x^2 - 9y^2 &= 144 \quad \textit{Add the equations.} \\ \hline 41x^2 \qquad &= 369 \end{aligned}$$

$x^2 = 9$, so $x = \pm 3$ *Solve for x.*

Step 2 Find the values for y.

$25(9) + 9y^2 = 225$ *Substitute 9 for x^2.*

$225 + 9y^2 = 225$ *Simplify.*

$y = 0$

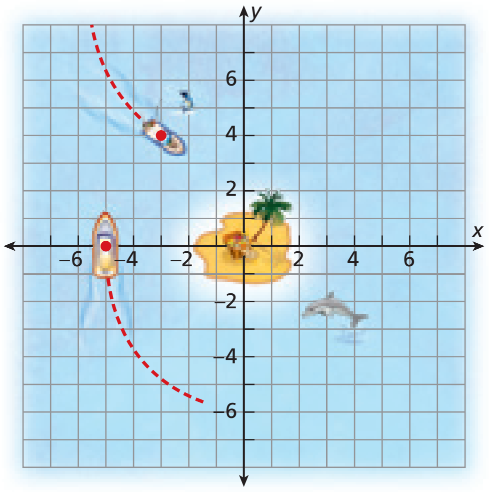
The solution set of the system is $\{(3, 0), (-3, 0)\}$.

CHECK IT OUT! **3.** Solve $\begin{cases} 25x^2 + 9y^2 = 225 \\ 25x^2 - 16y^2 = 400 \end{cases}$ by using the elimination method.

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A tour boat travels around a small island in a pattern that can be modeled by the equation $36x^2 + 25y^2 = 900$, with the island at the origin. Suppose that a fishing boat approaches the island on a path that can be modeled by the equation $y - 3 = \frac{1}{5}x^2$. Is there any danger of collision?



1 Understand the Problem

There is a potential danger of a collision if the two paths cross. The paths will cross if the graphs of the equations intersect.

List the important information:

- $36x^2 + 25y^2 = 900$ represents the path of the tour boat.
- $y - 3 = \frac{1}{5}x^2$ represents the path of the fishing boat.

2 Make a Plan

To see if the graphs intersect, solve the system $\begin{cases} 36x^2 + 25y^2 = 900 \\ y - 3 = \frac{1}{5}x^2 \end{cases}$

3 Solve

The graph of the first equation is an ellipse, and the graph of the second equation is a parabola. There may be as many as four points of intersection.

$x^2 = 5y - 15$ — *Solve the second equation for x^2.*

$36(5y - 15) + 25y^2 = 900$ — *Substitute this value into the first equation.*

$25y^2 + 180y - 1440 = 0$ — *Simplify, and set equal to 0.*

$$y = \frac{-180 \pm \sqrt{180^2 - 4(25)(-1440)}}{2(25)}$$ *Use the quadratic formula.*

$y = \frac{-180 \pm 420}{50}$, or $y = 4.8$ and $y = -12$

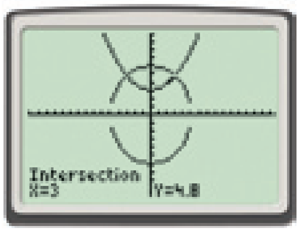
Substitute $y = 4.8$ and $y = -12$ into $x^2 = 5y - 15$ to find the values for x.

$x^2 = 5(4.8) - 15$ $\quad$ $x^2 = 5(-12) - 15$

$x^2 = 9$, or $x = \pm 3$ $\quad$ $x^2 = -75$ $\quad$ *There are no real values of $\sqrt{-75}$.*

The real solutions to the system are $(3, 4.8)$ and $(-3, 4.8)$.

4 Look Back



The graph supports that there are two points of intersection. Because the paths intersect, the boats are in danger of colliding if they arrive at the intersections $(3, 4.8)$ or $(-3, 4.8)$ at the same time.

CHECK IT OUT!

4. What if...? Suppose the paths of the boats can be modeled by the system $\begin{cases} 36x^2 + 25y^2 = 900 \\ y + 2 = -\frac{1}{10}x^2 \end{cases}$. Is there any danger of collision?

THINK AND DISCUSS

1. What can you tell about the graphs if the system has no solution?

2. Describe the steps for solving a nonlinear system of equations by graphing.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Use the table to record information on the intersection of a hyperbola and a circle.

	Graph	Example
No Solution		
One Solution		
Two Solutions		
Three Solutions		
Four Solutions		

10-7 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 10-7
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How is a *nonlinear system of equations* different from a linear system of equations?

SEE EXAMPLE 1 p. 768

Solve each system of equations by graphing.

2. $\begin{cases} y + 3x = 0 \\ y - 6 = -3x^2 \end{cases}$

3. $\begin{cases} y + 2 = \frac{1}{4}(x-4)^2 \\ x - y = 6 \end{cases}$

4. $\begin{cases} y + 2x = 10 \\ x = \frac{1}{8}(y-2)^2 \end{cases}$

SEE EXAMPLE 2 p. 769

Solve each system of equations by using the substitution method.

5. $\begin{cases} y + x = 17 \\ x^2 + y^2 = 169 \end{cases}$

6. $\begin{cases} x^2 + y^2 = 25 \\ y - x = 7 \end{cases}$

7. $\begin{cases} x^2 + y^2 = 36 \\ x + 2y = 16 \end{cases}$

8. $\begin{cases} x^2 + y^2 = 100 \\ x + 2 = \frac{1}{8}y^2 \end{cases}$

9. $\begin{cases} x^2 + y^2 = 36 \\ y + 6 = \frac{1}{3}x^2 \end{cases}$

10. $\begin{cases} x^2 + y^2 = 25 \\ y - 6.25 = -\frac{1}{4}x^2 \end{cases}$

SEE EXAMPLE 3 p. 770

Solve each system of equations by using the elimination method.

11. $\begin{cases} x^2 + y^2 = 20 \\ 4x^2 + y^2 = 68 \end{cases}$

12. $\begin{cases} 9x^2 + 5y^2 = 45 \\ 6y^2 - 27x^2 = 54 \end{cases}$

13. $\begin{cases} 4x^2 + 3y^2 = 12 \\ 5x^2 + 6y^2 = 30 \end{cases}$

SEE EXAMPLE 4 p. 770

14. **Radio** The range of a radio station is bounded by the circle with equation $x^2 + y^2 = 2025$. A stretch of highway near the station is modeled by the equation $y - 15 = \frac{1}{20}x^2$. At what points does a car on the highway enter or exit the broadcast range of the station?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–17	1
18–23	2
24–29	3
30	4

Extra Practice

Skills Practice p. S23
Application Practice p. S41

Solve each system of equations by graphing.

15. $\begin{cases} 2y - x = 10 \\ y - 3 = \frac{1}{8}(x+4)^2 \end{cases}$

16. $\begin{cases} x - 6 = -\frac{1}{6}y^2 \\ 2x + y = 6 \end{cases}$

17. $\begin{cases} y^2 - x^2 = 36 \\ 2x + y = -\frac{3}{2} \end{cases}$

Solve each system of equations by using the substitution method.

