7 Right Triangles and Trigonometry

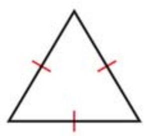
Before

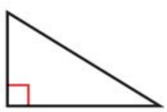
In previous courses and in Chapters 1–6, you learned the following skills, which you'll use in Chapter 7: classifying triangles, simplifying radicals, and solving proportions.

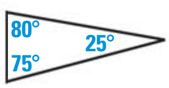
Prerequisite Skills

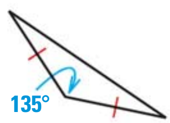
VOCABULARY CHECK

Name the triangle shown.

1. 

2. 

3. 

4. 

SKILLS AND ALGEBRA CHECK

Simplify the radical. *(Review p. 874 for 7.1, 7.2, 7.4.)*

5. $\sqrt{45}$ **6.** $(3\sqrt{7})^2$ **7.** $\sqrt{3} \cdot \sqrt{5}$ **8.** $\frac{7}{\sqrt{2}}$

Solve the proportion. *(Review p. 356 for 7.3, 7.5–7.7.)*

9. $\frac{3}{x} = \frac{12}{16}$ **10.** $\frac{2}{3} = \frac{x}{18}$ **11.** $\frac{x+5}{4} = \frac{1}{2}$ **12.** $\frac{x+4}{x-4} = \frac{6}{5}$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 493. You will also use the key vocabulary listed below.

Big Ideas

1. **Using the Pythagorean Theorem and its converse**
2. **Using special relationships in right triangles**
3. **Using trigonometric ratios to solve right triangles**

KEY VOCABULARY

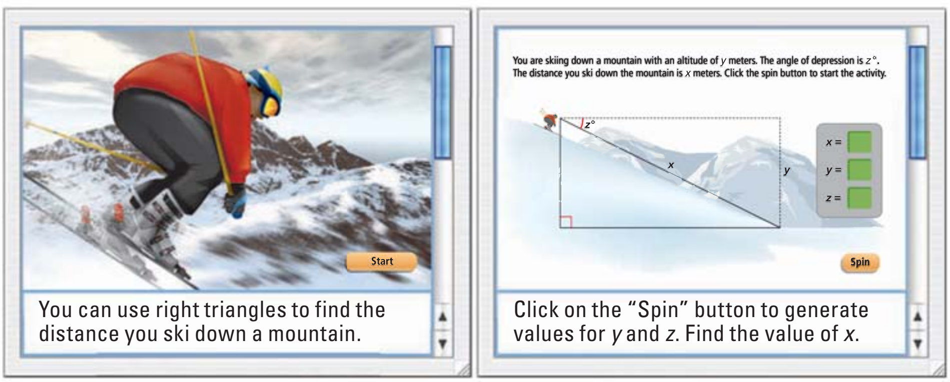
- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

Why?

You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

Animated Geometry

The animation illustrated below for Example 4 on page 475 helps you answer this question: How far will you ski down the mountain?



Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 7: pages 434, 442, 450, 460, and 462

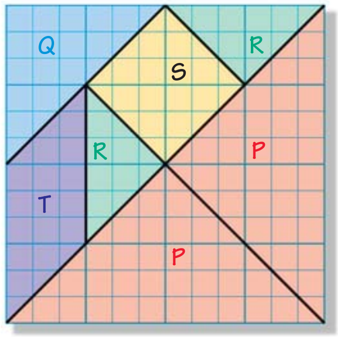
7.1 Pythagorean Theorem

MATERIALS • graph paper • ruler • pencil • scissors

QUESTION What relationship exists among the sides of a right triangle?

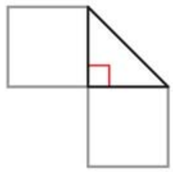
Recall that a square is a four sided figure with four right angles and four congruent sides.

EXPLORE Make and use a tangram set

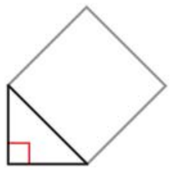


STEP 1 ***Make a tangram set*** On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

STEP 2 ***Trace a triangle*** On another piece of paper, trace one of the large triangles P of the tangram set.



STEP 3 ***Assemble pieces along the legs*** Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.



STEP 4 ***Assemble pieces along the hypotenuse*** Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle P?
2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle P?
3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.
4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? *Justify* your answers.

7.1 Apply the Pythagorean Theorem

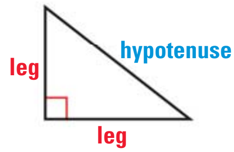
Before You learned about the relationships within triangles.

Now You will find side lengths in right triangles.

Why? So you can find the shortest distance to a campfire, as in Ex. 35.

Key Vocabulary
- **Pythagorean triple**
- **right triangle,** *p. 217*
- **leg of a right triangle,** *p. 241*
- **hypotenuse,** *p. 241*

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.

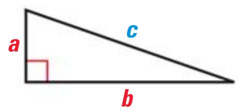


THEOREM — For Your Notebook

THEOREM 7.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

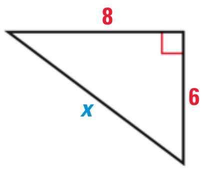
Proof: p. 434; Ex. 32, p. 455



$c^2 = a^2 + b^2$

EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



Solution

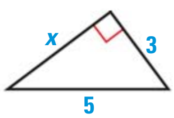
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$x^2 = 6^2 + 8^2$	**Substitute.**
$x^2 = 36 + 64$	**Multiply.**
$x^2 = 100$	**Add.**
$x = 10$	**Find the positive square root.**

ABBREVIATE
In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

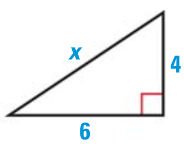
✓ GUIDED PRACTICE for Example 1

Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1.



2.

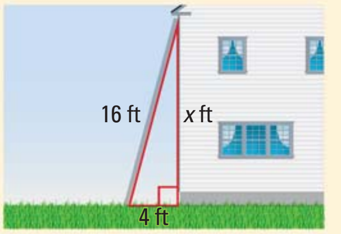


EXAMPLE 2 Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

Ⓐ 240 feet Ⓑ 20 feet

Ⓒ 16.5 feet Ⓓ 15.5 feet



Solution

$$\left(\begin{array}{c}\text{Length}\\\text{of ladder}\end{array}\right)^2 = \left(\begin{array}{c}\text{Distance}\\\text{from house}\end{array}\right)^2 + \left(\begin{array}{c}\text{Height}\\\text{of ladder}\end{array}\right)^2$$

$16^2 = 4^2 + x^2$ **Substitute.**

$256 = 16 + x^2$ **Multiply.**

$240 = x^2$ **Subtract 16 from each side.**

$\sqrt{240} = x$ **Find positive square root.**

$15.491 \approx x$ **Approximate with a calculator.**

The ladder is resting against the house at about 15.5 feet above the ground.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

APPROXIMATE

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

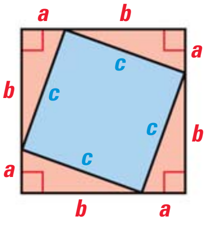
✓ GUIDED PRACTICE for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder?

4. The Pythagorean Theorem is only true for what type of triangle?

PROVING THE PYTHAGOREAN THEOREM There are many proofs of the Pythagorean Theorem. An informal proof is shown below. You will write another proof in Exercise 32 on page 455.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.



$$\text{Area of large square} = \text{Area of four triangles} + \text{Area of smaller square}$$

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$ **Use area formulas.**

$a^2 + 2ab + b^2 = 2ab + c^2$ **Multiply.**

$a^2 + b^2 = c^2$ **Subtract $2ab$ from each side.**

REVIEW AREA

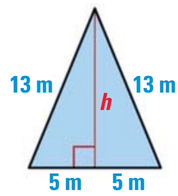
Recall that the area of a square with side length s is $A = s^2$. The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.

Animated Geometry at classzone.com

EXAMPLE 3 Find the area of an isosceles triangle

Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.

Solution



STEP 1 **Draw** a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.

STEP 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$13^2 = 5^2 + h^2$	Substitute.
$169 = 25 + h^2$	Multiply.
$144 = h^2$	Subtract 25 from each side.
$12 = h$	Find the positive square root.

STEP 3 **Find** the area.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2$$

▶ The area of the triangle is 60 square meters.

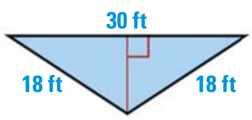
READ TABLES

You may find it helpful to use the Table of Squares and Square Roots on p. 924.

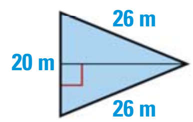
✓ GUIDED PRACTICE for Example 3

Find the area of the triangle.

5.



6.



PYTHAGOREAN TRIPLES A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

STANDARDIZED TESTS

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

KEY CONCEPT *For Your Notebook*

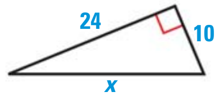
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $5 \cdot 2 = 10$ and $12 \cdot 2 = 24$. So, the length of the hypotenuse is $13 \cdot 2 = 26$.

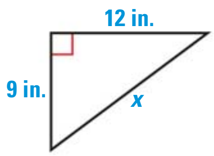
Method 2: Use the Pythagorean Theorem.

$x^2 = 10^2 + 24^2$	**Pythagorean Theorem**
$x^2 = 100 + 576$	**Multiply.**
$x^2 = 676$	**Add.**
$x = 26$	**Find the positive square root.**

✓ GUIDED PRACTICE for Example 4

Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7.



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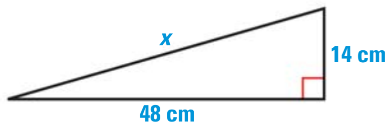


7.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 27, 33, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

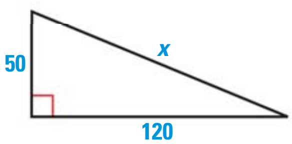
1. **VOCABULARY** Copy and complete: A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$ is called a _?_.

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle.

EXAMPLE 1 on p. 433 for Exs. 3–7

ALGEBRA **Find the length of the hypotenuse of the right triangle.**

3.



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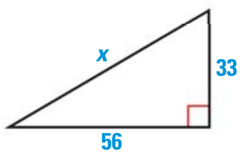

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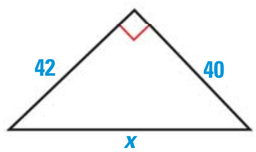
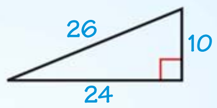


ERROR ANALYSIS *Describe* and correct the error in using the Pythagorean Theorem.

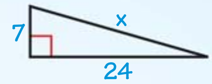
6.



$a^2 + b^2 = c^2$

$10^2 + 26^2 = 24^2$

7.



$x^2 = 7^2 + 24^2$

$x^2 = (7 + 24)^2$

$x^2 = 31^2$

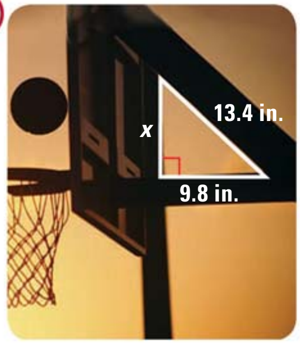
$x = 31$

EXAMPLE 2
on p. 434
for Exs. 8–10

FINDING A LENGTH Find the unknown leg length x.

8.

9.

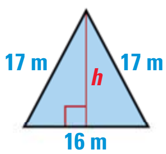


10.

EXAMPLE 3
on p. 435
for Exs. 11–13

FINDING THE AREA Find the area of the isosceles triangle.

11.



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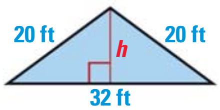

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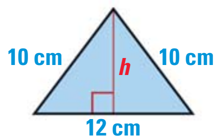

EXAMPLE 4
on p. 436
for Exs. 14–17

FINDING SIDE LENGTHS Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.

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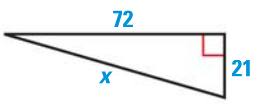

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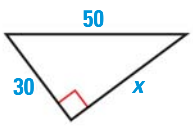

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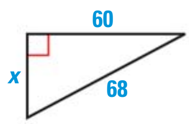


17. ★ **MULTIPLE CHOICE** What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches?

Ⓐ 13 inches Ⓑ 17 inches Ⓒ 21 inches Ⓓ 25 inches

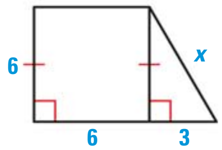
PYTHAGOREAN TRIPLES The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.

18. 24 and 51 **19.** 20 and 25 **20.** 28 and 96

21. 20 and 48 **22.** 75 and 85 **23.** 72 and 75

FINDING SIDE LENGTHS **Find the unknown side length x. Write your answer in simplest radical form.**

24.



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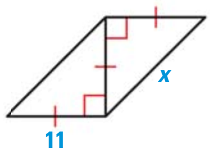

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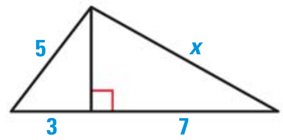


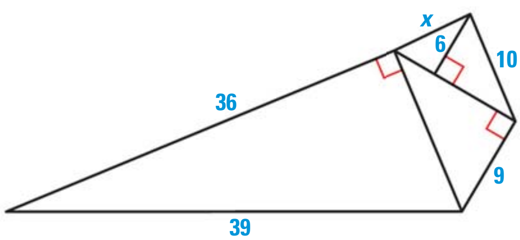
27. ★ **MULTIPLE CHOICE** What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?

Ⓐ 270 ft^2 Ⓑ 292.5 ft^2 Ⓒ 540 ft^2 Ⓓ 585 ft^2

28. **xy ALGEBRA** Solve for x if the lengths of the two legs of a right triangle are $2x$ and $2x + 4$, and the length of the hypotenuse is $4x - 4$.

CHALLENGE **In Exercises 29 and 30, solve for x.**

29.



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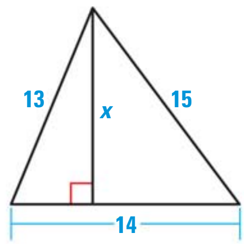
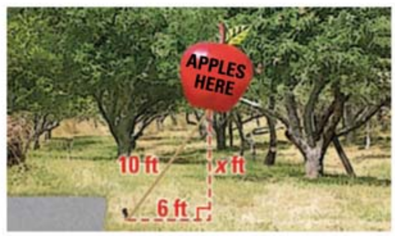


PROBLEM SOLVING

EXAMPLE 2
on p. 434
for Exs. 31–32

31. **BASEBALL DIAMOND** In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base?

@HomeTutor for problem solving help at classzone.com



32. **APPLE BALLOON** You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now?

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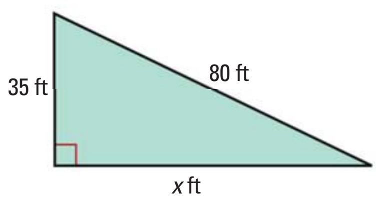
33. ★ **SHORT RESPONSE** Three side lengths of a right triangle are 25, 65, and 60. *Explain* how you know which side is the hypotenuse.

34. **MULTI-STEP PROBLEM** In your town, there is a field that is in the shape of a right triangle with the dimensions shown.

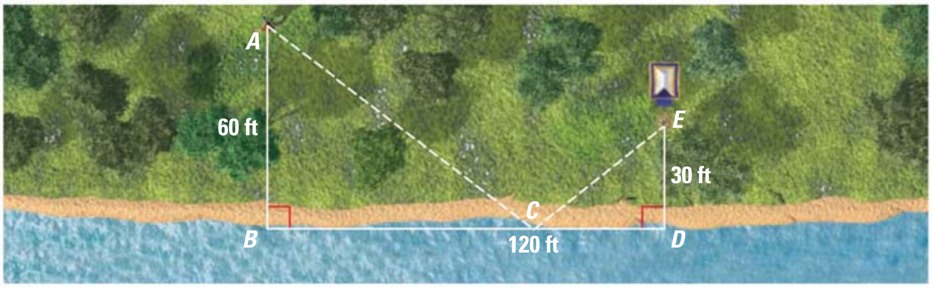
a. Find the perimeter of the field.

b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need?

c. If each dogwood seedling sells for $12, how much will the trees cost?



35. **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



a. **Making a Table** Make a table with columns labeled BC, AC, CE, and $AC + CE$. Enter values of BC from 10 to 120 in increments of 10.

b. **Calculating Values** Calculate AC, CE, and $AC + CE$ for each value of BC, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. **Drawing a Picture** Draw an accurate picture to scale of the shortest distance.

36. ★ **SHORT RESPONSE** *Justify* the Distance Formula using the Pythagorean Theorem.

37. **PROVING THEOREM 4.5** Find the Hypotenuse-Leg (HL) Congruence Theorem on page 241. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove Theorem 4.5.

38. **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.2 in Exs. 39–42.

Evaluate the expression. *(p. 874)*

39. $(\sqrt{7})^2$ **40.** $(4\sqrt{3})^2$ **41.** $(-6\sqrt{81})^2$ **42.** $(-8\sqrt{2})^2$

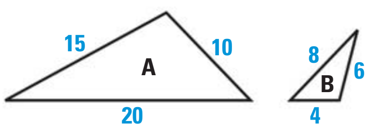
Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.** *(p. 328)*

43. 3 feet, 6 feet **44.** 5 inches, 11 inches **45.** 14 meters, 21 meters

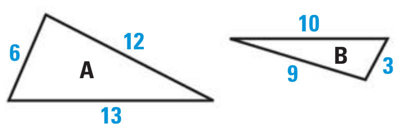
46. 12 inches, 27 inches **47.** 18 yards, 18 yards **48.** 27 meters, 39 meters

Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A. *(p. 388)*

49.