18. $\begin{cases} x^2 + y^2 = 13 \\ x - y = 1 \end{cases}$

19. $\begin{cases} y^2 - 4x^2 = 16 \\ y - x = 4 \end{cases}$

20. $\begin{cases} x^2 - y^2 = 16 \\ x + y^2 = 4 \end{cases}$

21. $\begin{cases} y = \frac{1}{4}(x-3)^2 \\ 3x - 2y = 13 \end{cases}$

22. $\begin{cases} -3 = 2x^2 - y \\ x^2 - 36 = 9y^2 \end{cases}$

23. $\begin{cases} x^2 + y^2 = 8 \\ x^2 - y = 6 \end{cases}$

Solve each system of equations by using the elimination method.

24. $\begin{cases} 2x^2 + 3y^2 = 83 \\ 4x^2 - 2y^2 = -34 \end{cases}$

25. $\begin{cases} \frac{x^2}{5} + \frac{y^2}{3} = 15 \\ x^2 + y^2 = 20 \end{cases}$

26. $\begin{cases} x^2 + y^2 = 16 \\ y^2 - 2x^2 = 16 \end{cases}$

27. $\begin{cases} x - y = 7 \\ x^2 - y = 7 \end{cases}$

28. $\begin{cases} 4x^2 + y^2 = 1 \\ -x^2 + y^2 = 1 \end{cases}$

29. $\begin{cases} x^2 + y^2 = 9 \\ x^2 - 4y^2 = 4 \end{cases}$

30. **Multi-Step** While waiting to land, an airplane is traveling above the airport in a holding pattern that can be modeled by the equation $49x^2 + 64y^2 = 3136$, with the air traffic control tower at the origin. Suppose that another plane approaches the airport at the same altitude as the first plane on a path that can be modeled by the equation $y - 6 = \frac{1}{4}x^2$. Should the air traffic controller at the airport be concerned? If so, what are the possible points of collision?

Solve each system of equations by using any method.

31. $\begin{cases} y = x^2 \\ x = y^2 \end{cases}$

32. $\begin{cases} 5x^2 + 4y^2 = 216 \\ 3x^2 + 6y^2 = 162 \end{cases}$

33. $\begin{cases} 8y - x = 2 \\ x - 10 = -4y^2 \end{cases}$

34. $\begin{cases} x^2 - 4y^2 = 9 \\ x - 4y = -3 \end{cases}$

35. $\begin{cases} x^2 + 4y^2 = 36 \\ x^2 + y^2 = 9 \end{cases}$

36. $\begin{cases} \frac{x^2}{16} - \frac{y^2}{9} = 1 \\ \frac{y^2}{25} - \frac{x^2}{4} = 1 \end{cases}$

37. $\begin{cases} x + 6 = \frac{1}{2}y^2 \\ x - 4 = -\frac{1}{8}y^2 \end{cases}$

38. $\begin{cases} \frac{x^2}{16} + \frac{y^2}{25} = 1 \\ \frac{x^2}{16} + \frac{y^2}{4} = 1 \end{cases}$

39. $\begin{cases} x - 3 = 2y^2 \\ y^2 - 9x^2 = 36 \end{cases}$

40. $\begin{cases} x^2 + y^2 = 100 \\ x + 5y = 10 \end{cases}$

41. $\begin{cases} 3x^2 - 6y^2 = 204 \\ 4x^2 - 2y^2 = 368 \end{cases}$

42. $\begin{cases} 4x^2 + 9y^2 = 36 \\ 2x + 3y = 6 \end{cases}$

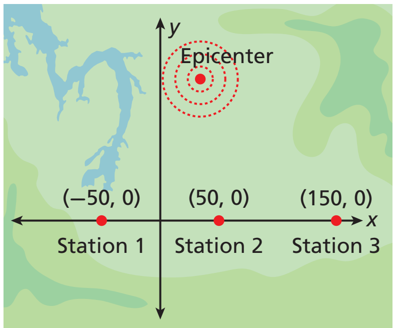
LINK

Geology

One of the largest earthquakes ever recorded occurred in December 2004 off the coast of Indonesia. The quake caused a giant tsunami that wreaked havoc in Sri Lanka, Thailand, and other countries.

43. **Physics** A speeding driver sees a parked police car at time $t = 0$ and starts to decelerate while the police car accelerates as the officer chases the driver. The distance that the driver has traveled in feet after t seconds can be modeled by the function $d(t) = 250 + 125t - 1.2t^2$. The distance that the police car has traveled in feet after t seconds can be modeled by the function $d(t) = 4.2t^2$. How long does it take the police car to catch the driver?

44. **Geology** Three seismic monitoring stations, located as shown, detect an earthquake.



a. Suppose that the epicenter of the earthquake is 30 miles closer to station 2 than to station 1. Use 30 as the constant difference and Stations 1 and 2 as the foci to write an equation for the possible locations of the earthquake.

b. Suppose that the epicenter of the earthquake is 40 miles closer to station 2 than to station 3. Use 40 as the constant difference and stations 2 and 3 as the foci to write an equation for the possible locations of the earthquake.

c. Find the coordinates of the epicenter of the earthquake to the nearest mile.

45. **Estimation** Use your graphing calculator to estimate the points of intersection of $x^2 - 4y^2 + 7x = 0$ and $x^2 - y + 5x - 24 = 0$.

46. **Critical Thinking** What must be true in order for two parabolas to have exactly four points of intersection? Explain.

MULTI-STEP TEST PREP

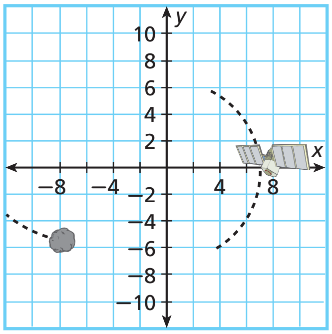
47. This problem will prepare you for the Multi-Step Test Prep on page 776.

A water-skiing exhibition takes place in a body of water modeled by the first and second quadrants of the coordinate plane. A water-skier is towed along a path that can be modeled by $-x^2 + 4y^2 + 8x - 8y - 16 = 0$.

a. What is the shape of the water-skier's path?

b. A second water-skier's path is modeled by $x^2 + y^2 - 8x - 8y + 23 = 0$. Is there a chance that the two water-skiers will collide? If so, where?

48. Astronomy An asteroid is traveling toward Earth on a path that can be modeled by the equation $y = \frac{1}{28}x^2 - 7$. It approaches a satellite in orbit on a path that can be modeled by the equation $\frac{x^2}{49} + \frac{y^2}{51} = 1$. What are the coordinates of the points where the satellite and asteroid might collide?



49. Recreation Ice skaters Bianca and Mark are performing a routine in which Bianca skates in a path that can be modeled by the equation $x + 2 = \frac{y^2}{4}$ and Mark skates in a path that can be modeled by the equation $\frac{(x+4)^2}{4} + \frac{y^2}{9} = 1$. When the pair meets, they will perform a lift. What are the coordinates of the point where the pair will perform a lift?

50. Multi-Step The lake at a resort has an island near the center. A tour boat's path on the lake can be modeled by the equation $16x^2 + 9y^2 = 36$, with the island at the origin. If a canoe's path on the lake can be modeled by the equation $8x + 5y^2 = 20$, find the coordinates of the points on the lake where the boats might meet.

51. Write About It How would the value of a in the system $\begin{cases} x^2 - y^2 = 25 \\ \frac{x^2}{a^2} + \frac{y^2}{9} = 1 \end{cases}$ affect the number of solutions for the system?

TEST PREP

52. Which of the following points is a solution to the system $\begin{cases} 4x^2 + 5y^2 = 189 \\ 8y^2 - 2x = 60 \end{cases}$?

Ⓐ $(3, -6)$ Ⓑ $(-3, -6)$ Ⓒ $(6, -3)$ Ⓓ $(-6, -3)$

53. How many solutions does the system $\begin{cases} \frac{x^2}{16} + \frac{y^2}{9} = 144 \\ x = 3(y-2)^2 \end{cases}$ have?

Ⓕ 1 Ⓖ 2 Ⓗ 3 Ⓙ 4

54. For which value of k will the system $\begin{cases} x^2 + y^2 = 25 \\ 5(y + k) = x^2 \end{cases}$ have exactly one solution?

Ⓐ $k = -5$ Ⓑ $k = 0$ Ⓒ $k = 5$ Ⓓ $k = 25$

CHALLENGE AND EXTEND

Solve each system of equations by any method.

55. $\begin{cases} x^2 + y^2 = 25 \\ 3x^2 + 2y^2 = 66 \\ x + y^2 = 13 \end{cases}$

56. $\begin{cases} 6x^2 - 3y^2 = 204 \\ y + 10 = \frac{1}{3}x^2 \\ 25x^2 - 36(y-2)^2 = 900 \end{cases}$

57. $\begin{cases} x^2 + y^2 = 25 \\ xy = 12 \\ y^2 - x - 8y + 19 = 0 \end{cases}$

Graph each system of inequalities.

58. $\begin{cases} y - 5 < -\frac{1}{8}x^2 \\ y + 5 \geq \frac{1}{6}x^2 \end{cases}$

59. $\begin{cases} x^2 + y^2 \leq 36 \\ y + 6 > x^2 \end{cases}$

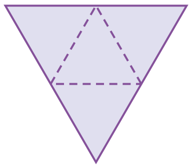
60. $\begin{cases} \frac{x^2}{9} + \frac{y^2}{36} \leq 1 \\ \frac{x^2}{25} + \frac{y^2}{9} \geq 1 \end{cases}$

61. **Economics** Industry analysts predict that the demand curve for a new software product can be modeled by the function $D(p) = 5000 - 0.2p^2$, where p is the price of the product and $D(p)$ is the number of products that can be sold at p. The analysts also predict that the supply curve for the new product can be modeled by the function $S(p) = 0.3p^2$, where p is the price of the product and $S(p)$ is the number of products that companies will supply at p. Predict the price for the new product.

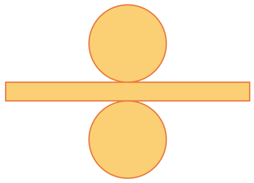
SPIRAL REVIEW

Describe the three-dimensional figure that can be made from the given net. *(Previous course)*

62.



63.



Evaluate each expression for the given values of the variable. *(Lesson 1-4)*

64. $2x^2 + 3y - 6$ for $x = -1, y = 4$

65. $\frac{a^2 - b^2}{a - b}$ for $a = -2, b = 6$

66. $\frac{5s + 2t + st}{s + t}$ for $s = 7, t = -3$

67. $\frac{3w^2 - 4z}{2wz}$ for $w = 2, z = 5$

Write a function that models the given data. *(Lesson 9-6)*

68.

x	−1	0	1	2	3	4
y	8	4	2	2	4	8

69.

x	−2	−1	0	1	2	3
y	−12	−7	−2	3	8	13

Career Path

go.hrw.com
Career Resources Online
KEYWORD: MB7 Career

Shawn Innes
Actuarial Science major

Q: What math classes did you take in high school?

A: In high school, I took algebra, geometry, and precalculus.

Q: What do actuaries do?

A: Basically, actuaries evaluate the likelihood of certain events and try to find creative ways to reduce the chances of undesirable outcomes. Actuaries are involved in many different industries such as business and finance, health, retirement planning, and insurance.

Q: How do you become an actuary?

A: To become a full actuary, a series of exams must be completed. These exams cover topics like calculus, economics, and finance.

Q: What are your future plans?

A: I'd like to work at the consulting firm where I interned. They specialize in retirement planning and benefits. There, I tested formulas used to calculate pensions for client companies.

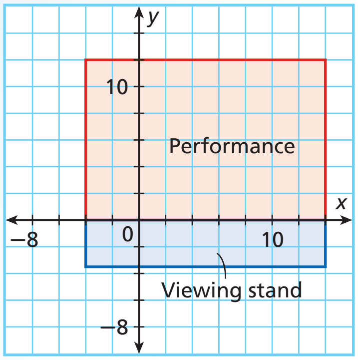
CHAPTER 10

SECTION 10B

MULTI-STEP TEST PREP

Applying Conic Sections

Water-skiing A water-skiing team is planning an exhibition on a lake. The figure shows the performance area that has been roped off on the lake and the location of the viewing stand. Each unit of the coordinate plane represents 10 ft.



1. The first water-skier enters the performance area, and the boat tows her along a path modeled by $16x^2 + 25y^2 - 96x - 300y + 644 = 0$. What is the shape of the path? Explain.
2. How close to the viewing stand does the water-skier pass?
3. During this routine, the water-skier will pass directly in front of what percentage of the viewers?
4. A second water-skier enters the performance area. The boat tows him along a path modeled by $x^2 + y^2 - 20x - 12y + 132 = 0$. What is the shape of the path? Explain.
5. Is there a chance that the second water-skier will collide with the first? If so, where?
6. A third water-skier is towed along a path modeled by $x^2 - 14x - 8y + 129 = 0$. At what points does the water-skier enter and exit the performance area?
7. Is there a chance that this water-skier will collide with the others? If so, where?

READY TO GO ON?

Quiz for Lessons 10-6 Through 10-7

10-6 Identifying Conic Sections

Identify the conic section that each equation represents.

1. $\frac{6x^2}{9} + \frac{8y^2}{12} = 1$
2. $8(y - 4) - 3(x + 4)^2 = 1$
3. $\frac{(y - 2)^2}{9} = \frac{(x + 5)^2}{16} + 1$
4. $(y - 2)^2 - 3(x + 7) = 0$
5. $2x^2 + 4y^2 - 12y = 18$
6. $7x^2 - 5xy - 3y^2 + 7x - 6 = 0$
7. $9x^2 + 12xy + 16y^2 - 5x + 2y = 0$
8. $x^2 + y^2 - 4x + 6y - 11 = 0$

Write each equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

9. $y - 5 = \frac{1}{4}(x + 8)^2$
10. $\frac{(y - 3)^2}{9} - \frac{(x + 5)^2}{5} = 1$

Find the standard form of each equation by completing the square. Then identify the conic.

11. $x^2 + y^2 - 6x - 8y + 15 = 0$
12. $3x^2 + 4y^2 - 18x + 8y + 19 = 0$
13. $5y^2 - x - 60y + 176 = 0$
14. $2x^2 - 6y^2 - 16x - 24y = 4$

10-7 Solving Nonlinear Systems

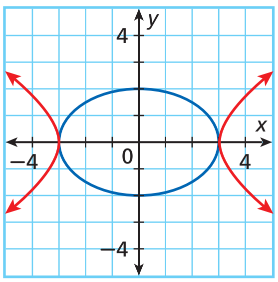
Solve each system of equations by graphing.