50.



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Keystrokes

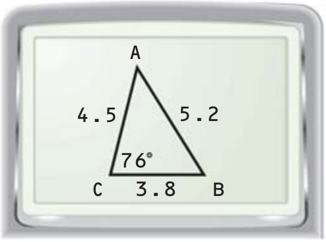
7.2 Converse of the Pythagorean Theorem

MATERIALS • graphing calculator or computer

QUESTION How can you use the side lengths in a triangle to classify the triangle by its angle measures?

You can use geometry drawing software to construct and measure triangles.

EXPLORE Construct a triangle



STEP 1 ***Draw a triangle*** Draw any $\triangle ABC$ with the largest angle at C. Measure $\angle C$, $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$.

STEP 2 ***Calculate*** Use your measurements to calculate AB^2, AC^2, CB^2, and $(AC^2 + CB^2)$.

STEP 3 ***Complete a table*** Copy the table below and record your results in the first row. Then move point A to different locations and record the values for each triangle in your table. Make sure $\overline{AB}$ is always the longest side of the triangle. Include triangles that are acute, right, and obtuse.

$m\angle C$	AB	AB^2	AC	CB	$AC^2 + CB^2$
76°	5.2	27.04	4.5	3.8	34.69
?	?	?	?	?	?
?	?	?	?	?	?

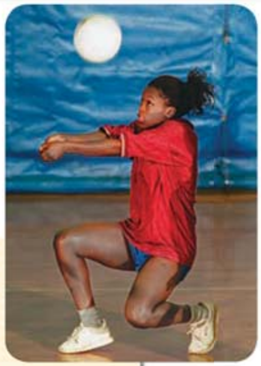
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The Pythagorean Theorem states that "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." Write the Pythagorean Theorem in if-then form. Then write its converse.
2. Is the converse of the Pythagorean Theorem true? *Explain*.
3. Make a conjecture about the relationship between the measure of the largest angle in a triangle and the squares of the side lengths.

Copy and complete the statement.

4. If $AB^2 > AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
5. If $AB^2 < AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
6. If $AB^2 = AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.

7.2 Use the Converse of the Pythagorean Theorem



Before You used the Pythagorean Theorem to find missing side lengths.

Now You will use its converse to determine if a triangle is a right triangle.

Why? So you can determine if a volleyball net is set up correctly, as in Ex. 38.

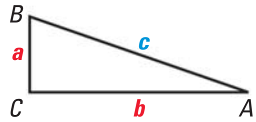
Key Vocabulary
- **acute triangle,** *p. 217*
- **obtuse triangle,** *p. 217*

The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

THEOREM *For Your Notebook*

THEOREM 7.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



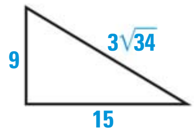
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Proof: Ex. 42, p. 446

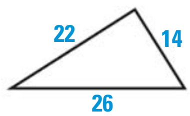
EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

b. $26^2 \stackrel{?}{=} 22^2 + 14^2$

$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

REVIEW ALGEBRA

Use a square root table or a calculator to find the decimal representation. So, $3\sqrt{34} \approx 17.493$ is the length of the longest side in part (a).

✓ **GUIDED PRACTICE** for Example 1

Tell whether a triangle with the given side lengths is a right triangle.

1. $4, 4\sqrt{3}, 8$

2. 10, 11, and 14

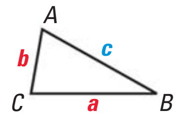
3. 5, 6, and $\sqrt{61}$

CLASSIFYING TRIANGLES The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.

THEOREMS *For Your Notebook*

THEOREM 7.3

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an acute triangle.

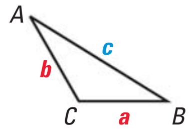


If $c^2 < a^2 + b^2$, then the triangle ABC is acute.

Proof: Ex. 40, p. 446

THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an obtuse triangle.



If $c^2 > a^2 + b^2$, then triangle ABC is obtuse.

Proof: Ex. 41, p. 446

EXAMPLE 2 Classify triangles

Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?

APPLY THEOREMS
The Triangle Inequality Theorem on page 330 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution

STEP 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$4.3 + 5.2 = 9.5$ $\qquad$ $4.3 + 6.1 = 10.4$ $\qquad$ $5.2 + 6.1 = 11.3$

$9.5 > 6.1$ $\qquad$ $10.4 > 5.2$ $\qquad$ $11.3 > 4.3$

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

STEP 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

c^2	$\underline{\ ?\ }$	$a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
6.1^2	$\underline{\ ?\ }$	$4.3^2 + 5.2^2$	**Substitute.**
37.21	$\underline{\ ?\ }$	$18.49 + 27.04$	**Simplify.**
37.21	$<$	45.53	**c^2 is less than $a^2 + b^2$.**

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

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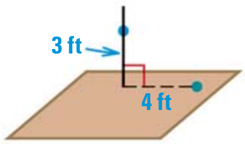
EXAMPLE 3 Use the Converse of the Pythagorean Theorem

CATAMARAN You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

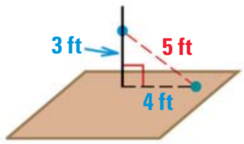
Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

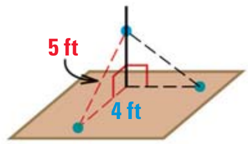
Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.



First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast.



Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.



Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

✓ GUIDED PRACTICE for Example 2 and 3

4. Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as *acute, right,* or *obtuse.*

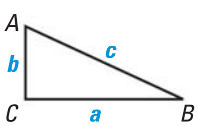
5. **WHAT IF?** In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? *Explain.*

CLASSIFYING TRIANGLES You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

CONCEPT SUMMARY *For Your Notebook*

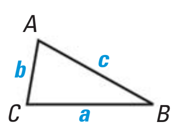
Methods for Classifying a Triangle by Angles Using its Side Lengths

Theorem 7.2



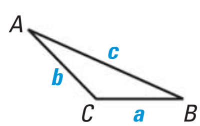
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Theorem 7.4



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

7.2 EXERCISES

HOMEWORK KEY

= **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 38, 39, and 43

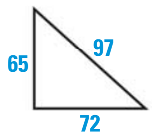
SKILL PRACTICE

1. **VOCABULARY** What is the longest side of a right triangle called?

2. ★ **WRITING** *Explain* how the side lengths of a triangle can be used to classify it as acute, right, or obtuse.

EXAMPLE 1 on p. 441 for Exs. 3–14

VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

3.


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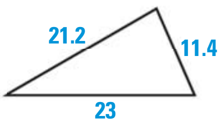

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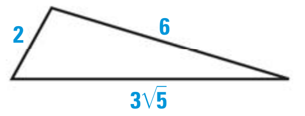

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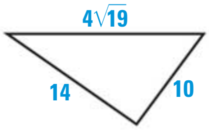

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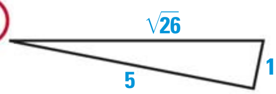

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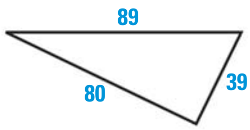


VERIFYING RIGHT TRIANGLES **Tell whether the given side lengths of a triangle can represent a right triangle.**

9. 9, 12, and 15
10. 9, 10, and 15
11. 36, 48, and 60
12. 6, 10, and $2\sqrt{34}$
13. 7, 14, and $7\sqrt{5}$
14. 10, 12, and 20

EXAMPLE 2 on p. 442 for Exs. 15–23

CLASSIFYING TRIANGLES **In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be *acute, right,* or *obtuse*?**

15. 10, 11, and 14
16. 10, 15, and $5\sqrt{13}$
17. 24, 30, and $6\sqrt{43}$
18. 5, 6, and 7
19. 12, 16, and 20
20. 8, 10, and 12
21. 15, 20, and 36
22. 6, 8, and 10
23. 8.2, 4.1, and 12.2

24. ★ **MULTIPLE CHOICE** Which side lengths do not form a right triangle?

   (A) 5, 12, 13 (B) 10, 24, 28 (C) 15, 36, 39 (D) 50, 120, 130

25. ★ **MULTIPLE CHOICE** What type of triangle has side lengths of 4, 7, and 9?

   (A) Acute scalene (B) Right scalene
   (C) Obtuse scalene (D) None of the above

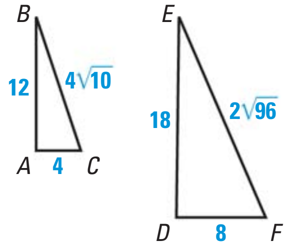
26. **ERROR ANALYSIS** A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. *Explain* why this statement is incorrect.

GRAPHING TRIANGLES **Graph points *A*, *B*, and *C*. Connect the points to form $\triangle ABC$. Decide whether $\triangle ABC$ is *acute, right,* or *obtuse*.**

27. $A(-2, 4)$, $B(6, 0)$, $C(-5, -2)$
28. $A(0, 2)$, $B(5, 1)$, $C(1, -1)$

29. **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is *acute*, *right*, or *obtuse*.

USING DIAGRAMS In Exercises 30 and 31, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.



30. $m\angle A$ __?__ $m\angle D$

31. $m\angle B + m\angle C$ __?__ $m\angle E + m\angle F$

32. ★ **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and x (where $x > 0$). What are the values of x that make the triangle a right triangle? an acute triangle? an obtuse triangle?

33. **ALGEBRA** The sides of a triangle have lengths x, $x + 4$, and 20. If the length of the longest side is 20, what values of x make the triangle acute?

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of x make the triangle obtuse?

PROBLEM SOLVING

EXAMPLE 3 on p. 443 for Ex. 35

35. **PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

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36. **WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain.*

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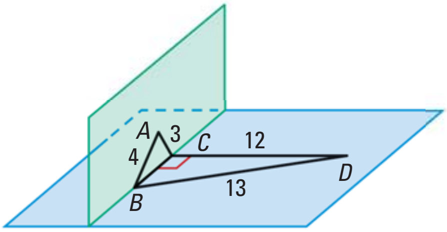
37. **MULTI-STEP PROBLEM** Use the diagram shown.

a. Find BC.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not.



38. ★ **SHORT RESPONSE** You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 4 feet from the pole. The rope between the pole and stake is about 8 feet 4 inches long. Is the pole perpendicular to the ground? *Explain*. If it is not, how can you fix it?

39. ★ **EXTENDED RESPONSE** You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? *Explain*.

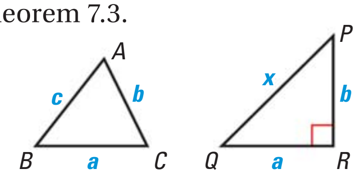
b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? *Explain*.

c. The car owner says the car was never in an accident. Do you believe this claim? *Explain*.

40. **PROVING THEOREM 7.3** Copy and complete the proof of Theorem 7.3.

GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.

PROVE ▶ $\triangle ABC$ is an acute triangle.



Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x, where $\angle R$ is a right angle and x is the length of the longest side. Compare lengths c and x.

STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** _?_
2. $a^2 + b^2 = x^2$	**2.** _?_
3. $c^2 < x^2$	**3.** _?_
4. $c < x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** _?_
6. $m\angle C < m\angle$ _?_	**6.** Converse of the Hinge Theorem
7. $m\angle C < 90°$	**7.** _?_
8. $\angle C$ is an acute angle.	**8.** _?_
9. $\triangle ABC$ is an acute triangle.	**9.** _?_

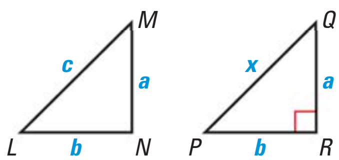
41. **PROVING THEOREM 7.4** Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (*Hint*: Look back at Exercise 40.)

42. **PROVING THEOREM 7.2** Prove the Converse of the Pythagorean Theorem.

GIVEN ▶ In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.

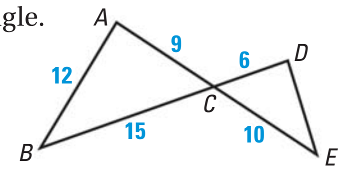
PROVE ▶ $\triangle LMN$ is a right triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



★ = STANDARDIZED TEST PRACTICE

43. ★ **SHORT RESPONSE** *Explain* why $\angle D$ must be a right angle.



44. **COORDINATE PLANE** Use graph paper.

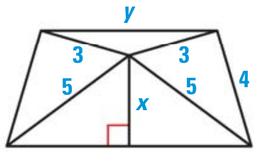
a. Graph $\triangle ABC$ with $A(-7, 2)$, $B(0, 1)$ and $C(-4, 4)$.

b. Use the slopes of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

c. Use the lengths of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

d. Did you get the same answer in parts (b) and (c)? If not, *explain* why.

45. **CHALLENGE** Find the values of x and y.

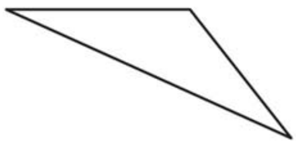


MIXED REVIEW

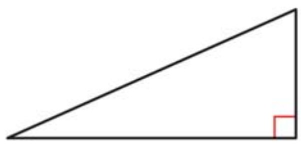
PREVIEW
Prepare for Lesson 7.3 in Exs. 46–48.

In Exercises 46–48, copy the triangle and draw one of its altitudes. *(p. 319)*

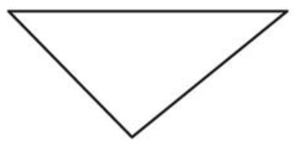
46.



47.



48.



Copy and complete the statement. *(p. 364)*

49. If $\frac{10}{x} = \frac{7}{y}$, then $\frac{10}{7} = \frac{?}{?}$.

50. If $\frac{x}{15} = \frac{y}{2}$, then $\frac{x}{y} = \frac{?}{?}$.

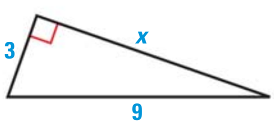
51. If $\frac{x}{8} = \frac{y}{9}$, then $\frac{x+8}{8} = \frac{?}{?}$.

52. The perimeter of a rectangle is 135 feet. The ratio of the length to the width is 8 : 1. Find the length and the width. *(p. 372)*

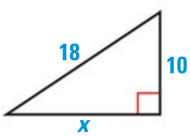
QUIZ *for Lessons 7.1–7.2*

Find the unknown side length. Write your answer in simplest radical form. *(p. 433)*

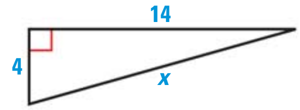
1.



2.



3.



Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*. *(p. 441)*

4. 6, 7, and 9

5. 10, 12, and 16

6. 8, 16, and $8\sqrt{6}$

7. 20, 21, and 29

8. 8, 3, $\sqrt{73}$

9. 8, 10, and 12

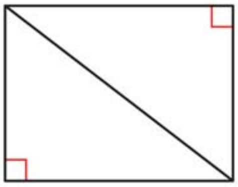
7.3 Similar Right Triangles

MATERIALS • rectangular piece of paper • ruler • scissors • colored pencils

QUESTION How are geometric means related to the altitude of a right triangle?

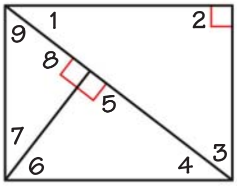
EXPLORE Compare right triangles

STEP 1



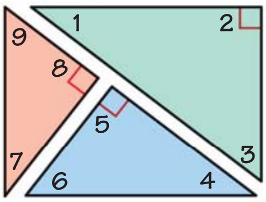
Draw a diagonal Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

STEP 2



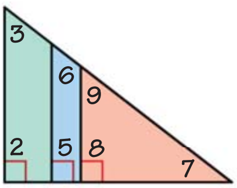
Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.

STEP 3



Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP 4



Arrange the triangles Arrange the triangles so ∠1, ∠4, and ∠7 are on top of each other as shown.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. How are the two smaller right triangles related to the large triangle?
2. *Explain* how you would show that the green triangle is similar to the red triangle.
3. *Explain* how you would show that the red triangle is similar to the blue triangle.
4. The *geometric mean* of a and b is x if $\frac{a}{x} = \frac{x}{b}$. Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion.

7.3 Use Similar Right Triangles

Before	You identified the altitudes of a triangle.
Now	You will use properties of the altitude of a right triangle.
Why?	So you can determine the height of a wall, as in Example 4.

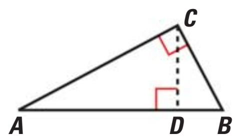
Key Vocabulary
- **altitude of a triangle,** *p. 320*
- **geometric mean,** *p. 359*
- **similar polygons,** *p. 372*

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

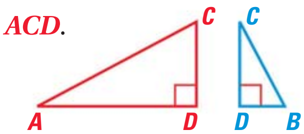
THEOREM *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



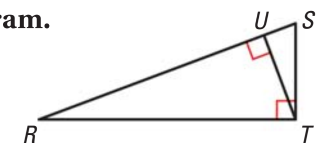
Proof: below; Ex. 35, p. 456

Plan for Proof of Theorem 7.5 First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle and each triangle includes $\angle B$. The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that $\triangle ACD \sim \triangle ABC$.