15. $\begin{cases} 8y + 3x^2 = 56 \\ y = \frac{1}{4}x^2 - 3 \end{cases}$
16. $\begin{cases} 2x - 8 = y^2 \\ 3x - 3y = -12 \end{cases}$
17. $\begin{cases} x^2 + y^2 = 169 \\ 5y - 12x = 0 \end{cases}$

Solve each system by using the substitution or elimination method.

18. $\begin{cases} x^2 - 2y^2 = 28 \\ 3y - x = 0 \end{cases}$
19. $\begin{cases} 2x^2 + 3y^2 = 21 \\ x^2 - 9y = 0 \end{cases}$
20. $\begin{cases} 8x^2 + 4y^2 = 32 \\ 10x^2 + 6y^2 = 60 \end{cases}$

21. A team of stunt racing boats is performing a series of stunts along paths shown in the graph. During the performance, the lead boat moves in a path that can be modeled by the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Two other boats race in formation along each of the branches of the equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$. At what points are the boats in danger of colliding?



22. Find n so that the system $\begin{cases} \frac{x^2}{9} + \frac{y^2}{16} = 1 \\ y - n = x^2 \end{cases}$ has exactly three solutions.

CHAPTER 10

Study Guide: Review

Vocabulary

branch of a hyperbola 744
circle 729
conic section 722
conjugate axis 744
co-vertices of an ellipse 736
co-vertices of a hyperbola . . . 744
directrix 751
ellipse 736
foci of an ellipse 736
foci of a hyperbola 744
focus of a parabola 751
hyperbola 744
major axis 736
minor axis 736
nonlinear system of equations 768
tangent 731
transverse axis 744
vertices of an ellipse 736
vertices of a hyperbola 744

Complete the sentences below with vocabulary words from the list above.

1. The line containing the vertices and the foci of a hyperbola is the ___?___ of symmetry of the hyperbola.

2. A line in the same plane as a circle that intersects the circle in exactly one point is a(n) ___?___.

3. A parabola is the set of all points P(x, y) that are equidistant from both a fixed point, called the ___?___, and a fixed line, called the ___?___.

4. A(n) ___?___ is formed by the intersection of a double right cone and a plane.

10-1 Introduction to Conic Sections *(pp. 722–728)*

EXAMPLE

■ **Graph $4x^2 + 25y^2 = 100$ on a graphing calculator. Identify and describe the conic section.**

Solve for y so that the expression can be used in a graphing calculator.

$25y^2 = 100 - 4x^2$ *Subtract $4x^2$ from both sides.*

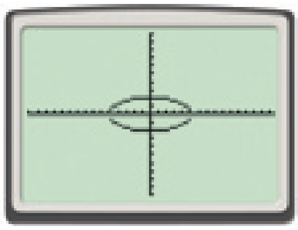
$y^2 = \dfrac{100 - 4x^2}{25}$ *Divide both sides by 25.*

$y = \pm\sqrt{\dfrac{100 - 4x^2}{25}}$ *Take the square root of both sides.*

Use two equations to see the complete graph.

$y_1 = \sqrt{\dfrac{100 - 4x^2}{25}}$ and $y_2 = -\sqrt{\dfrac{100 - 4x^2}{25}}$

The graph is an ellipse with center $(0, 0)$, y-intercepts 2 and -2, and x-intercepts 5 and -5.



EXERCISES

Graph each equation on a graphing calculator. Identify and describe the conic section.

5. $x^2 + y^2 = 81$

6. $\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

7. $x = \frac{1}{4}(y + 1)^2$

8. $8x^2 + 25y^2 = 98$

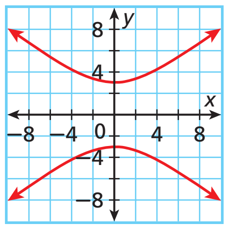
9. Which equation is represented by the graph?

A. $16x^2 - 16y^2 = 256$

B. $16y^2 = 9x^2 + 144$

C. $9x^2 + 16y^2 = 256$

D. $9y^2 - 16x^2 = 144$



Find the center and the radius of a circle that has a diameter with the given endpoints.

10. $(-9, -3)$ and $(15, -3)$

11. $(-4, 1)$ and $(20, -6)$

10-2 Circles *(pp. 729–734)*

EXAMPLES

■ **Write the equation of the circle with center $(-5, 9)$ and radius $r = 16$.**

Substitute into the general equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

$$(x-(-5))^2 + (y-9)^2 = 16^2$$

$$(x+5)^2 + (y-9)^2 = 256 \qquad \textit{Simplify.}$$

■ **Write an equation of the line that is tangent at (12, 9) to the circle with equation $x^2 + y^2 = 225$.**

The circle has center $(0, 0)$. The tangent is perpendicular to the radius at the point of tangency.

Find the slope of the radius and the slope of the tangent.

$$m_r = \frac{9-0}{12-0} = \frac{9}{12} = \frac{3}{4} \qquad \textit{The slope of the radius is } \tfrac{3}{4}.$$

$$m_t = -\frac{4}{3} \qquad \textit{Use the negative reciprocal.}$$

$$y - 9 = -\frac{4}{3}(x-12) \qquad \textit{Use point-slope form.}$$

EXERCISES

Find the center and the radius of each circle.

12. $(x-6)^2 + y^2 = 361$

13. $(x+12)^2 + (y-4)^2 = 15$

Write the equation of each circle.

14. center $(8, -7)$ and radius $r = 14$

15. center $(3, 6)$ and containing the point $(7, -2)$

16. diameter with endpoints $(2, 5)$ and $(-8, 11)$

Write an equation of the line that is tangent to the given circle at the given point.

17. $x^2 + y^2 = 34$ at $(3, 5)$

18. $(x+3)^2 + y^2 = 16$ at $(-3, 4)$

19. $(x-2)^2 + (y+7)^2 = 44$ at $(6, -2)$

20. $(x+4)^2 + (y-1)^2 = 89$ at $(1, -7)$

10-3 Ellipses *(pp. 736–742)*

EXAMPLE

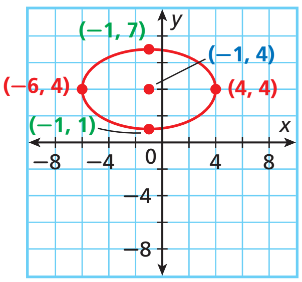
■ **Graph $\frac{(x+1)^2}{25} + \frac{(y-4)^2}{9} = 1$. Then find the foci of the ellipse.**

Rewrite the equation as $\frac{(x+1)^2}{5^2} + \frac{(y-4)^2}{3^2} = 1$.

The center is $(-1, 4)$. Because $5 > 3$, the major axis is horizontal, $a = 5$ and $b = 3$. The vertices are $(-1 \pm 5, 4)$, or $(-6, 4)$ and $(4, 4)$. The co-vertices are $(-1, 4 \pm 3)$ or $(-1, 7)$ and $(-1, 1)$.

In an ellipse, $c^2 = a^2 - b^2$. In this ellipse, $c^2 = 5^2 - 3^2 = 16$, so $c = 4$.

The foci are $(-1 \pm 4, 4)$, or $(-5, 4)$ and $(3, 4)$.



EXERCISES

Find the center, vertices, co-vertices, and foci of each ellipse. Then graph.

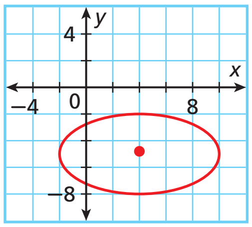
21. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

22. $25x^2 + 64y^2 = 1600$

23. $\frac{(x-3)^2}{49} + \frac{(y+2)^2}{64} = 1$

Find the equation of each ellipse.

24.



25. co-vertices at $(12, 0)$ and $(-12, 0)$ and major axis length 30

26. vertices at $(-8, 3)$ and $(4, 3)$ and foci at $(-5, 3)$ and $(1, 3)$

10-4 Hyperbolas *(pp. 744–750)*

EXAMPLE

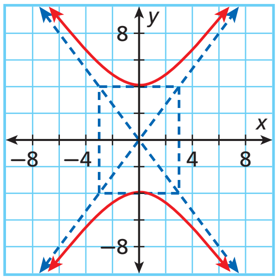
■ **Find the center, vertices, co-vertices, foci, and asymptotes of $\frac{y^2}{16} - \frac{x^2}{9} = 1$. Then graph.**

The equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2}$, so the transverse axis is vertical. The center is $(0, 0)$.

Because $a = 4$ and $b = 3$, the vertices are $(0, 4)$ and $(0, -4)$ and the co-vertices are $(3, 0)$ and $(-3, 0)$. The equations of the asymptotes are $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.

In a hyperbola, $c^2 = a^2 + b^2$. In this hyperbola, $c^2 = 4^2 + 3^2 = 25$, so $c = 5$ and the foci are $(0, 5)$ and $(0, -5)$.

Draw a box by using the vertices and co-vertices. Draw the asymptotes through the corners of the box. Draw the hyperbola by using the vertices and the asymptotes.



EXERCISES

Find the center, vertices, co-vertices, foci, and asymptotes of each hyperbola, and then graph.

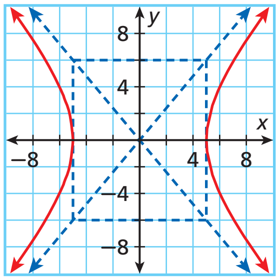
27. $\frac{x^2}{25} - \frac{y^2}{49} = 1$

28. $64y^2 - 36x^2 = 2304$

29. $\frac{(x-3)^2}{4} - \frac{(y+6)^2}{49} = 1$

Write an equation in standard form for each hyperbola.

30.



31. vertices $(11, 0)$ and $(-11, 0)$ and conjugate axis length 8

32. co-vertices $(6, 0)$ and $(-6, 0)$ and asymptotes $y = \frac{5}{6}x$ and $y = -\frac{5}{6}x$

33. length of transverse axis 10 and foci at $(-7, 18)$ and $(-7, -8)$

10-5 Parabolas *(pp. 751–757)*

EXAMPLE

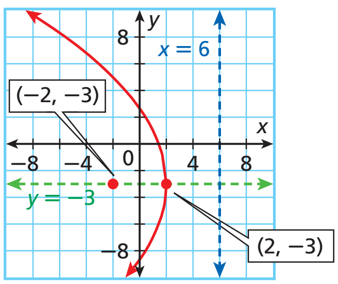
■ **Find the vertex, value of p, axis of symmetry, focus, and directrix of $x - 2 = -\frac{1}{16}(y + 3)^2$. Then graph.**

The equation is in the form $x - h = \frac{1}{4p}(y - k)^2$ with $p < 0$, so the graph opens to the left.

The vertex is $(2, -3)$, and the axis of symmetry is $y = -3$.

Because $\frac{1}{4p} = -\frac{1}{16}$, $p = -4$. The focus is $(2 - 4, -3)$, or $(-2, -3)$.

The directrix is $x = 2 + 4$, or $x = 6$.



EXERCISES

Find the vertex, value of p, axis of symmetry, focus, and directrix for each parabola. Then graph.

34. $y = -\frac{1}{12}x^2$

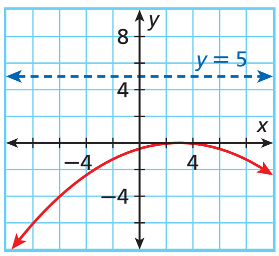
35. $x = 2y^2$

36. $y - 5 = (x + 4)^2$

37. $x - 4 = -\frac{1}{6}(y + 2)^2$

Write the equation in standard form for each parabola.

38.



39. vertex $(4, 6)$, axis of symmetry $y = 6$, $p = -2.5$

40. focus $(12, -4)$ and directrix $x = 6$

10-6 Identifying Conic Sections *(pp. 760–766)*

EXAMPLES

- **Identify the conic section represented by $3x^2 + 5xy - 8y^2 + 3x - 5y = 2$.**

$A = 3, B = 5, C = -8$ *Identify values of A, B, and C.*

$B^2 - 4AC = 5^2 - 4(3)(-8) = 121$ *Substitute.*

Because $B^2 - 4AC > 0$, the equation represents a hyperbola.

- **Find the standard form of the equation by completing the square. Then identify the conic.**

$y^2 - 4x - 10y = -13$

$y^2 - 10y + \blacksquare = 4x - 13 + \blacksquare$ *Rearrange.*

$y^2 - 10y + \left(\frac{10}{2}\right)^2 = 4x - 13 + \left(\frac{10}{2}\right)^2$ *Add $\left(\frac{10}{2}\right)^2$ to both sides.*

$(y - 5)^2 = 4x + 12$ *Factor, and simplify.*

$x + 3 = \frac{1}{4}(y - 5)^2$ *Rewrite in standard form.*

The equation represents a parabola.

EXERCISES

Identify the conic section that each equation represents.

41. $\frac{x^2}{12} = 1 - \frac{y^2}{9}$

42. $(x - 5)^2 = \frac{2}{3}(y + 4)^2 + 1$

43. $(x - 8)^2 = \frac{1}{12}(y + 5)$

44. $7x^2 + 7y^2 - 15x = 25$

45. $15x^2 - 6xy + 9y^2 - 12x - 12y + 15 = 0$

Find the standard form of each equation by completing the square. Then identify and graph each conic.

46. $y^2 - 4x + 12y = -24$

47. $2x^2 + 6y^2 + 16x = -20$

48. $x^2 + y^2 + 10x - 8y + 5 = 0$

49. $4x^2 - 8y^2 + 8x - 48y - 100 = 0$

10-7 Solving Nonlinear Systems *(pp. 768–775)*

EXAMPLE

- **Solve $\begin{cases} x^2 - y^2 = 16 \\ y^2 - x = 4 \end{cases}$ by using the substitution method.**

The graph of the first equation is a hyperbola. The graph of the second equation is a parabola. There may be as many as four points of intersection.

It is simplest to solve for y^2 because both equations have y^2 terms.

$y^2 = x + 4$ *Solve the second equation for y^2.*

$x^2 - (x + 4) = 16$ *Substitute this value into the first equation.*

$(x - 5)(x + 4) = 0$ *Simplify, and factor.*

$x = 5$ or $x = -4$

$y^2 = 5 + 4 = 9$ or $y^2 = -4 + 4 = 0$ *Substitute.*

$y = \pm 3$ when $x = 5$ and $y = 0$ when $x = -4$. The solution set is $\{(5, 3), (5, -3), (-4, 0)\}$.