To show $\angle CBD \sim \triangle ACD$, begin by showing $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Each triangle also has a right angle, so you can use the AA Similarity Postulate.

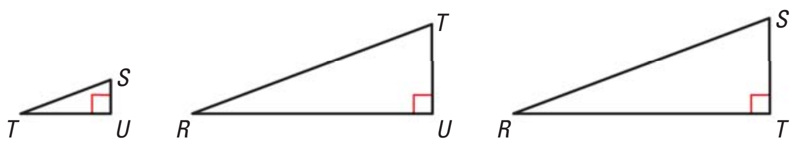
EXAMPLE 1 Identify similar triangles

Identify the similar triangles in the diagram.



Solution

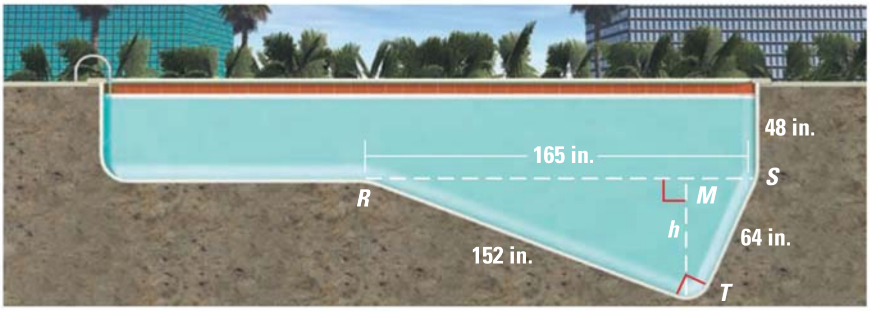
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

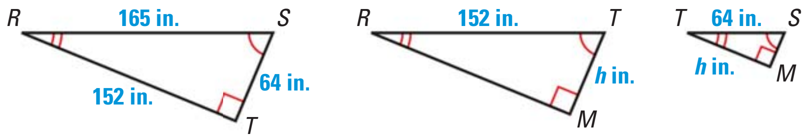
EXAMPLE 2 Find the length of the altitude to the hypotenuse

SWIMMING POOL The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?



Solution

STEP 1 **Identify** the similar triangles and sketch them.



$\triangle RST \sim \triangle RTM \sim \triangle TSM$

STEP 2 **Find** the value of h. Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

$\frac{TM}{ST} = \frac{TR}{SR}$	**Corresponding side lengths of similar triangles are in proportion.**
$\frac{h}{64} = \frac{152}{165}$	**Substitute.**
$165h = 64(152)$	**Cross Products Property**
$h \approx 59$	**Solve for *h*.**

STEP 3 **Read** the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

▶ The maximum depth of the pool is about 107 inches.

Animated Geometry at classzone.com

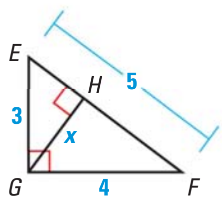
AVOID ERRORS

Notice that if you tried to write a proportion using $\triangle RTM$ and $\triangle TSM$, there would be two unknowns, so you would not be able to solve for *h*.

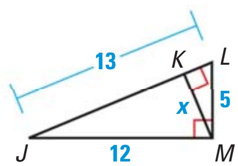
✓ GUIDED PRACTICE for Examples 1 and 2

Identify the similar triangles. Then find the value of *x*.

1.

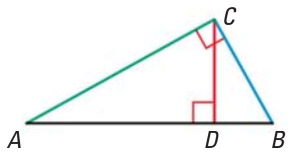


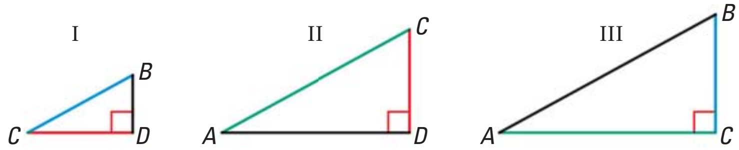
2.



READ SYMBOLS Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and CD refers to its length.

GEOMETRIC MEANS In Lesson 6.1, you learned that the *geometric mean* of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.





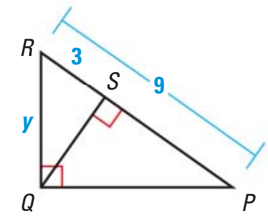
Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the geometric mean of BD and AD. As you see below, CB and AC are also geometric means of segment lengths in the diagram.

Proportions Involving Geometric Means in Right $\triangle ABC$

length of shorter leg of I length of shorter leg of II	→	$\frac{BD}{CD} = \frac{CD}{AD}$	← length of longer leg of I length of longer leg of II
length of hypotenuse of III length of hypotenuse of I	→	$\frac{AB}{CB} = \frac{CB}{DB}$	← length of shorter leg of III length of shorter leg of I
length of hypotenuse of III length of hypotenuse of II	→	$\frac{AB}{AC} = \frac{AC}{AD}$	← length of longer leg of III length of longer leg of II

EXAMPLE 3 Use a geometric mean

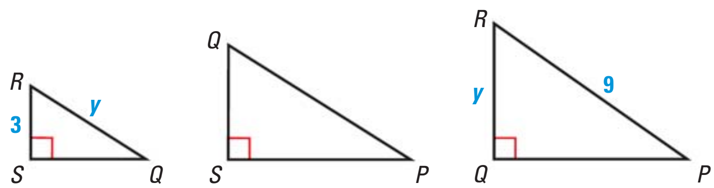
xy **Find the value of y. Write your answer in simplest radical form.**



Solution

REVIEW SIMILARITY Notice that $\triangle RQS$ and $\triangle RPQ$ both contain the side with length y, so these are the similar pair of triangles to use to solve for y.

STEP 1 **Draw** the three similar triangles.



STEP 2 **Write** a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$\frac{9}{y} = \frac{y}{3}$ **Substitute.**

$27 = y^2$ **Cross Products Property**

$\sqrt{27} = y$ **Take the positive square root of each side.**

$3\sqrt{3} = y$ **Simplify.**

WRITE PROOFS
In Exercise 32 on page 455, you will use the geometric mean theorems to prove the Pythagorean Theorem.

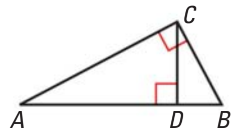
THEOREMS *For Your Notebook*

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

Proof: Ex. 36, p. 456



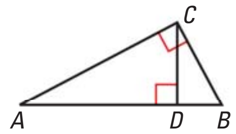
$\frac{BD}{CD} = \frac{CD}{AD}$

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Proof: Ex. 37, p. 456

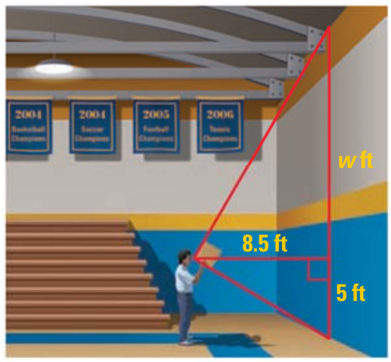


$\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$

EXAMPLE 4 Find a height using indirect measurement

ROCK CLIMBING WALL To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.



Solution

By Theorem 7.6, you know that 8.5 is the geometric mean of w and 5.

$\frac{w}{8.5} = \frac{8.5}{5}$ **Write a proportion.**

$w \approx 14.5$ **Solve for *w*.**

▶ So, the height of the wall is $5 + w \approx 5 + 14.5 = 19.5$ feet.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In Example 3, which theorem did you use to solve for y? *Explain*.

4. Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height?

7.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 20, 31, and 34

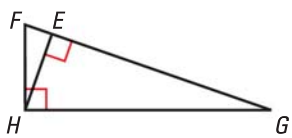
SKILL PRACTICE

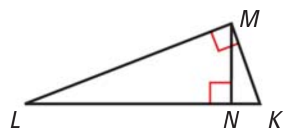
1. **VOCABULARY** Copy and complete: Two triangles are __?__ if their corresponding angles are congruent and their corresponding side lengths are proportional.

2. ★ **WRITING** In your own words, explain *geometric mean*.

EXAMPLE 1 on p. 449 for Exs. 3–4

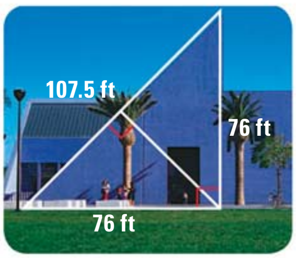
IDENTIFYING SIMILAR TRIANGLES **Identify the three similar right triangles in the given diagram.**

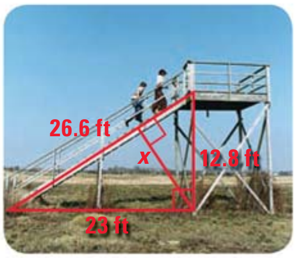
3. 

4. 

EXAMPLE 2 on p. 450 for Exs. 5–7

FINDING ALTITUDES **Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

5. 

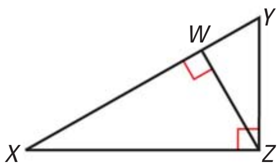
6. 

7. 

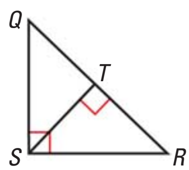
EXAMPLES 3 and 4 on pp. 451–452 for Exs. 8–18

COMPLETING PROPORTIONS **Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.**

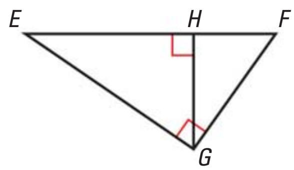
8. $\frac{XW}{?} = \frac{ZW}{YW}$



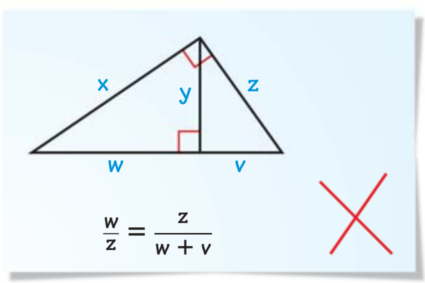
9. $\frac{?}{SQ} = \frac{SQ}{TQ}$

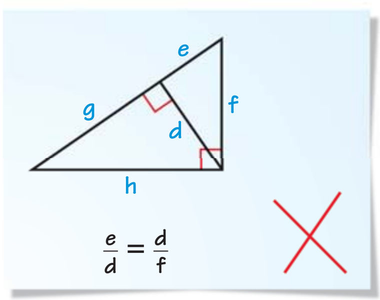


10. $\frac{EF}{EG} = \frac{EG}{?}$



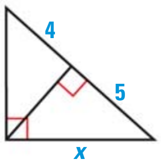
ERROR ANALYSIS ***Describe*** **and correct the error in writing a proportion for the given diagram.**

11. 

12. 

FINDING LENGTHS **Find the value of the variable. Round decimal answers to the nearest tenth.**

13.



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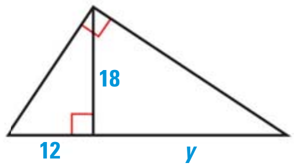

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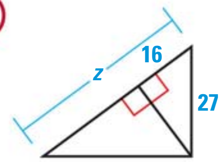

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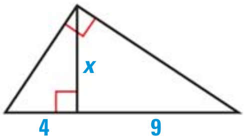
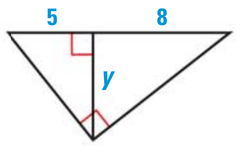


17.



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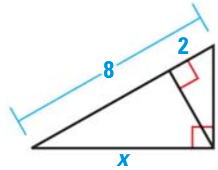
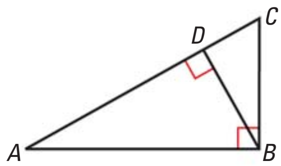


19. ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false.

Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{CA}{AB} = \frac{AB}{AD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DC}{BC} = \frac{BC}{CA}$

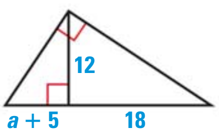


20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, $AC = 36$ and $BC = 18$. Find AD. If necessary, round to the nearest tenth.

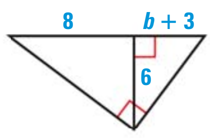
Ⓐ 9 Ⓑ 15.6 Ⓒ 27 Ⓓ 31.2

xy ALGEBRA **Find the value(s) of the variable(s).**

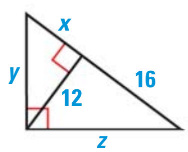
21.



22.

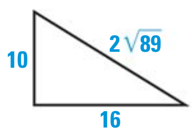


23.

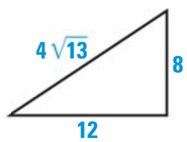


USING THEOREMS **Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

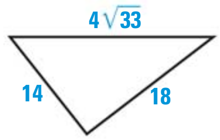
24.



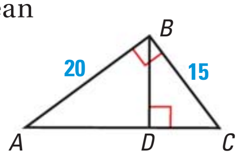
25.



26.



27. **FINDING LENGTHS** Use the Geometric Mean Theorems to find AC and BD.



28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now draw the three similar triangles and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? *Explain.*

○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

29. **DOGHOUSE** The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof.

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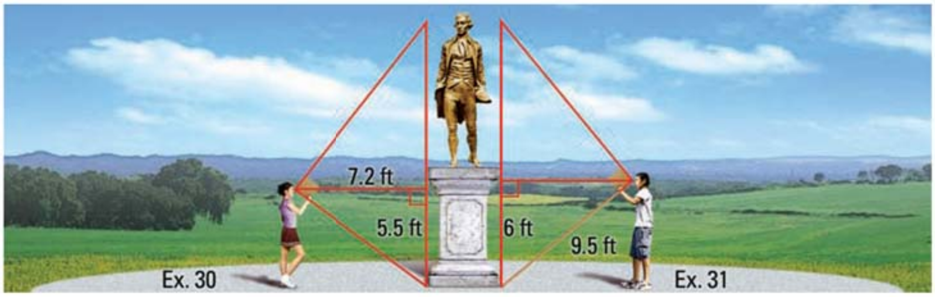
EXAMPLE 4
on p. 452
for Exs. 30–31

30. **MONUMENT** You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below).



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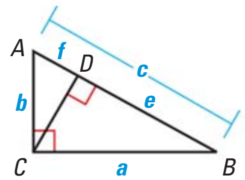
31. ★ **SHORT RESPONSE** Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? *Explain*.

32. **PROVING THEOREM 7.1** Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.

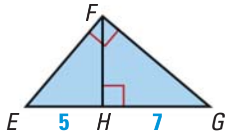


GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $c^2 = a^2 + b^2$

STATEMENTS	REASONS
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	**1.** ___?___
2. Draw a perpendicular from C to $\overline{AB}$.	**2.** Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**3.** ___?___
4. $ce = a^2$ and $cf = b^2$	**4.** ___?___
5. $ce + b^2 =$ ___?___ $+ b^2$	**5.** Addition Property of Equality
6. $ce + cf = a^2 + b^2$	**6.** ___?___
7. $c(e + f) = a^2 + b^2$	**7.** ___?___
8. $e + f =$ ___?___	**8.** Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	**9.** ___?___
10. $c^2 = a^2 + b^2$	**10.** Simplify.

33. **MULTI-STEP PROBLEM** Use the diagram.

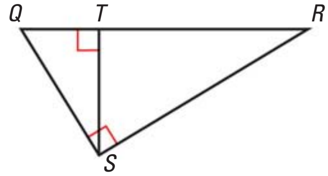


a. Name all the altitudes in $\triangle EGF$. *Explain.*

b. Find FH.

c. Find the area of the triangle.

34. ★ **EXTENDED RESPONSE** Use the diagram.

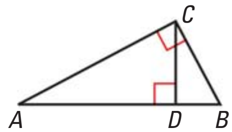


a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.

b. Write similarity statements for the three triangles.

c. Which segment's length is the geometric mean of RT and RQ? *Explain* your reasoning.

PROVING THEOREMS In Exercises 35–37, use the diagram and GIVEN statements below.



GIVEN ▶ $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

35. Prove Theorem 7.5 by using the Plan for Proof on page 449.

36. Prove Theorem 7.6 by showing $\frac{BD}{CD} = \frac{CD}{AD}$.

37. Prove Theorem 7.7 by showing $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

38. **CHALLENGE** The *harmonic mean* of a and b is $\frac{2ab}{a+b}$. The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.

a. Find the harmonic mean of 10 and 15.

b. Find the harmonic mean of 6 and 14.

c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.4 in Exs. 39–46.

Simplify the expression. *(p. 874)*

39. $\sqrt{27} \cdot \sqrt{2}$
40. $\sqrt{8} \cdot \sqrt{10}$
41. $\sqrt{12} \cdot \sqrt{7}$
42. $\sqrt{18} \cdot \sqrt{12}$

43. $\frac{5}{\sqrt{7}}$
44. $\frac{8}{\sqrt{11}}$
45. $\frac{15}{\sqrt{27}}$
46. $\frac{12}{\sqrt{24}}$

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

47. **Line 1:** (2, 4), (4, 2)
Line 2: (3, 5), (−1, 1)

48. **Line 1:** (0, 2), (−1, −1)
Line 2: (3, 1), (1, −5)

49: **Line 1:** (1, 7), (4, 7)
Line 2: (5, 2), (7, 4)

7.4 Special Right Triangles

Before You found side lengths using the Pythagorean Theorem.

Now You will use the relationships among the sides in special right triangles.

Why? So you can find the height of a drawbridge, as in Ex. 28.

Key Vocabulary
- **isosceles triangle,** *p. 217*

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.

USE RATIOS
The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

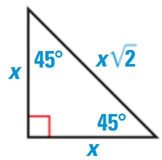
THEOREM *For Your Notebook*

THEOREM 7.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$

Proof: Ex. 30, p. 463



EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle

Find the length of the hypotenuse.

a.

b.

Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$	**45°-45°-90° Triangle Theorem**
$= 8\sqrt{2}$	**Substitute.**

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$	**45°-45°-90° Triangle Theorem**
$= 3\sqrt{2} \cdot \sqrt{2}$	**Substitute.**
$= 3 \cdot 2$	**Product of square roots**
$= 6$	**Simplify.**

REVIEW ALGEBRA
Remember the following properties of radicals:

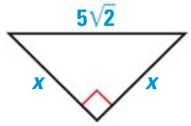
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

$\sqrt{a \cdot a} = a$

For a review of radical expressions, see p. 874.

EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle

Find the lengths of the legs in the triangle.



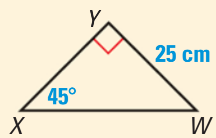
Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$5\sqrt{2} = x \cdot \sqrt{2}$	**Substitute.**
$\dfrac{5\sqrt{2}}{\sqrt{2}} = \dfrac{x\sqrt{2}}{\sqrt{2}}$	**Divide each side by $\sqrt{2}$.**
$5 = x$	**Simplify.**

EXAMPLE 3 Standardized Test Practice

Triangle *WXY* is a right triangle. Find the length of $\overline{WX}$.



Ⓐ 50 cm

Ⓑ $25\sqrt{2}$ cm

Ⓒ 25 cm

Ⓓ $\dfrac{25\sqrt{2}}{2}$ cm

ELIMINATE CHOICES
You can eliminate choices C and D because the hypotenuse has to be longer than the leg.

Solution

By the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

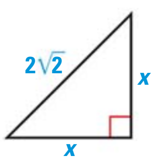
hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$WX = 25\sqrt{2}$	**Substitute.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

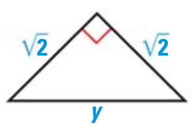
GUIDED PRACTICE for Examples 1, 2, and 3

Find the value of the variable.

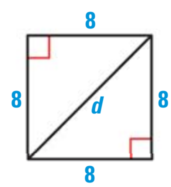
1.



2.



3.



4. Find the leg length of a 45°-45°-90° triangle with a hypotenuse length of 6.

A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

USE RATIOS

The extended ratio of the side lengths of a 30°-60°-90° triangle is $1 : \sqrt{3} : 2$.

THEOREM *For Your Notebook*

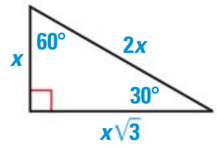
THEOREM 7.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse $= 2 \cdot$ shorter leg

longer leg $=$ shorter leg $\cdot \sqrt{3}$

Proof: Ex. 32, p. 463



EXAMPLE 4 Find the height of an equilateral triangle

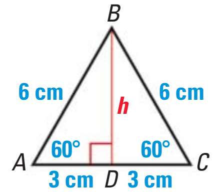
LOGO The logo on the recycling bin at the right resembles an equilateral triangle with side lengths of 6 centimeters. What is the approximate height of the logo?

Solution

REVIEW MEDIAN

Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30°-60°-90° triangles. The length h of the altitude is approximately the height of the logo.

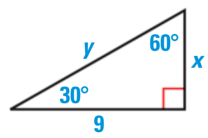


$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$

$$h = 3 \cdot \sqrt{3} \approx 5.2 \text{ cm}$$

EXAMPLE 5 Find lengths in a 30°-60°-90° triangle

xy Find the values of x and y. Write your answer in simplest radical form.



STEP 1 **Find** the value of x.

longer leg $=$ shorter leg $\cdot \sqrt{3}$	**30°-60°-90° Triangle Theorem**
$9 = x\sqrt{3}$	**Substitute.**
$\frac{9}{\sqrt{3}} = x$	**Divide each side by $\sqrt{3}$.**
$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$	**Multiply numerator and denominator by $\sqrt{3}$.**
$\frac{9\sqrt{3}}{3} = x$	**Multiply fractions.**
$3\sqrt{3} = x$	**Simplify.**

STEP 2 **Find** the value of y.

hypotenuse $= 2 \cdot$ shorter leg	**30°-60°-90° Triangle Theorem**
$y = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$	**Substitute and simplify.**

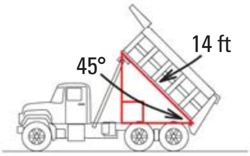
EXAMPLE 6 Find a height

DUMP TRUCK The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. 45° angle **b.** 60° angle

Solution

a. When the body is raised 45° above the frame, the height h is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 14 feet.



$14 = h \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

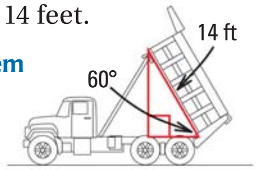
$\frac{14}{\sqrt{2}} = h$ **Divide each side by $\sqrt{2}$.**

$9.9 \approx h$ **Use a calculator to approximate.**

REWRITE MEASURES
To write 9.9 ft in feet and inches, multiply the decimal part by 12.
$12 \cdot 0.9 = 10.8$
So, 9.9 ft is about 9 feet 11 inches.

▶ When the angle of elevation is 45°, the body is about 9 feet 11 inches above the frame.

b. When the body is raised 60°, the height h is the length of the longer leg of a 30°-60°-90° triangle. The length of the hypotenuse is 14 feet.



hypotenuse $= 2 \cdot$ shorter leg **30°-60°-90° Triangle Theorem**

$14 = 2 \cdot s$ **Substitute.**

$7 = s$ **Divide each side by 2.**

longer leg $=$ shorter leg $\cdot \sqrt{3}$ **30°-60°-90° Triangle Theorem**

$h = 7\sqrt{3}$ **Substitute.**

$h \approx 12.1$ **Use a calculator to approximate.**

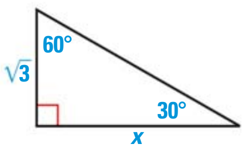
▶ When the angle of elevation is 60°, the body is about 12 feet 1 inch above the frame.

Animated Geometry at classzone.com

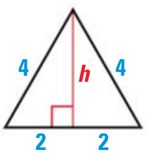
✓ GUIDED PRACTICE for Examples 4, 5, and 6

Find the value of the variable.

5.



6.



7. **WHAT IF?** In Example 6, what is the height of the body of the dump truck if it is raised 30° above the frame?

8. In a 30°-60°-90° triangle, *describe* the location of the shorter side. *Describe* the location of the longer side?

7.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 19, 22, 29, and 34

SKILL PRACTICE

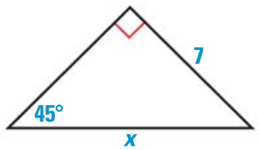
1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called _?_.

2. ★ **WRITING** *Explain* why the acute angles in an isosceles right triangle always measure 45°.

EXAMPLES 1 and 2 on pp. 457–458 for Exs. 3–5

45°-45°-90° TRIANGLES **Find the value of x. Write your answer in simplest radical form.**

3.



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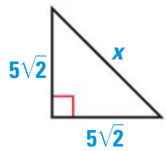

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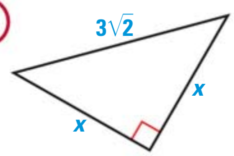
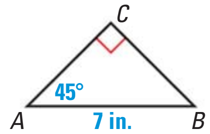


EXAMPLE 3 on p. 458 for Exs. 6–7

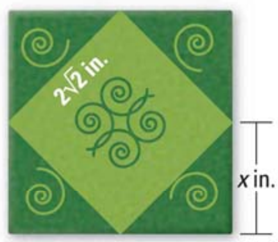
6. ★ **MULTIPLE CHOICE** Find the length of $\overline{AC}$.



Ⓐ $7\sqrt{2}$ in. Ⓑ $2\sqrt{7}$ in.

Ⓒ $\frac{7\sqrt{2}}{2}$ in. Ⓓ $\sqrt{14}$ in.

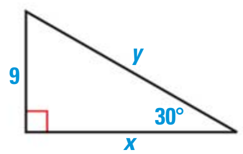
7. **ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of x? What is the side length of the tile?



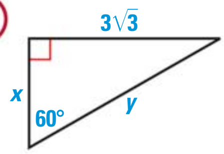
EXAMPLES 4 and 5 on p. 459 for Exs. 8–10

30°-60°-90° TRIANGLES **Find the value of each variable. Write your answers in simplest radical form.**

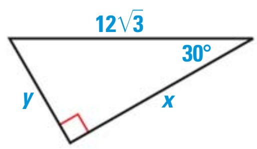
8.



9.

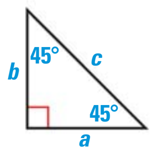


10.



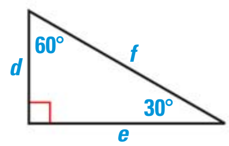
SPECIAL RIGHT TRIANGLES **Copy and complete the table.**

11.



a	7	?	?	?	$\sqrt{5}$
b	?	11	?	?	?
c	?	?	10	$6\sqrt{2}$	?

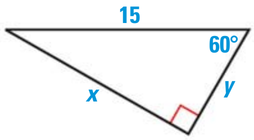
12.



d	5	?	?	?	?
e	?	?	$8\sqrt{3}$	?	12
f	?	14	?	$18\sqrt{3}$	?

ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

13.



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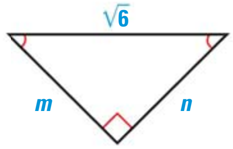

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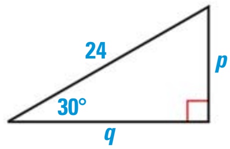

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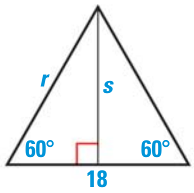
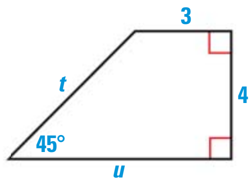
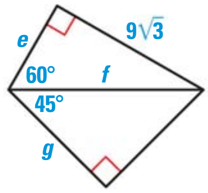


17.



18.



Animated Geometry at classzone.com

19. ★ **MULTIPLE CHOICE** Which side lengths do *not* represent a 30°-60°-90° triangle?

Ⓐ $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$

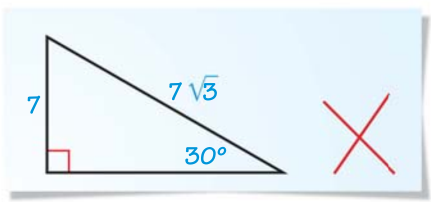
Ⓑ $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$

Ⓒ $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$

Ⓓ $3, 3\sqrt{3}, 6$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the length of the hypotenuse.**

20.



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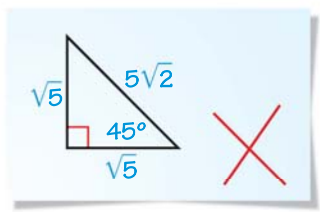
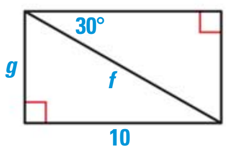


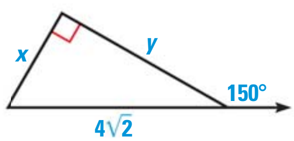
22. ★ **WRITING** Abigail solved Example 5 on page 459 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? *Explain* why or why not.

ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

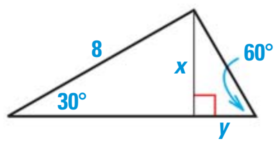
23.



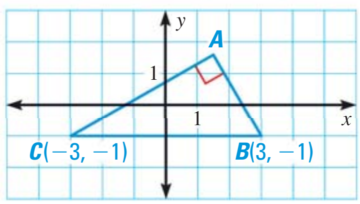
24.



25.



26. **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of A.



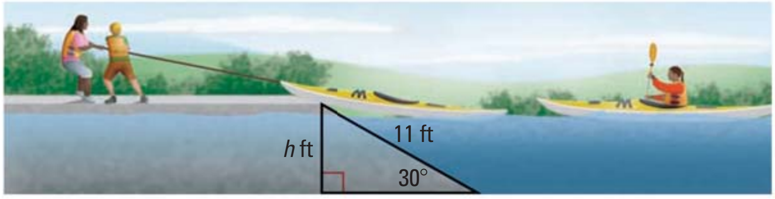
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 6 on p. 460 for Ex. 27

27. **KAYAK RAMP** A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown?



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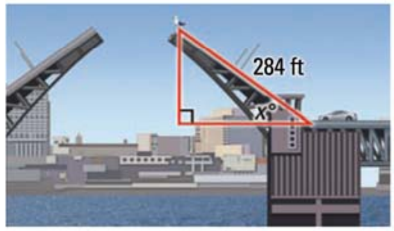
28. **DRAWBRIDGE** Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull rise who is on the end of the drawbridge when the angle with measure $x°$ is 30°? 45°? 60°?



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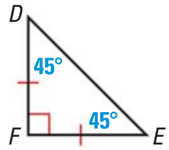
29. ★ **SHORT RESPONSE** *Describe* two ways to show that all isosceles right triangles are similar to each other.

30. **PROVING THEOREM 7.8** Write a paragraph proof of the 45°-45°-90° Triangle Theorem.

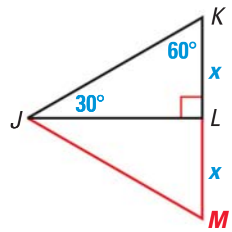


GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

31. **EQUILATERAL TRIANGLE** If an equilateral triangle has a side length of 20 inches, find the height of the triangle.

32. **PROVING THEOREM 7.9** Write a paragraph proof of the 30°-60°-90° Triangle Theorem.

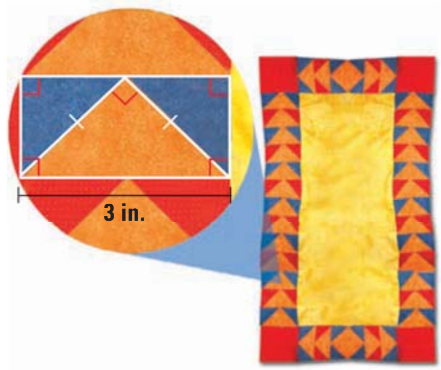


GIVEN ▶ $\triangle JKL$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Plan for Proof Construct $\triangle JML$ congruent to $\triangle JKL$. Then prove that $\triangle JKM$ is equilateral. Express the lengths of $\overline{JK}$ and $\overline{JL}$ in terms of x.

33. **MULTI-STEP PROBLEM** You are creating a quilt that will have a traditional "flying geese" border, as shown below.

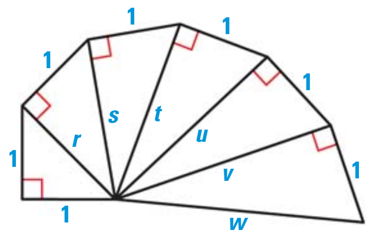


a. Find all the angle measures of the small blue triangles and the large orange triangles.

b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need?

c. What size square do you need to create each small triangle?

34. ★ **EXTENDED RESPONSE** Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.



a. Find the values of r, s, t, u, v, and w. *Explain* the procedure you used to find the values.

b. Which of the triangles, if any, is a 45°-45°-90° triangle? *Explain*.

c. Which of the triangles, if any, is a 30°-60°-90° triangle? *Explain*.

35. **CHALLENGE** In quadrilateral $QRST$, $m\angle R = 60°$, $m\angle T = 90°$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point Z.

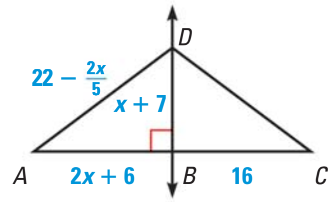
a. Draw a diagram.

b. *Explain* why $\triangle RQT \cong \triangle RST$.

c. Which is longer, QS or RT? *Explain*.

MIXED REVIEW

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. *(p. 303)*



36. Which pairs of segment lengths are equal?

37. What is the value of x?

38. Find CD.

Is it possible to build a triangle using the given side lengths? *(p. 328)*

39. 4, 4, and 7 **40.** 3, 3, and $9\sqrt{2}$ **41.** 7, 15, and 21

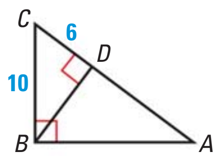
PREVIEW
Prepare for Lesson 7.5 in Exs. 42–44.