EXERCISES

Solve each system of equations by graphing.

50. $\begin{cases} y + 6 = \frac{1}{2}(x - 2)^2 \\ y + 2x = -2 \end{cases}$

51. $\begin{cases} 25x^2 + 16y^2 = 400 \\ 16y = -5(x - 4)^2 \end{cases}$

Solve each system by using the substitution method.

52. $\begin{cases} 2x^2 - 2y^2 = 56 \\ x^2 + y^2 = 100 \end{cases}$

53. $\begin{cases} 2x^2 - y^2 = 14 \\ y - 2x = -4 \end{cases}$

Solve each system by using the elimination method.

54. $\begin{cases} 4y^2 - 8x^2 = 16 \\ 4x^2 + 5y^2 = 20 \end{cases}$

55. $\begin{cases} 3x^2 - 2y^2 = 76 \\ 5x^2 + 3y^2 = 228 \end{cases}$

Solve each system by using any method.

56. $\begin{cases} 3x^2 + 5y^2 = 192 \\ 3y - x = 16 \end{cases}$

57. $\begin{cases} \frac{x^2}{25} - \frac{y^2}{16} = 1 \\ 30x^2 + 20y^2 = 600 \end{cases}$

CHAPTER TEST

1. The transmission of a radio signal can be received at the locations $(1, -10)$ and $(-11, 6)$. Write an equation for the range of the signal if a line between the locations represents a diameter of the range.
2. Write the equation of the line that is tangent to $(x + 2)^2 + (y - 8)^2 = 40$ at $(3, -1)$.
3. Find the center, vertices, co-vertices, and foci of the ellipse with equation $49(x + 4)^2 + 16(y - 2)^2 = 784$. Then graph.
4. A shelter for a patch of young strawberry plants is constructed in the form of an ellipse. If the shelter is 4.5 feet high at its highest point and the patch is 19 feet wide, write an equation for the ellipse.
5. Find the center, vertices, co-vertices, foci, and asymptotes of the hyperbola with equation $\frac{x^2}{25} - \frac{y^2}{144} = 1$. Then graph.
6. Write the equation of the hyperbola with vertices $(0, 7)$ and $(0, -7)$ and conjugate axis length 28.
7. Find the vertex, value of p, axis of symmetry, focus, and directrix of the parabola with equation $y + 4 = \frac{1}{24}(x - 2)^2$. Then graph.
8. The filament of a flashlight bulb is located at the focus, which is 0.75 centimeters from the vertex of the flashlight's parabolic reflector. Write an equation for the cross section of the parabolic reflector if the vertex is at the origin and the reflector is pointed to the left.

Identify the conic section that each equation represents.

9. $\frac{x - 2}{4} = \frac{(y + 5)^2}{12}$
10. $1 - \frac{(x + 5)^2}{8} = \frac{(y - 4)^2}{8}$
11. $7x^2 + 5xy - 2y^2 + 8x - 26 = 0$

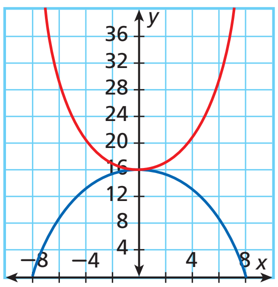
Find the standard form of each equation by completing the square. Then identify the conic.

12. $x^2 + y^2 - 16x + 20y + 124 = 0$
13. $6x^2 + 4y^2 + 84x - 24y + 306 = 0$

Find the solutions to the system by using the substitution or elimination method.

14. $\begin{cases} 2y - 3x = 1 \\ x + 4 = \frac{1}{4}(y - 2)^2 \end{cases}$
15. $\begin{cases} y + x = 2 \\ x^2 + y^2 = 52 \end{cases}$
16. $\begin{cases} 3x^2 - 4y^2 = 143 \\ 5x^2 - 5y^2 = 280 \end{cases}$

17. Two trapeze artists are swinging through the air along the paths shown in the graph. One performer releases the swing and travels in a path that can be modeled by the equation $y = -\frac{1}{4}x^2 + 16$. The performer's partner moves along a path that can be modeled by the equation $y = \frac{1}{2}x^2 + 16$. At what point will the performer be caught by his partner?



College Entrance Exam Practice

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The topics covered on each SAT Mathematics Subject Tests vary only slightly each time the test is administered. You can find out the general distribution of questions across topics and then determine which areas need more of your attention when you are studying for the test.

HOT TIP! To prepare for the SAT Mathematics Subject Tests, start reviewing course material a couple of months before your test date. Take sample tests to find the areas you might need to focus on more. Remember that you are not expected to have studied all of the topics on the test.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. The graph of the equation $x^2 + y^2 - 2x + 3y + 8 = 0$ is which of the following?

(A) Parabola

(B) Circle

(C) Hyperbola

(D) Ellipse

(E) Point

2. What is the length of the major axis of the ellipse with equation $\frac{(x-1)^2}{4} + (y+3)^2 = 9$?

(A) 2

(B) 3

(C) 4

(D) 6

(E) 12

3. What is the distance from the focus to the vertex of a parabola with equation $x = \frac{1}{12}(y-1)^2$?

(A) 3

(B) 6

(C) 12

(D) 48

(E) 144

4. Which of the following is the equation of an asymptote of the graph of $\frac{(y+2)^2}{9} - \frac{(x-5)^2}{4} = 1$?

(A) $y + 2 = \frac{3}{2}(x-5)$

(B) $y + 2 = \frac{2}{3}(x-5)$

(C) $y + 2 = \frac{9}{4}(x-5)$

(D) $y = \frac{3}{2}x$

(E) $y = \frac{2}{3}x$

5. The circle with equation $x^2 + y^2 + sx + ty + 33 = 0$ has center $(4, 5)$. What is $\frac{s}{t}$?

(A) $-\frac{5}{4}$

(B) $-\frac{4}{5}$

(C) $\frac{4}{5}$

(D) $\frac{5}{4}$

(E) There is not enough information to determine the answer.

CHAPTER 10

TEST TACKLER

Standardized Test Strategies

Multiple Choice: Context-Based Test Items

You will encounter some multiple-choice test items where the problem statement does not give you an actual problem to solve but requires you to use the answer choices provided to determine which choice fits the context of the problem statement. Depending on the problem, you can use a variety of methods, such as substitution, graphing, or elimination, to obtain the correct answer.

EXAMPLE 1

Which of the following equations, when graphed, has x-intercepts at $(5, 0)$ and $(-5, 0)$?

Ⓐ $2x^2 + 25y^2 = 150$

Ⓑ $8x^2 + 50y^2 = 200$

Ⓒ $5x^2 + 5y^2 = 100$

Ⓓ $4x^2 + 5y^2 = 50$

Although there are many equations that have x-intercepts at $(5, 0)$ and $(-5, 0)$, you need to select the equation from the four choices given. For this problem, you can use either of these two methods:

Substitution Method Substitute $x = 5$ and $y = 0$ into the equation, and simplify. Then substitute $x = -5$ and $y = 0$ into the equation, and simplify. Find which equation makes a true statement with the given x-intercepts.