Tell whether the given side lengths form a right triangle. *(p. 441)*

42. 21, 22, and $5\sqrt{37}$ **43.** $\frac{3}{2}$, 2, and $\frac{5}{2}$ **44.** 8, 10, and 14

QUIZ *for Lessons 7.3–7.4*

In Exercises 1 and 2, use the diagram. *(p. 449)*

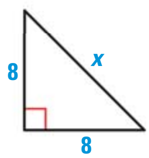


1. Which segment's length is the geometric mean of AC and CD?

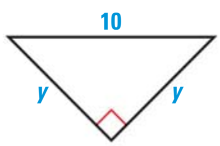
2. Find BD, AD, and AB.

Find the values of the variable(s). Write your answer(s) in simplest radical form. *(p. 457)*

3.



4.



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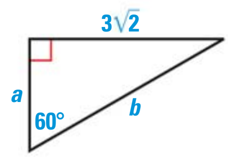

STATE TEST PRACTICE
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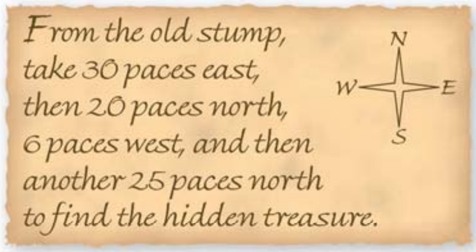
Lessons 7.1–7.4

1. **GRIDDED ANSWER** Find the direct distance, in paces, from the treasure to the stump.



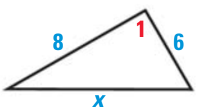
2. **MULTI-STEP PROBLEM** On a map of the United States, you put a pushpin on three state capitols you want to visit: Jefferson City, Missouri; Little Rock, Arkansas; and Atlanta, Georgia.

   a. Draw a diagram to model the triangle.

   b. Do the pushpins form a right triangle? If not, what type of triangle do they form?

3. **SHORT RESPONSE** Bob and John started running at 10 A.M. Bob ran east at 4 miles per hour while John ran south at 5 miles per hour. How far apart were they at 11:30 A.M.? *Describe* how you calculated the answer.

4. **EXTENDED RESPONSE** Give all values of x that make the statement true for the given diagram.



   a. $\angle 1$ is a right angle. *Explain*.

   b. $\angle 1$ is an obtuse angle. *Explain*.

   c. $\angle 1$ is an acute angle. *Explain*.

   d. The triangle is isosceles. *Explain*.

   e. No triangle is possible. *Explain*.

5. **EXTENDED RESPONSE** A Chinese checker board is made of triangles. Use the picture below to answer the questions.

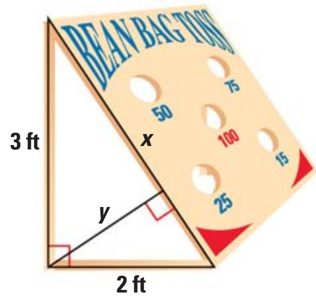
   a. Count the marble holes in the purple triangle. What kind of triangle is it?

   b. If a side of the purple triangle measures 8 centimeters, find the area of the purple triangle.

   c. How many marble holes are in the center hexagon? Assuming each marble hole takes up the same amount of space, what is the relationship between the purple triangle and center hexagon?

   d. Find the area of the center hexagon. *Explain* your reasoning.

6. **MULTI-STEP PROBLEM** You build a beanbag toss game. The game is constructed from a sheet of plywood supported by two boards. The two boards form a right angle and their lengths are 3 feet and 2 feet.



   a. Find the length x of the plywood.

   b. You put in a support that is the altitude y to the hypotenuse of the right triangle. What is the length of the support?

   c. Where does the support attach to the plywood? *Explain*.

7.5 Apply the Tangent Ratio

Before You used congruent or similar triangles for indirect measurement.

Now You will use the tangent ratio for indirect measurement.

Why? So you can find the height of a roller coaster, as in Ex. 32.

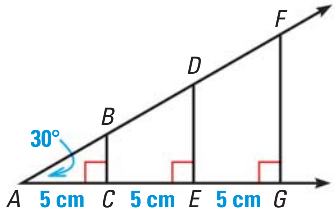
Key Vocabulary
- **trigonometric ratio**
- **tangent**

ACTIVITY RIGHT TRIANGLE RATIO

Materials: metric ruler, protractor, calculator

STEP 1 **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

STEP 2 **Measure** the legs of each right triangle. Copy and complete the table.

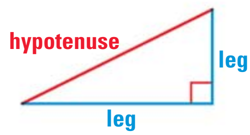


Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	?	?
$\triangle ADE$	10 cm	?	?
$\triangle AFG$	15 cm	?	?

STEP 3 **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

STEP 4 **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.



The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

ABBREVIATE

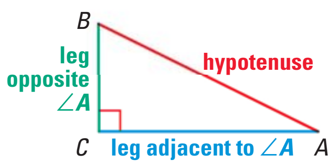
Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

KEY CONCEPT *For Your Notebook*

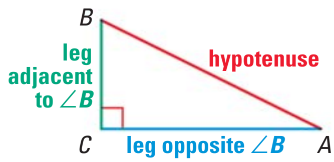
Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

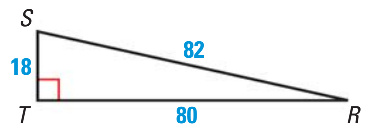


COMPLEMENTARY ANGLES In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.



EXAMPLE 1 Find tangent ratios

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places.



APPROXIMATE
Unless told otherwise, you should round the values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

Solution

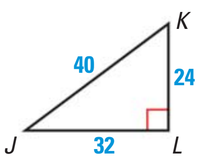
$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

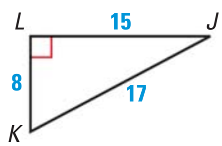
✓ GUIDED PRACTICE for Example 1

Find tan J and tan K. Round to four decimal places.

1.

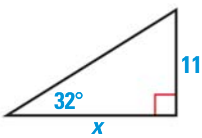


2.



EXAMPLE 2 Find a leg length

ALGEBRA **Find the value of x.**



Solution

Use the tangent of an acute angle to find a leg length.

$\tan 32° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32° = \frac{11}{x}$	Substitute.
$x \cdot \tan 32° = 11$	Multiply each side by x.
$x = \frac{11}{\tan 32°}$	Divide each side by tan 32°.
$x \approx \frac{11}{0.6249}$	Use a calculator to find tan 32°.
$x \approx 17.6$	Simplify.

ANOTHER WAY
You can also use the Table of Trigonometric Ratios on p. 925 to find the decimal values of trigonometric ratios.

EXAMPLE 3 Estimate height using tangent

LAMPPOST **Find the height h of the lamppost to the nearest inch.**

$\tan 70° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 70°.

$\tan 70° = \frac{h}{40}$ Substitute.

$40 \cdot \tan 70° = h$ Multiply each side by 40.

$109.9 \approx h$ Use a calculator to simplify.

▶ The lamppost is about 110 inches tall.

SPECIAL RIGHT TRIANGLES You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a 60° angle.

SIMILAR TRIANGLES

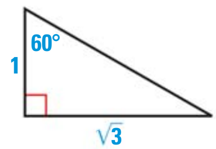
The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

STEP 1 **Because** all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

longer leg = shorter leg $\cdot \sqrt{3}$ 30°-60°-90° Triangle Theorem

$x = 1 \cdot \sqrt{3}$ Substitute.

$x = \sqrt{3}$ Simplify.



STEP 2 **Find** tan 60°.

$\tan 60° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 60°.

$\tan 60° = \frac{\sqrt{3}}{1}$ Substitute.

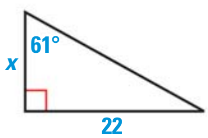
$\tan 60° = \sqrt{3}$ Simplify.

▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

Find the value of x. Round to the nearest tenth.

3.



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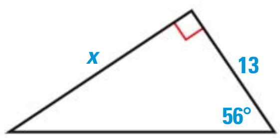


5. **WHAT IF?** In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

7.5 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 17, 35, and 37

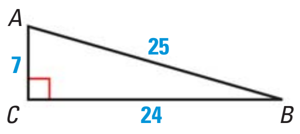
SKILL PRACTICE

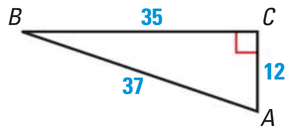
1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of __?__ to the length of __?__.

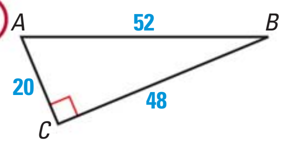
2. ★ **WRITING** *Explain* how you know that all right triangles with an acute angle measuring $n°$ are similar to each other.

EXAMPLE 1 on p. 467 for Exs. 3–5

FINDING TANGENT RATIOS **Find tan *A* and tan *B*. Write each answer as a fraction and as a decimal rounded to four places.**

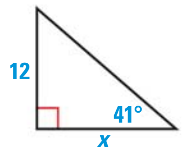
3. 

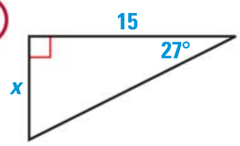
4. 

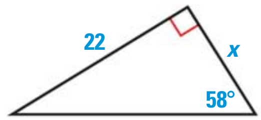
5. 

EXAMPLE 2 on p. 467 for Exs. 6–8

FINDING LEG LENGTHS **Find the value of *x* to the nearest tenth.**

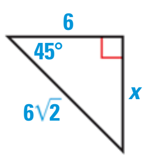
6. 

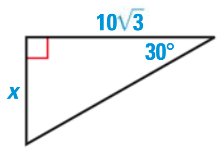
7. 

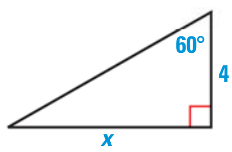
8. 

EXAMPLE 4 on p. 468 for Exs. 9–12

FINDING LEG LENGTHS **Find the value of *x* using the definition of tangent. Then find the value of *x* using the 45°-45°-90° Theorem or the 30°-60°-90° Theorem. *Compare* the results.**

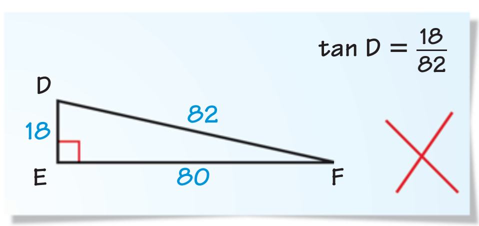
9. 

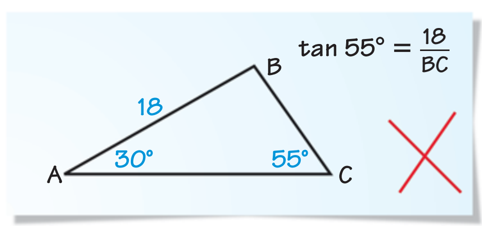
10. 

11. 

12. **SPECIAL RIGHT TRIANGLES** Find tan 30° and tan 45° using the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem.

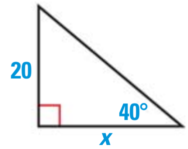
ERROR ANALYSIS ***Describe*** **the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write *not possible*.**

13. 

14. 

15. ★ **WRITING** *Describe* what you must know about a triangle in order to use the tangent ratio.

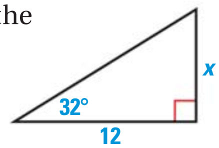
16. ★ **MULTIPLE CHOICE** Which expression can be used to find the value of x in the triangle shown?



Ⓐ $x = 20 \cdot \tan 40°$ Ⓑ $x = \dfrac{\tan 40°}{20}$

Ⓒ $x = \dfrac{20}{\tan 40°}$ Ⓓ $x = \dfrac{20}{\tan 50°}$

17. ★ **MULTIPLE CHOICE** What is the approximate value of x in the triangle shown?

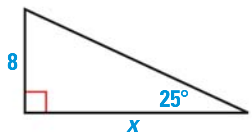


Ⓐ 0.4 Ⓑ 2.7

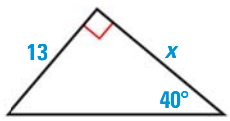
Ⓒ 7.5 Ⓓ 19.2

FINDING LEG LENGTHS **Use a tangent ratio to find the value of x. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

18.



19.



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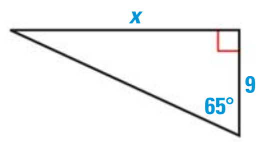
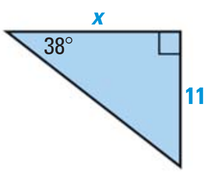
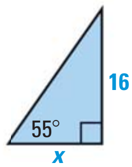


FINDING AREA **Find the area of the triangle. Round to the nearest tenth.**

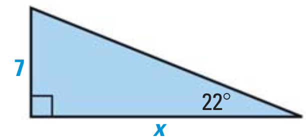
21.



22.

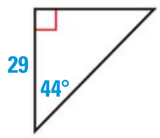


23.



FINDING PERIMETER **Find the perimeter of the triangle. Round to the nearest tenth.**

24.



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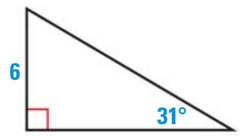

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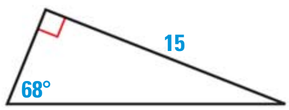
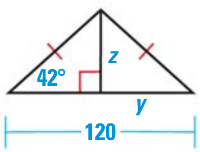
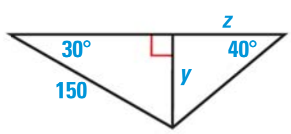


FINDING LENGTHS **Find y. Then find z. Round to the nearest tenth.**

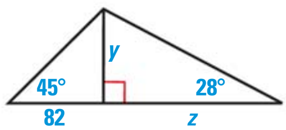
27.



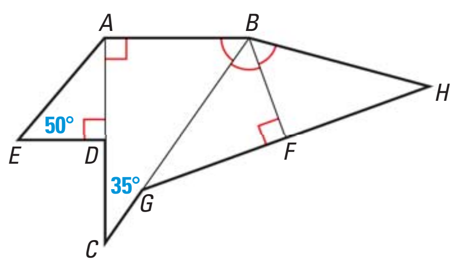
28.



29.



30. **CHALLENGE** Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.



◯ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 3
on p. 468
for Exs. 31–32

31. WASHINGTON MONUMENT A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot.

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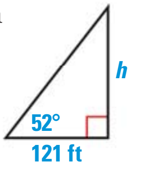
32. ROLLER COASTERS A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height h of the roller coaster to the nearest foot.



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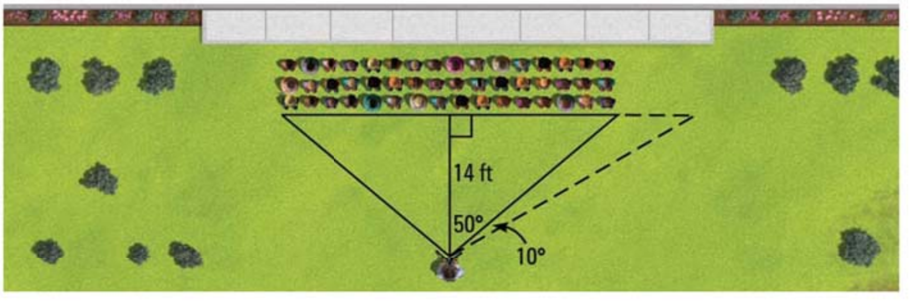
CLASS PICTURE Use this information and diagram for Exercises 33 and 34.

Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.



33. ISOSCELES TRIANGLE What is the distance between the ends of the class?

34. MULTI-STEP PROBLEM The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.

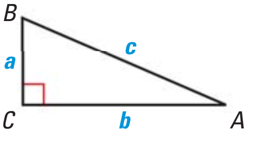
a. Find the distance from the center to the last student in the row.

b. Find the distance from the center to the end of the camera range.

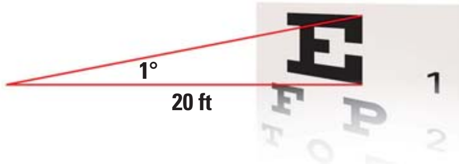
c. Use the results of parts (a) and (b) to estimate the length of the empty space.

d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning.

35. ★ **SHORT RESPONSE** Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are $\angle A$ and $\angle B$?

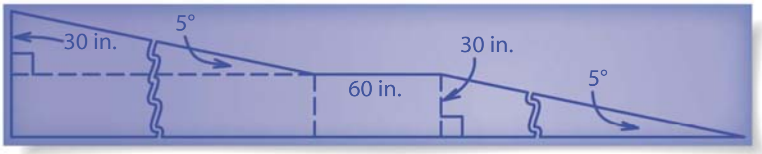


36. EYE CHART You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the "E" on the chart. To see the top of the "E," you look up 1°. How tall is the "E"?



Not drawn to scale

37. ★ EXTENDED RESPONSE According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

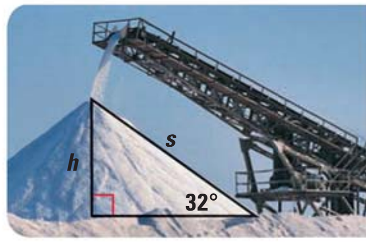


a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot.

b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to *justify* your answer.

c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)?

38. CHALLENGE The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height h of the cone. Then find the length s of the cone-shaped pile.



MIXED REVIEW

The expressions given represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

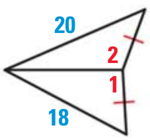
39. $m\angle A = x°$
$m\angle B = 4x°$
$m\angle C = 4x°$

40. $m\angle A = x°$
$m\angle B = x°$
$m\angle C = (5x - 60)°$

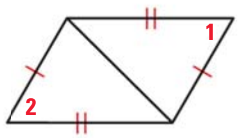
41. $m\angle A = (x + 20)°$
$m\angle B = (3x + 15)°$
$m\angle C = (x - 30)°$

Copy and complete the statement with <, >, or =. *Explain.* *(p. 335)*

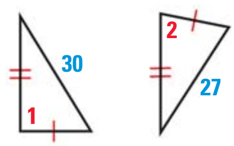
42. $m\angle 1$ __?__ $m\angle 2$



43. $m\angle 1$ __?__ $m\angle 2$



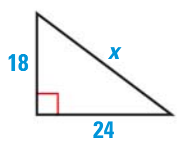
44. $m\angle 1$ __?__ $m\angle 2$



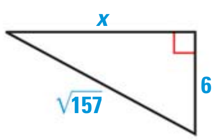
PREVIEW
Prepare for Lesson 7.6 in Exs. 45–47.

Find the unknown side length of the right triangle. *(p. 433)*

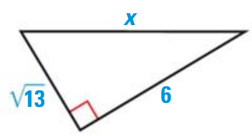
45.



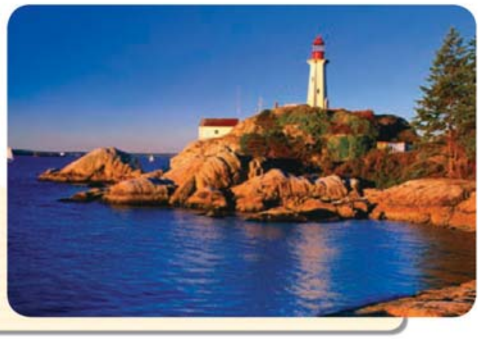
46.



47.



7.6 Apply the Sine and Cosine Ratios



Before You used the tangent ratio.
Now You will use the sine and cosine ratios.
Why So you can find distances, as in Ex. 39.

Key Vocabulary
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

ABBREVIATE
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

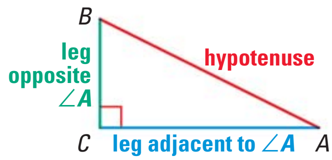
KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

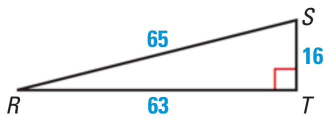
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



EXAMPLE 1 Find sine ratios

Find sin S and sin R. Write each answer as a fraction and as a decimal rounded to four places.



Solution

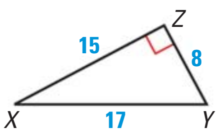
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

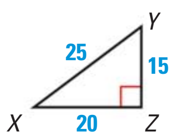
✓ GUIDED PRACTICE for Example 1

Find sin X and sin Y. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1.

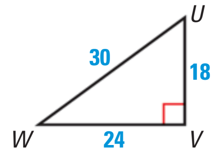


2.



EXAMPLE 2 Find cosine ratios

Find cos U and cos W. Write each answer as a fraction and as a decimal.



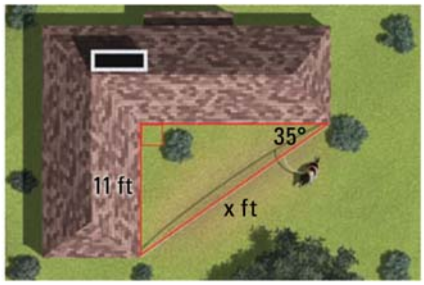
Solution

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

DOG RUN You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.



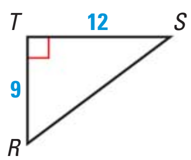
Solution

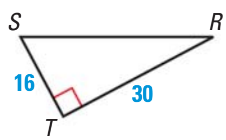
$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$	Write ratio for sine of 35°.
$\sin 35° = \frac{11}{x}$	Substitute.
$x \cdot \sin 35° = 11$	Multiply each side by x.
$x = \frac{11}{\sin 35°}$	Divide each side by sin 35°.
$x \approx \frac{11}{0.5736}$	Use a calculator to find sin 35°.
$x \approx 19.2$	Simplify.

▶ You will need a little more than 19 feet of cable.

✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, find cos R and cos S. Write each answer as a decimal. Round to four decimal places, if necessary.

3. 

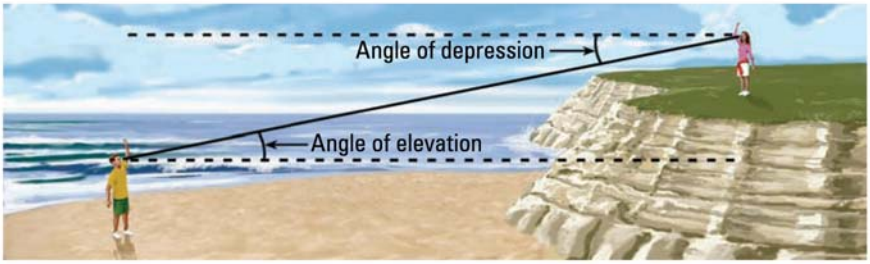
4. 

5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed.

ANGLES If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

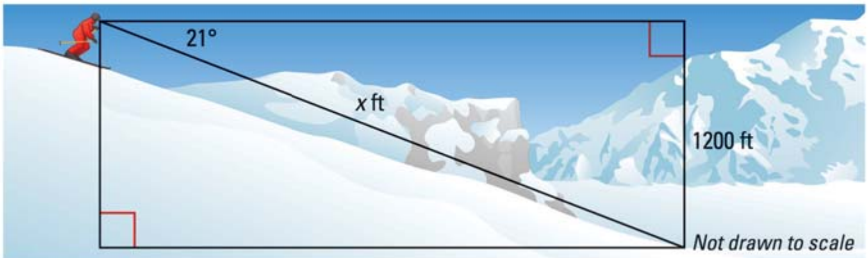
APPLY THEOREMS

Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem on page 155.



EXAMPLE 4 Find a hypotenuse using an angle of depression

SKIING You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



Solution

$\sin 21° = \dfrac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 21°.**

$\sin 21° = \dfrac{1200}{x}$ **Substitute.**

$x \cdot \sin 21° = 1200$ **Multiply each side by x.**

$x = \dfrac{1200}{\sin 21°}$ **Divide each side by sin 21°.**

$x \approx \dfrac{1200}{0.3584}$ **Use a calculator to find sin 21°.**

$x \approx 3348.2$ **Simplify.**

▶ You ski about 3348 meters down the mountain.

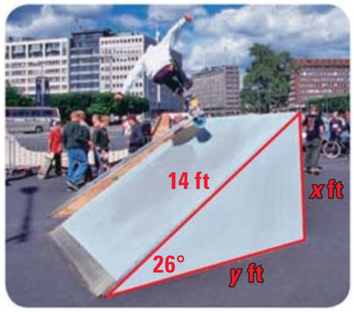
AnimatedGeometry at classzone.com

✓ **GUIDED PRACTICE** for Example 4

6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski?

EXAMPLE 5 Find leg lengths using an angle of elevation

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.



ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 481 for the **Problem Solving Workshop**.

Solution

STEP 1 **Find** the height.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 26°.

$\sin 26° = \frac{x}{14}$ Substitute.

$14 \cdot \sin 26° = x$ Multiply each side by 14.

$6.1 \approx x$ Use a calculator to simplify.

▶ The height is about 6.1 feet.

STEP 2 **Find** the length of the base.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ Write ratio for cosine of 26°.

$\cos 26° = \frac{y}{14}$ Substitute.

$14 \cdot \cos 26° = y$ Multiply each side by 14.

$12.6 \approx y$ Use a calculator to simplify.

▶ The length of the base is about 12.6 feet.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a 60° angle.

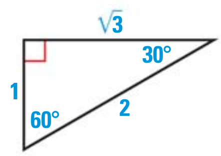
DRAW DIAGRAMS
As in Example 4 on page 468, to simplify calculations you can choose 1 as the length of the shorter leg.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$



✓ GUIDED PRACTICE for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp?

8. Use a special right triangle to find the sine and cosine of a 30° angle.

7.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

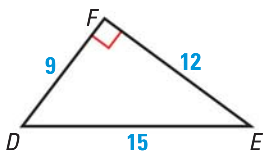
SKILL PRACTICE

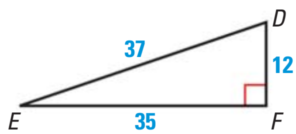
1. **VOCABULARY** Copy and complete: The sine ratio compares the length of _?_ to the length of _?_.

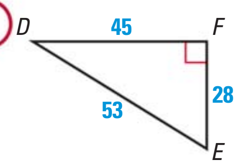
2. ★ **WRITING** *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

EXAMPLE 1 on p. 473 for Exs. 3–6

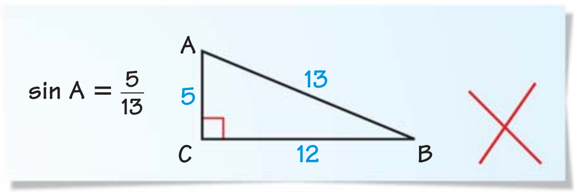
FINDING SINE RATIOS **Find sin *D* and sin *E*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

3. 

4. 

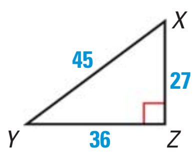
5. 

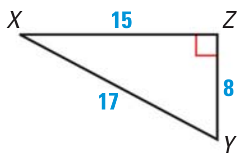
6. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle.

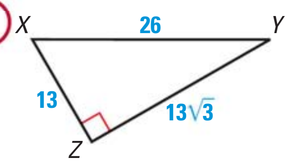


EXAMPLE 2 on p. 474 for Exs. 7–9

FINDING COSINE RATIOS **Find cos *X* and cos *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

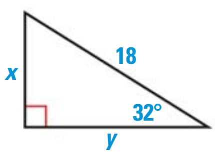
7. 

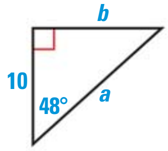
8. 

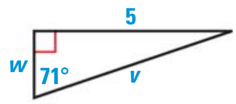
9. 

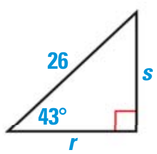
EXAMPLE 3 on p. 474 for Exs. 10–15

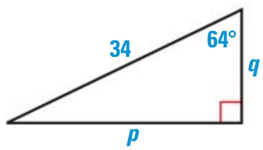
USING SINE AND COSINE RATIOS **Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.**

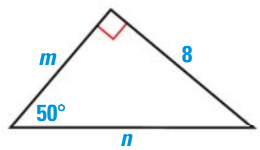
10. 

11. 

12. 

13. 

14. 

15. 

EXAMPLE 6 on p. 476 for Ex. 16

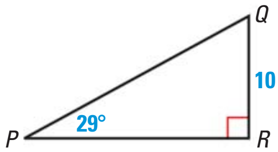
16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ?

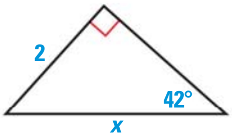
Ⓐ $10 \cdot \cos 29°$ Ⓑ $10 \cdot \sin 29°$

Ⓒ $\dfrac{10}{\sin 29°}$ Ⓓ $\dfrac{10}{\cos 29°}$



ALGEBRA **Find the value of x. Round decimals to the nearest tenth.**

19.



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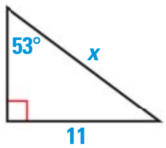

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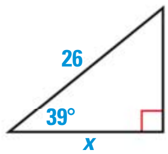

FINDING SINE AND COSINE RATIOS **Find the unknown side length. Then find sin X and cos X. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.**

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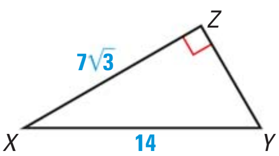

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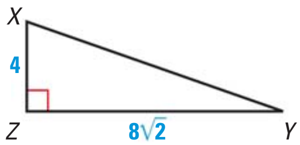

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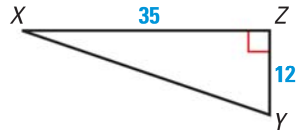

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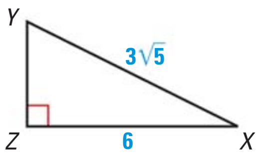

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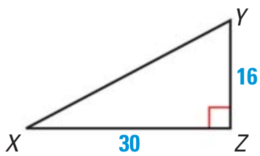

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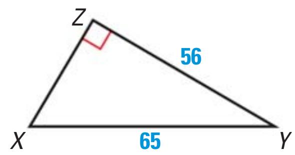


28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

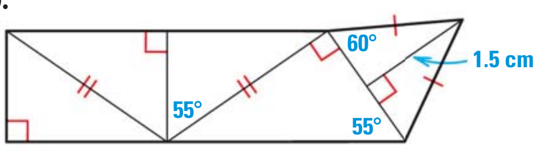
29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true?

Ⓐ $\sin J = 0.5$ Ⓑ $\sin J = 0.1071$

Ⓒ $\sin J = 0.8660$ Ⓓ $\sin J = 1.1$

PERIMETER **Find the approximate perimeter of the figure.**

30.



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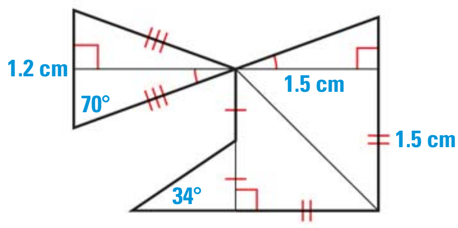

32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \dfrac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 475–476 for Exs. 33–36

33. **AIRPLANE RAMP** The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp?

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34. **BLEACHERS** Find the horizontal distance h the bleachers cover. Round to the nearest foot.

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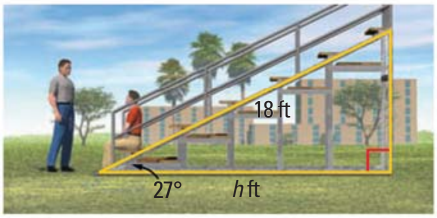
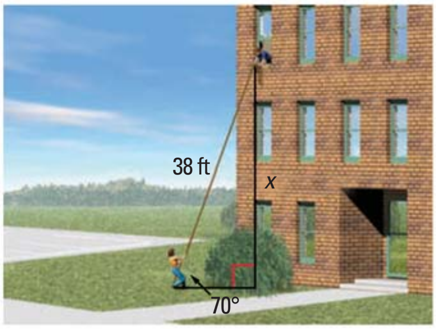


35. ★ **SHORT RESPONSE** You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.

   a. Draw and label a diagram to represent the situation.

   b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite.

36. **MULTI-STEP PROBLEM** You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.

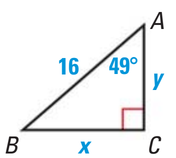


   a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?

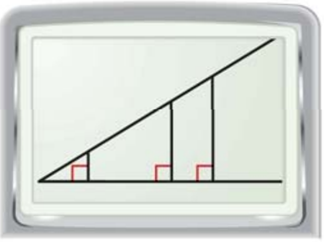
   b. The bush is 6 feet tall. Will your banner fit above the bush?

   c. **What If?** Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?

37. ★ **SHORT RESPONSE** Nick uses the equation $\sin 49° = \frac{x}{16}$ to find BC in $\triangle ABC$. Tim uses the equation $\cos 41° = \frac{x}{16}$. Which equation produces the correct answer? *Explain*.



38. **TECHNOLOGY** Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?



39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.

a. **Drawing a Diagram** Draw and label a diagram of the situation.

b. **Making a Table** Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. **Drawing a Graph** Graph the values you found in part (b), with the angle measures on the x-axis.

d. **Making a Prediction** Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.

b. Compare the sine and cosine values. What pattern(s) do you notice?

c. Make a conjecture about the sine and cosine values in part (b).

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain.*

MIXED REVIEW

Rewrite the equation so that x is a function of y. *(p. 877)*

42. $y = \sqrt{x}$

43. $y = 3x - 10$

44. $y = \frac{x}{9}$

PREVIEW
Prepare for Lesson 7.7 in Exs. 45–47.

Copy and complete the table. *(p. 884)*

45.

x	$\sqrt{x}$
?	0
?	1
?	$\sqrt{2}$
?	2
?	4

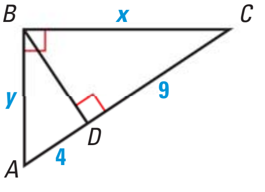
46.

x	$\frac{1}{x}$
?	1
?	$\frac{1}{2}$
?	3
?	$\frac{2}{7}$
?	7

47.

x	$\frac{2}{7}x + 4$
?	0
?	2
?	6
?	8
?	10

48. Find the values of x and y in the triangle at the right. *(p. 449)*



PROBLEM SOLVING WORKSHOP
LESSON 7.6

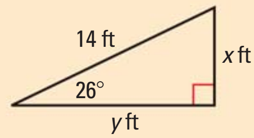
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 476

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$26° + 90° + m\angle 3 = 180°$	**Triangle Sum Theorem**
$116° + m\angle 3 = 180°$	**Combine like terms.**
$m\angle 3 = 64°$	**Subtact 116° from each side.**

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$	**Write ratio for cosine of 64°.**
$\cos 64° = \frac{x}{14}$	**Substitute.**
$14 \cdot \cos 64° = x$	**Multiply each side by 14.**
$6.1 \approx x$	**Use a calculator to simplify.**

▶ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$14^2 = 6.1^2 + y^2$	**Substitute.**
$196 = 37.21 + y^2$	**Multiply.**
$158.79 = y^2$	**Subtract 37.21 from each side.**
$12.6 \approx y$	**Find the positive square root.**

▶ The length of the base is about 12.6 feet.