Try choice A: $2x^2 + 25y^2 = 150$; $2(5)^2 + 25(0)^2 = 50$
Because the first equation does not make a true statement, $150 \neq 50$, choice A is incorrect.

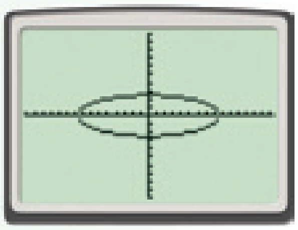
Try choice B: $8x^2 + 50y^2 = 200$; $8(5)^2 + 50(0)^2 = 200$; $8(-5)^2 + 50(0)^2 = 200$
Because both equations make a true statement, choice B is correct.

Try choice C and choice D to confirm that you found the correct answer.

Graphing Method Solve each equation in the answer choices for y. Then graph each equation on a graphing calculator. Look for the graph that intersects the x-axis at $(5, 0)$ and $(-5, 0)$.

Try choice A: $2x^2 + 25y^2 = 150$

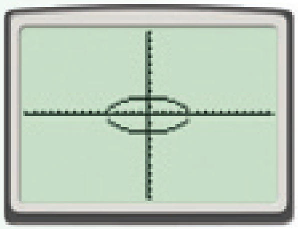
$$y = \pm\sqrt{\frac{(150 - 2x^2)}{25}}$$



When graphed on a calculator, the graph crosses the x-axis at about (8.5, 0) and (−8.5, 0). Choice A is incorrect.

Try choice B: $8x^2 + 50y^2 = 200$

$$y = \pm\sqrt{\frac{(200 - 8x^2)}{50}}$$



When graphed on a calculator, the graph crosses the x-axis at (5, 0) and (−5, 0). Choice B is the correct answer.

Try choice C and choice D to confirm that you found the correct answer.

HOT TIP! Underline the context of the problem statement to make sure that you are clear about what is being asked.

Read each test item and answer the questions that follow.

Item A

Which equation, when graphed, is a parabola that opens to the right?

(A) $4y - 2x^2 = 6$

(B) $4y + 2x^2 = 6$

(C) $4x - 2y^2 = 6$

(D) $4x + 2y^2 = 6$

1. On a coordinate grid, sketch two or three parabolas that open to the right. Can they all be represented by the same equation? If not, what do these equations have in common?
2. From what you know about parabolas, can any of the answer choices be eliminated? Explain.
3. Describe how to determine which answer choice is correct.

Item B

The graph of which of the following ellipses has the smallest distance between foci?

(F) $\frac{(x+16)^2}{64} + \frac{(y-9)^2}{25} = 1$

(G) $\frac{x^2}{4} + \frac{y^2}{81} = 1$

(H) $\frac{(x-1)^2}{1} + \frac{(y-1)^2}{100} = 1$

(J) $\frac{x^2}{289} + \frac{y^2}{169} = 1$

4. If you read only the problem statement and not the answer choices, can you solve the problem? Explain.
5. How can you find the distance between foci if you know *a* and *b*?

Item C

Which of the following points is inside the circle described by the following equation?

$$(x-2)^2 + (y-6)^2 = 9$$

(A) $(0, 0)$

(B) $(-2, 4)$

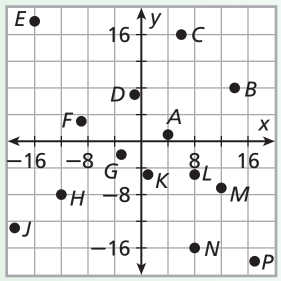
(C) $(5, 6)$

(D) $(3, 5)$

6. Describe how you can use your graphing calculator to determine the correct answer.
7. Can you use algebra to determine the correct answer? If so, describe your method.

Item D

A power outage affects areas *L*, *M*, and *N*. Which of the following best describes the power outage?



(F) The main generator is located at $(12, -9)$ and shuts down power up to 9 miles away.

(G) The main generator is located at $(4, -6)$ and shuts down power up to 8 miles away.

(H) The main generator is located at $(10, -3)$ and shuts down power up to 5 miles away.

(J) The main generator is located at $(-8, -15)$ and shuts down power up to 15 miles away.

8. What does the problem state about areas *L*, *M*, and *N*? What can you interpret about the areas not mentioned?
9. A student found that areas *L* and *M* are within the circle described by choice H. Can the student stop working and select choice H as the correct response? Explain.
10. Describe a method that you can use to determine the correct answer.

CHAPTER 10

STANDARDIZED TEST PREP

go.hrw.com
State Test Practice Online
KEYWORD: MB7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–10

Multiple Choice

1. Which conic section does the equation $\frac{x^2}{20} + \frac{y^2}{52} = 1$ represent?

(A) Circle

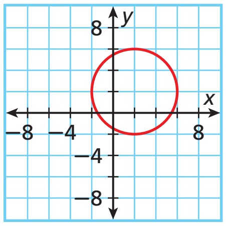
(B) Parabola

(C) Hyperbola

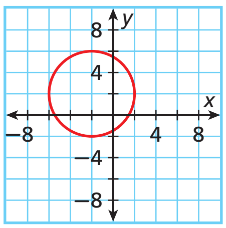
(D) Ellipse

2. Which is the graph of $(x + 2)^2 + (y - 2)^2 = 16$?

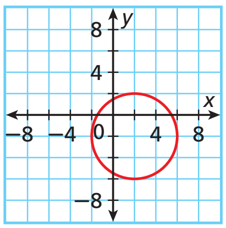
(F)



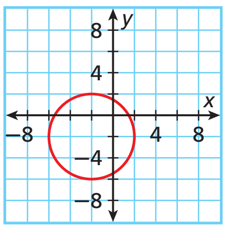
(H)



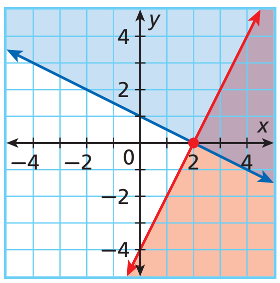
(G)



(J)



3. What system of linear inequalities can be used to represent the graph?



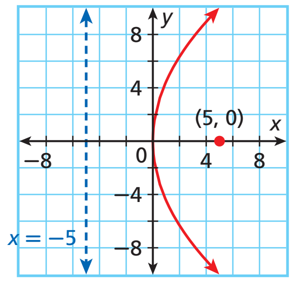
(A) $\begin{cases} y \le 2x - 4 \\ 2y \ge -x + 2 \end{cases}$

(B) $\begin{cases} y < 2x - 4 \\ 2y > -x + 2 \end{cases}$

(C) $\begin{cases} y \le 2x - 4 \\ 2y \le -x + 2 \end{cases}$

(D) $\begin{cases} y < 2x - 4 \\ 2y < -x + 2 \end{cases}$

4. What equation can be used to represent the graph?



(F) $x = -\frac{1}{20}y^2$

(G) $x = \frac{1}{20}y^2$

(H) $y = \frac{1}{20}x^2$

(J) $y = -\frac{1}{20}x^2$

5. Solve $\begin{cases} x^2 + 4y^2 = 64 \\ x + 8 = \frac{1}{2}y^2 \end{cases}$ by using the substitution method.

(A) $\{(-8, 0), (0, 4), (0, -4)\}$

(B) $\{(0, -8), (4, 0), (-4, 0)\}$

(C) $\{(-8, 0), (0, 4)\}$

(D) $\{(0, 4), (0, -4)\}$

6. Find all of the roots of the polynomial equation $x^5 + x^4 - x^3 - x^2 - 20x - 20 = 0$.

(F) $\sqrt{5}, -\sqrt{5}, -1$

(G) $i, -i, \sqrt{5}, -\sqrt{5}, -1$

(H) $2i, -2i, \sqrt{5}, -\sqrt{5}, -1$

(J) $4i, -4i, \sqrt{5}, -\sqrt{5}, -1$

7. At age 20, Jon invested \$50 at 6.75% compounded continuously. Jon is now 50. What is the present value of Jon's investment?

(A) \$4,545.67

(B) \$378.81

(C) \$17,534.57

(D) \$2,314.46

HOT TIP! In Item 15, recall that the notation $(f \circ g)(x)$ is equivalent to $f(g(x))$. Begin by substituting the value for x into the function $g(x)$.

8. Simplify.

$\frac{x+1}{3x+4} + \frac{x-1}{4x-7}$

Ⓕ $\frac{2x}{7x-3}$

Ⓖ $\frac{7x^2 - 17x - 11}{(3x+4)(4x-7)}$

Ⓗ $\frac{7x^2 + 17x - 11}{(3x+4)(4x-7)}$

Ⓙ $\frac{7x^2 - 2x - 11}{(3x+4)(4x-7)}$

9. What is the inverse of the function $f(x) = \frac{7x-4}{3}$?

Ⓐ $f^{-1}(x) = \frac{3}{7x-4}$

Ⓑ $f^{-1}(x) = \frac{3x+4}{7}$

Ⓒ $f^{-1}(x) = \frac{3}{7}x - \frac{3}{4}$

Ⓓ $f^{-1}(x) = \frac{3}{7}x - \frac{4}{7}$

10. Solve for x : $3^{2x-1} = 27^{x+4}$.

Ⓕ 5

Ⓖ $\frac{7}{5}$

Ⓗ -2

Ⓙ -13

Gridded Response

11. What is the x-intercept of the axis of symmetry of the function $f(x) = 5x^2 - \frac{1}{2}x - 4$?

12. Evaluate.

$\log_{256} 16$

13. Solve for x.

$\frac{8}{x+4} - \frac{2}{x} = \frac{5}{x+4}$

14. Simplify.

$\left(\sqrt[3]{2^9}\right)^2$

15. Given $f(x) = 3x^2 - 1$ and $g(x) = \frac{1}{x+5}$, what is the value of $(f \circ g)(-6)$?

Short Response

16. A hyperbola has center $(3, -5)$, focus $(-10, -5)$, and vertex $(15, -5)$.

a. Write the equation for the hyperbola.

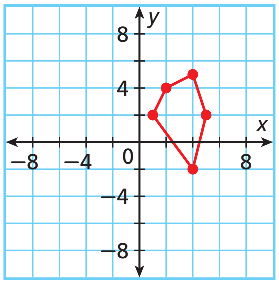
b. What are the equations of the asymptotes of the hyperbola?

17. The approximate heart rate of an adult can be modeled by $f(x) = -(x-5)^2 + 75$, where x is the age (in tens) of the person.

a. Find the inverse for the function, and explain what it represents.

b. Approximate the age of a person whose heart rate is 65.

18. The pentagon below has vertices at $(1, 2)$, $(2, 4)$, $(4, 5)$, $(5, 2)$, and $(4, -1)$.



a. Write the matrix used to transform the pentagon 3 units to the left and 5 units down.

b. What are the coordinates of the transformed pentagon?

Extended Response

19. Joan's grandmother gave her a diamond necklace valued at \$4500. The value of the necklace is predicted to appreciate 7.5% per year.

a. Write a function to model the predicted growth in the value of the necklace.

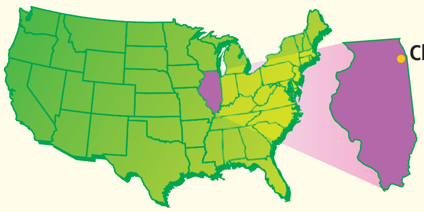
b. Graph the function.

c. What is the predicted value of the necklace in 10 years?

d. Based on the model, what was the value of the necklace 30 years ago?

Problem Solving on Location

ILLINOIS



The First Ferris Wheel

The organizers of the 1893 World's Fair in Chicago wanted an attraction that would outdo the Eiffel Tower, which had been built four years earlier for the Paris World's Fair. A bridge builder named George Ferris met the challenge by designing a colossal wheel that could carry more than 2100 passengers at a time. During the fair, 1.5 million visitors paid the 50-cent fee for a 20-minute ride on Ferris's wheel.

Choose one or more strategies to solve each problem. For 1, use the table.

Building Ferris's Wheel

Date	Total Number of Cars Attached
June 10, 1893	1
June 11, 1893	6
June 13, 1893	21

1. Each car of the wheel could carry up to 60 passengers. Because of the cars' enormous size, it took several days to hang all of the cars on the wheel. Develop a model to predict the number of cars that had been attached to the wheel by June 14, 1893.
2. As the wheel revolved, the paths of the cars could be modeled by $x^2 + y^2 - 250y = 0$. Find the diameter of the wheel.
3. Approximately how many feet has a car traveled after one complete revolution of the wheel?
4. A car starts at the bottom of the wheel. After 2 min 15 s, the car's horizontal distance to the central axle is 125 ft. How long does it take the car to make one complete revolution?

Problem Solving Strategies

- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Use a Venn Diagram
- Make an Organized List

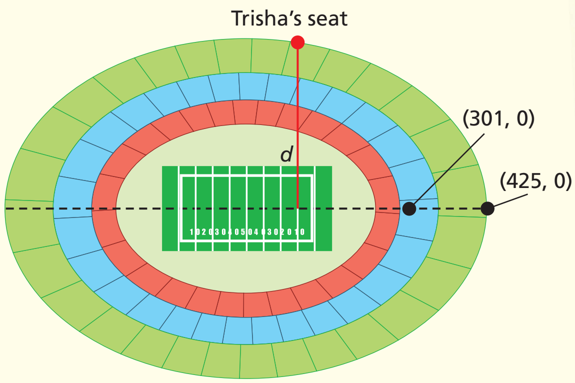
Soldier Field

Chicago's Soldier Field was built in 1924 as a multipurpose sports stadium. Since 1970, it has been the home of the National Football League's Chicago Bears. In 2003, a new 61,500-seat stadium was built within the shell of the original structure so that the historic colonnades and exterior walls of the old Soldier Field could be preserved.

Choose one or more strategies to solve each problem.

1. The renovated stadium can be modeled by an ellipse centered at the origin, a vertex at $(425, 0)$, and a focus at $(301, 0)$, measured in feet. Find the length and width of the stadium.

For 2, use the diagram.



2. Trisha bought tickets to a Bears game. As shown, her seat is on the 20-yard line, in the last row of the stadium. What is the horizontal distance d from her seat to the middle of the playing field?

3. During the game, a player makes a kick from the 40-yard line. The ball reaches a maximum height of 9 yards and lands 60 yards away. If the path of the ball is modeled by a parabola, does the ball clear the 10-foot-tall goalpost located 50 yards away? If so, by how many feet?