METHOD 2 **Using a Tangent Ratio**

Use the tangent ratio and $h = 6.1$ feet to find the length of the base of the ramp.

$$\tan 26° = \frac{\text{opp.}}{\text{adj.}}$$ **Write ratio for tangent of 26°.**

$$\tan 26° = \frac{6.1}{y}$$ **Substitute.**

$$y \cdot \tan 26° = 61$$ **Multiply each side by *y*.**

$$y = \frac{6.1}{\tan 26°}$$ **Divide each side by tan 26°.**

$$y \approx 12.5$$ **Use a calculator to simplify.**

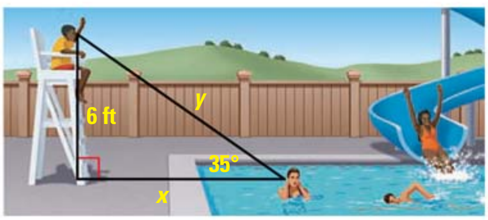
▶ The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

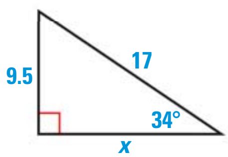
PRACTICE

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp.

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is 35°. Find the distance x from the swimmer to the base of the lifeguard chair. Find the distance y from the swimmer to the lifeguard.

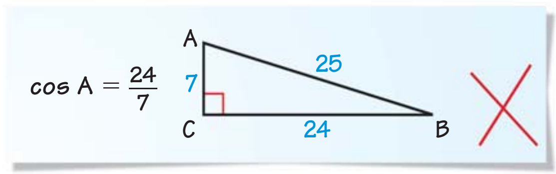


3. **ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length.



4. **SHORT RESPONSE** *Describe* how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle.

5. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the cosine of the angle.



6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree's shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be 75°.
   a. Find the height of the tree. *Explain* the method you chose to solve the problem.
   b. What else would you need to know to solve this problem using similar triangles.
   c. *Explain* why you cannot use the sine ratio to find the height of the tree.

7.7 Solve Right Triangles

Before You used tangent, sine, and cosine ratios.

Now You will use inverse tangent, sine, and cosine ratios.

Why? So you can build a saddlerack, as in Ex. 39.

Key Vocabulary
- **solve a right triangle**
- **inverse tangent**
- **inverse sine**
- **inverse cosine**

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

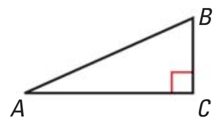
- Two side lengths
- One side length and the measure of one acute angle

In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

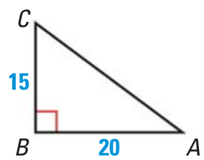
Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$

READ VOCABULARY
The expression "$\tan^{-1}x$" is read as "the inverse tangent of x."

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765 \cdots$

▶ So, the measure of $\angle A$ is approximately $36.9°$.

EXAMPLE 2 Use an inverse sine and an inverse cosine

ANOTHER WAY
You can use the Table of Trigonometric Ratios on p. 925 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and read the angle measure at the left.

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.87$

b. $\cos B = 0.15$

Solution

a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$

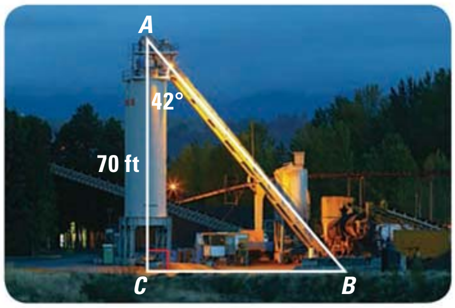
b. $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

✓ GUIDED PRACTICE for Examples 1 and 2

1. Look back at Example 1. Use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.
2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.54$.

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

STEP 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

$$180° = 90° + 42° + m\angle B$$
$$48° = m\angle B$$

STEP 2 **Approximate** BC by using a tangent ratio.

$\tan 42° = \frac{BC}{70}$ — Write ratio for tangent of 42°.

$70 \cdot \tan 42° = BC$ — Multiply each side by 70.

$70 \cdot 0.9004 \approx BC$ — Approximate tan 42°.

$63 \approx BC$ — Simplify and round answer.

ANOTHER WAY
You could also find AB by using the Pythagorean Theorem, or a sine ratio.

STEP 3 **Approximate** AB using a cosine ratio.

$\cos 42° = \frac{70}{AB}$ — Write ratio for cosine of 42°.

$AB \cdot \cos 42° = 70$ — Multiply each side by AB.

$AB = \frac{70}{\cos 42°}$ — Divide each side by cos 42°.

$AB \approx \frac{70}{0.7431}$ — Use a calculator to find cos 42°.

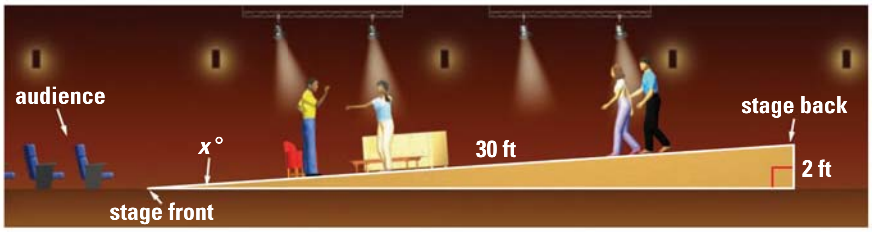
$AB \approx 94.2$ — Simplify .

▶ The angle measures are 42°, 48°, and 90°. The side lengths are 70 feet, about 63 feet, and about 94 feet.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

Use the sine and inverse sine ratios to find the degree measure x of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain*.

7.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

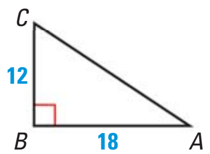
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__.

2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

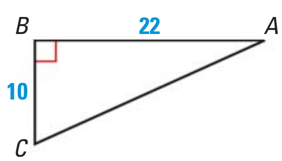
EXAMPLE 1
on p. 483
for Exs. 3–5

USING INVERSE TANGENTS **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

3.



4.



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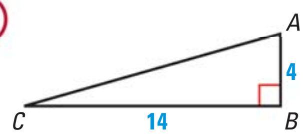
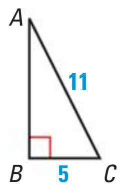


EXAMPLE 2
on p. 484
for Exs. 6–9

USING INVERSE SINES AND COSINES **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

6.



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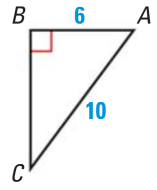

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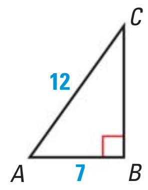
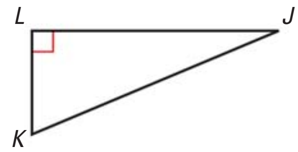


9. ★ **MULTIPLE CHOICE** Which expression is correct?



Ⓐ $\sin^{-1}\frac{JL}{JK} = m\angle J$

Ⓑ $\tan^{-1}\frac{KL}{JL} = m\angle J$

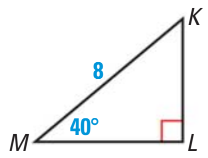
Ⓒ $\cos^{-1}\frac{JL}{JK} = m\angle K$

Ⓓ $\sin^{-1}\frac{JL}{KL} = m\angle K$

EXAMPLE 3
on p. 484
for Exs. 10–18

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimal answers to the nearest tenth.**

10.



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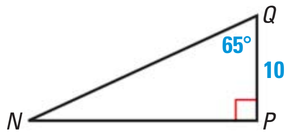

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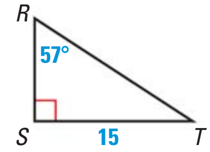

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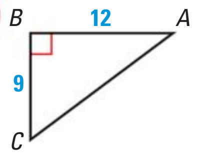

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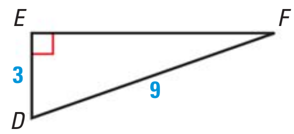

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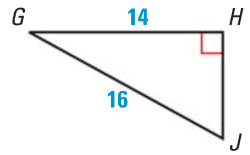

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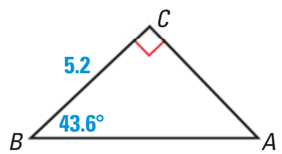
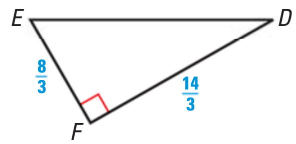
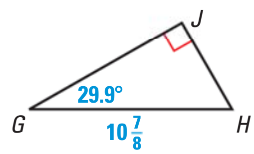


17.

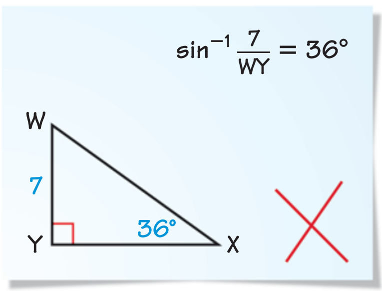


18.

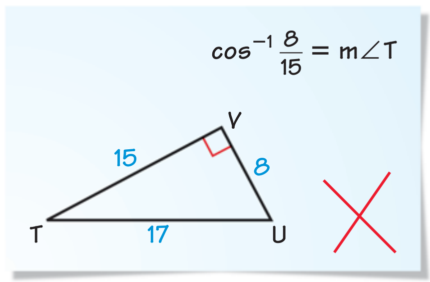


ERROR ANALYSIS ***Describe*** **and correct the student's error in using an inverse trigonometric ratio.**

19.



20.



CALCULATOR **Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.**

21. $\sin A = 0.5$

22. $\sin A = 0.75$

23. $\cos A = 0.33$

24. $\cos A = 0.64$

25. $\tan A = 1.0$

26. $\tan A = 0.28$

27. $\sin A = 0.19$

28. $\cos A = 0.81$

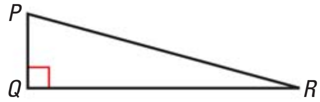
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

29. ★ **MULTIPLE CHOICE** Which additional information would *not* be enough to solve $\triangle PRQ$?

Ⓐ $m\angle P$ and PR Ⓑ $m\angle P$ and $m\angle R$

Ⓒ PQ and PR Ⓓ $m\angle P$ and PQ



30. ★ **WRITING** *Explain* why it is incorrect to say that $\tan^{-1} x = \frac{1}{\tan x}$.

31. **SPECIAL RIGHT TRIANGLES** If $\sin A = \frac{1}{2}\sqrt{2}$, what is $m\angle A$? If $\sin B = \frac{1}{2}\sqrt{3}$, what is $m\angle B$?

32. **TRIGONOMETRIC VALUES** Use the *Table of Trigonometric Ratios* on page 925 to answer the questions.

   a. What angles have nearly the same sine and tangent values?

   b. What angle has the greatest difference in its sine and tangent value?

   c. What angle has a tangent value that is double its sine value?

   d. Is $\sin 2x$ equal to $2 \cdot \sin x$?

33. **CHALLENGE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles.

PROBLEM SOLVING

EXAMPLE 4
on p. 485
for Exs. 34–36

34. **SOCCER** A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?

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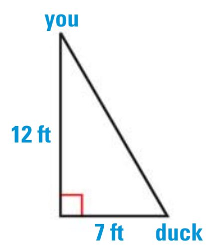
35. ★ **SHORT RESPONSE** You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? *Explain* your reasoning.



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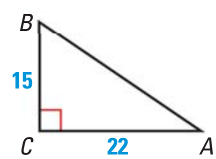
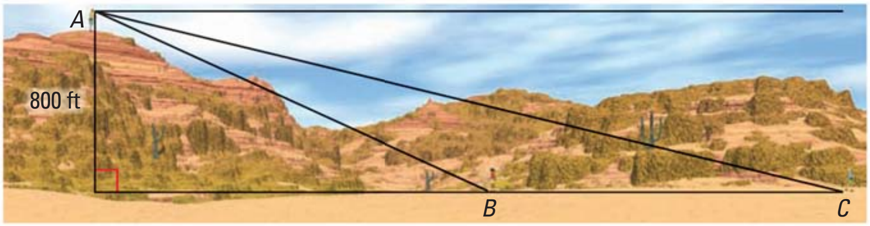
36. **CLAY** In order to unload clay easily, the body of a dump truck must be elevated to at least 55°. If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily?

37. **REASONING** For $\triangle ABC$ shown, each of the expressions $\sin^{-1}\frac{BC}{AB}$, $\cos^{-1}\frac{AC}{AB}$, and $\tan^{-1}\frac{BC}{AC}$ can be used to approximate the measure of $\angle A$. Which expression would you choose? *Explain* your choice.



38. **MULTI-STEP PROBLEM** You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

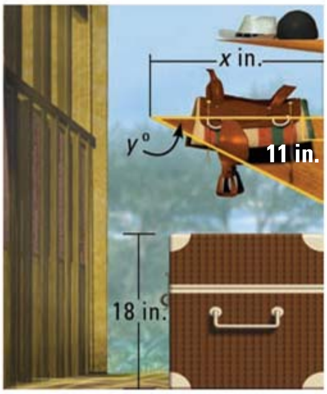


a. If the angle of depression from your line of sight to the hiker at B is 25°, how far is the hiker from the base of the plateau?

b. If the angle of depression from your line of sight to the hiker at C is 15°, how far is the hiker from the base of the plateau?

c. How far apart are the two hikers? *Explain.*

39. **MULTIPLE REPRESENTATIONS** A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.



a. **Making a Table** The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length x and the measure of the adjacent angle $y°$.

b. **Drawing a Graph** Use your table to draw a scatterplot.

c. **Making a Conjecture** Make a conjecture about the relationship between the length of the rack and the angle needed.

40. ★ **OPEN-ENDED MATH** *Describe* a real-world problem you could solve using a trigonometric ratio.

41. ★ **EXTENDED RESPONSE** Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire's connection with the ground should be between 50% to 75% of the height of the guy wire's connection with the tower.

a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire's connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire?

b. How long will a guy wire need to be that is attached 60 feet above the ground?

c. How long will a guy wire need to be that is attached 30 feet above the ground?

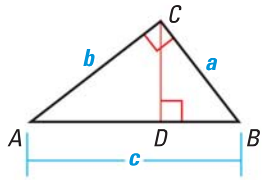
d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent*, *similar*, or *neither*? *Explain*.

e. *Explain* which trigonometric ratios you used to solve the problem.

42. CHALLENGE Use the diagram of $\triangle ABC$.

GIVEN ▶ $\triangle ABC$ with altitude $\overline{CD}$.

PROVE ▶ $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$



MIXED REVIEW

PREVIEW
Prepare for Lesson 8.1 in Ex. 43.

43. Copy and complete the table. *(p. 42)*

Number of sides	Type of polygon
5	?
12	?
?	Octagon
?	Triangle
7	?

Number of sides	Type of polygon
?	n-gon
?	Quadrilateral
10	?
9	?
?	Hexagon

A point on an image and the transformation are given. Find the corresponding point on the original figure. *(p. 272)*

44. Point on image: (5, 1); translation: $(x, y) \to (x + 3, y - 2)$

45. Point on image: (4, −6); reflection: $(x, y) \to (x, -y)$

46. Point on image: (−2, 3); translation: $(x, y) \to (x - 5, y + 7)$

Draw a dilation of the polygon with the given vertices using the given scale factor *k*. *(p. 409)*

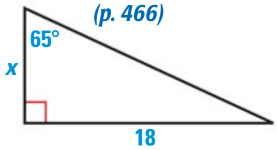
47. $A(2, 2), B(-1, -3), C(5, -3); k = 2$

48. $A(-4, -2), B(-2, 4), C(3, 6), D(6, 3); k = \frac{1}{2}$

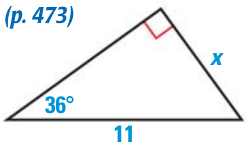
QUIZ *for Lessons 7.5–7.7*

Find the value of *x* to the nearest tenth.

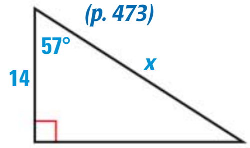
1. *(p. 466)*



2. *(p. 473)*

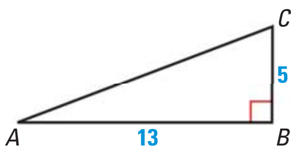


3. *(p. 473)*

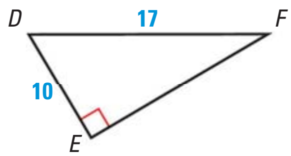


Solve the right triangle. Round decimal answers to the nearest tenth. *(p. 483)*

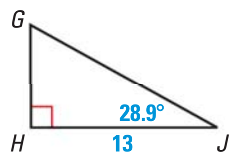
4.



5.



6.



Extension
Use after Lesson 7.7

Law of Sines and Law of Cosines

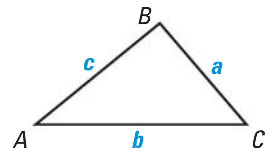
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT *For Your Notebook*

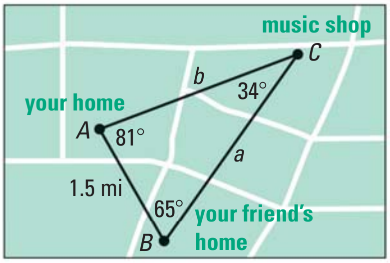
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE Use the information in the diagram to determine how much closer you live to the music store than your friend does.



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ **Write Law of Sines.**

$$\frac{\sin 81^\circ}{a} = \frac{\sin 34^\circ}{1.5}$$ **Substitute.**

$$a \approx 2.6$$ **Solve for *a*.**

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$ **Write Law of Sines.**

$$\frac{\sin 65^\circ}{b} = \frac{\sin 34^\circ}{1.5}$$ **Substitute.**

$$b \approx 2.4$$ **Solve for *b*.**

STEP 3 **Subtract** the distances.

$$a - b \approx 2.6 - 2.4 = 0.2$$

▶ You live about 0.2 miles closer to the music store.

LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT *For Your Notebook*

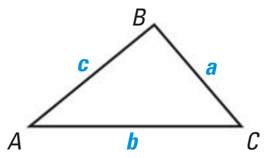
Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

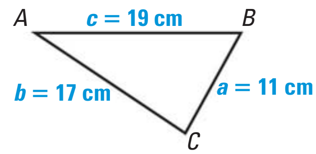
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



Solution

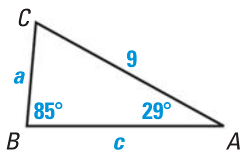
$c^2 = a^2 + b^2 - 2ab \cos C$	**Write Law of Cosines.**
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	**Substitute.**
$0.1310 = \cos C$	**Solve for cos C.**
$m\angle C \approx 82°$	**Find $\cos^{-1}$ (0.1310).**

PRACTICE

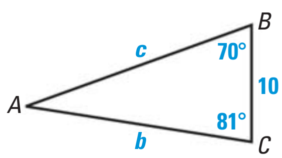
EXAMPLE 1 for Exs. 1–3

LAW OF SINES Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.

1.



2.



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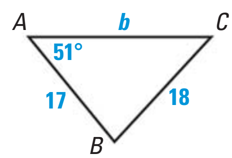
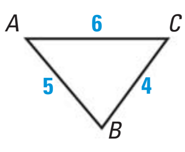


EXAMPLE 2 for Exs. 4–7

LAW OF COSINES Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.

4.



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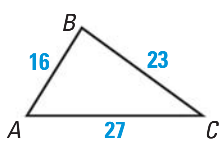

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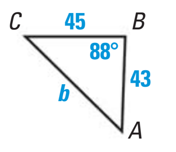
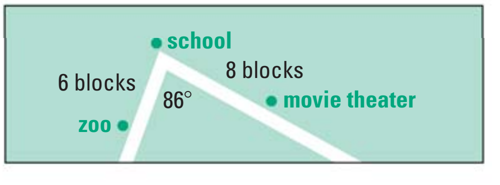


7. DISTANCE Use the diagram at the right. Find the straight distance between the zoo and movie theater.



STATE TEST PRACTICE
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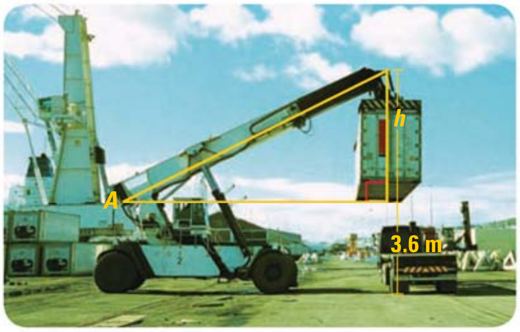
Lessons 7.5–7.7

1. **MULTI-STEP PROBLEM** A *reach stacker* is a vehicle used to lift objects and move them between ships and land.



   a. The vehicle's arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height h the arm can reach if the vehicle is 3.6 meters tall?

   b. The vehicle's arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach?

   c. What is the difference between the two heights the arm can reach above the ground?

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

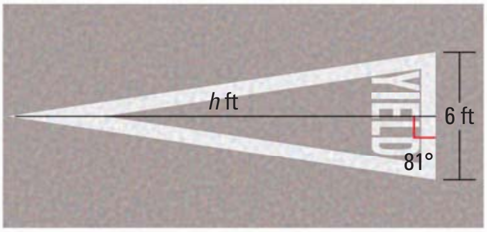
   a. Sketch the situation.

   b. *Explain* how to find the distance across the canyon.

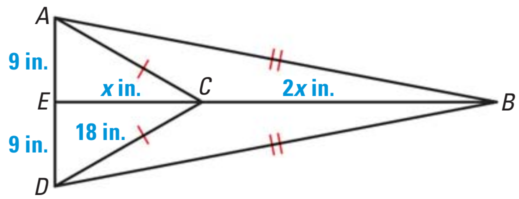
   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance?

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? *Explain* your reasoning.

4. **GRIDDED ANSWER** The specifications for a *yield ahead* pavement marking are shown. Find the height h in feet of this isosceles triangle.



5. **EXTENDED RESPONSE** Use the diagram to answer the questions.

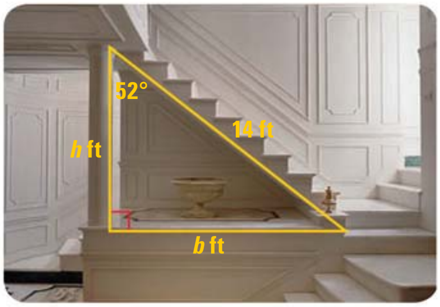


   a. Solve for x. *Explain* the method you chose.

   b. Find $m\angle ABC$. *Explain* the method you chose.

   c. *Explain* a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height h of the staircase? What is the length b of the base of the staircase?



7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree.

7 CHAPTER SUMMARY

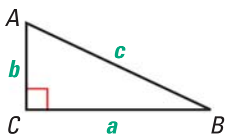
BIG IDEAS *For Your Notebook*

Big Idea 1

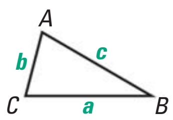
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

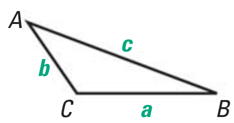
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



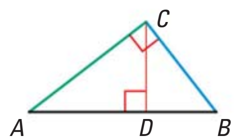
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

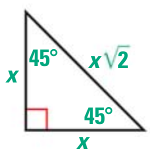
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



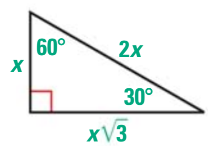
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg • $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

Big Idea 3

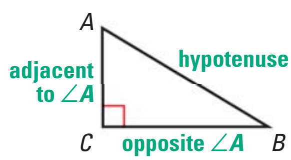
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\qquad \tan^{-1}\frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\qquad \sin^{-1}\frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\qquad \cos^{-1}\frac{AC}{AB} = m\angle A$



7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation _?_.
2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?
3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

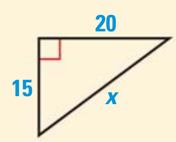
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem *pp. 433–439*

EXAMPLE

Find the value of x.



Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2 \quad \textbf{Pythagorean Theorem}$$

$$x^2 = 15^2 + 20^2 \quad \textbf{Substitute.}$$

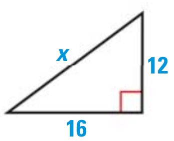
$$x^2 = 625 \quad \textbf{Simplify.}$$

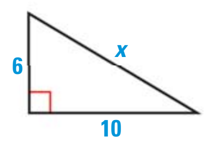
$$x = 25 \quad \textbf{Find the positive square root.}$$

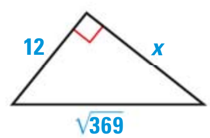
EXERCISES

EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

4. 

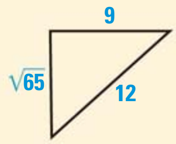
5. 

6. 

7.2 Use the Converse of the Pythagorean Theorem *pp. 441–447*

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$

$144 \stackrel{?}{=} 65 + 81$

$144 < 146$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2
on p. 442
for Exs. 7–12

Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*.

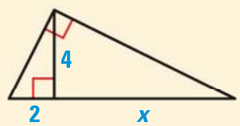
7. 6, 8, 9 **8.** 4, 2, 5 **9.** 10, $2\sqrt{2}$, $6\sqrt{3}$

10. 15, 20, 15 **11.** 3, 3, $3\sqrt{2}$ **12.** 13, 18, $3\sqrt{55}$

7.3 Use Similar Right Triangles *pp. 449–456*

EXAMPLE

Find the value of x.



By Theorem 7.6, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

$2x = 16$ **Cross Products Property**

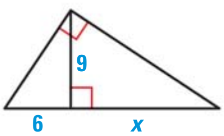
$x = 8$ **Divide.**

EXERCISES

EXAMPLES 2 and 3
on pp. 450–451
for Exs. 13–18

Find the value of x.

13.



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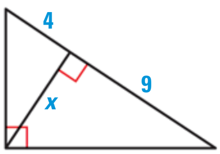
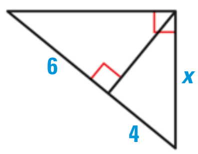
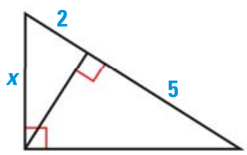


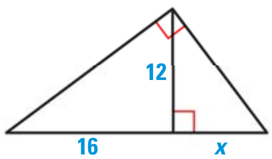
15.



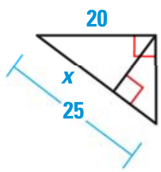
16.



17.



18.

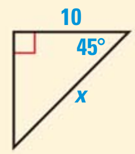


7 CHAPTER REVIEW

7.4 Special Right Triangles
pp. 457–464

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

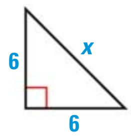
$x = 10\sqrt{2}$ **Substitute.**

EXERCISES

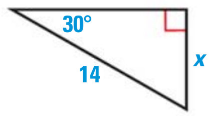
EXAMPLES 1, 2, and 5 on pp. 457–459 for Exs. 19–21

Find the value of *x*. Write your answer in simplest radical form.

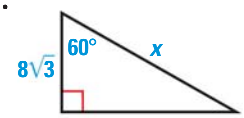
19.



20.



21.



7.5 Apply the Tangent Ratio
pp. 466–472

EXAMPLE

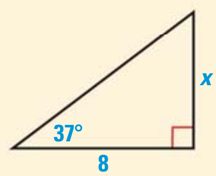
Find the value of *x*.

$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 37°.**

$\tan 37° = \frac{x}{8}$ **Substitute.**

$8 \cdot \tan 37° = x$ **Multiply each side by 8.**

$6 \approx x$ **Use a calculator to simplify.**



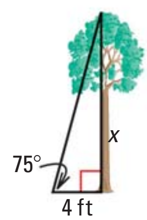
EXERCISES

EXAMPLE 2 on p. 467 for Exs. 22–26

In Exercises 22 and 23, use the diagram.

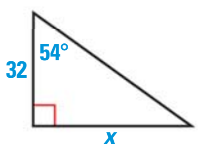
22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

23. In Exercise 22, how tall is the tree if the angle is 55°?



Find the value of *x* to the nearest tenth.

24.



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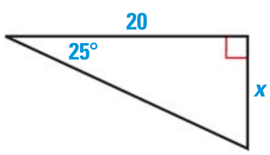
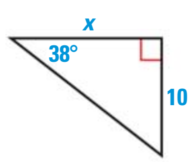


26.



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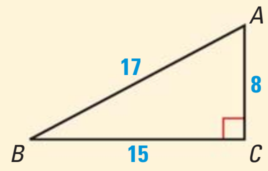
Chapter Review Practice

7.6 Apply the Sine and Cosine Ratios

pp. 473–480

EXAMPLE

Find sin A and sin B.



$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$

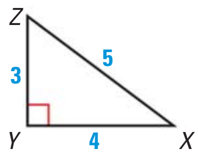
$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$

EXERCISES

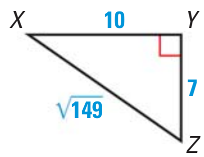
EXAMPLES 1 and 2 on pp. 473–474 for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27.



28.



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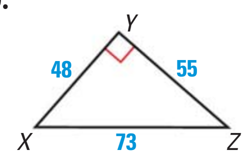
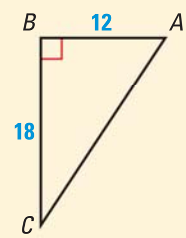


7.7 Solve Right Triangles

pp. 483–489

EXAMPLE

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324\ldots$

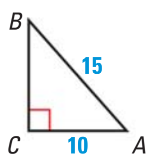
So, the measure of $\angle A$ is approximately 56.3°.

EXERCISES

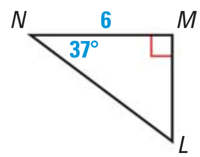
EXAMPLE 3 on p. 484 for Exs. 30–33

Solve the right triangle. Round decimal answers to the nearest tenth.

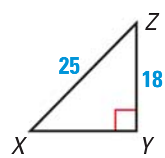
30.



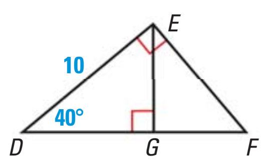
31.



32.



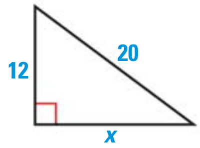
33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.



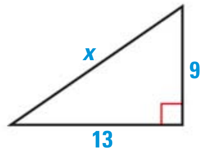
7 CHAPTER TEST

Find the value of *x*. Write your answer in simplest radical form.

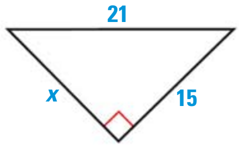
1.



2.



3.



Classify the triangle as *acute*, *right*, or *obtuse*.

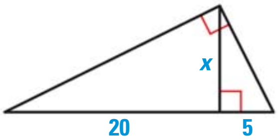
4. 5, 15, $5\sqrt{10}$

5. 4.3, 6.7, 8.2

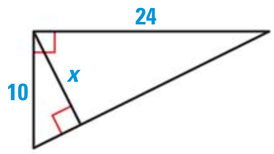
6. 5, 7, 8

Find the value of *x*. Round decimal answers to the nearest tenth.

7.



8.



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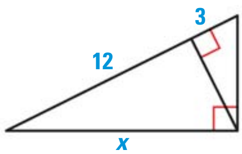
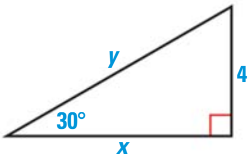


Find the value of each variable. Write your answer in simplest radical form.

10.



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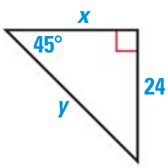

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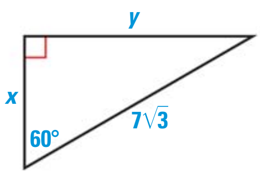
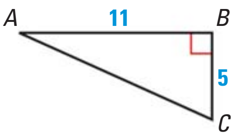


Solve the right triangle. Round decimal answers to the nearest tenth.

13.



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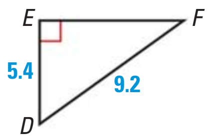

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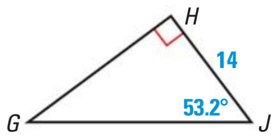
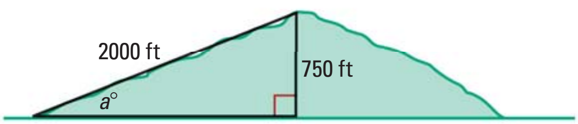


16. FLAGPOLE Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.

17. HILLS The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?



7 ALGEBRA REVIEW

Animated Algebra classzone.com

GRAPH AND SOLVE QUADRATIC EQUATIONS

The graph of $y = ax^2 + bx + c$ is a parabola that opens upward if $a > 0$ and opens downward if $a < 0$. The x-coordinate of the vertex is $-\frac{b}{2a}$. The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

EXAMPLE 1 *Graph a quadratic function*

Graph the equation $y = -x^2 + 4x - 3$.

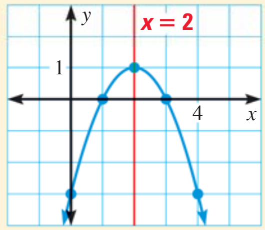
Because $a = -1$ and $-1 < 0$, the graph opens downward.

The vertex has x-coordinate $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex is $-(2)^2 + 4(2) - 3 = 1$.

So, the vertex is $(2, 1)$ and the axis of symmetry is $x = 2$.

Use a table of values to draw a parabola through the plotted points.



EXAMPLE 2 *Solve a quadratic equation by graphing*

Solve the equation $x^2 - 2x = 3$.

Write the equation in the standard form $ax^2 + bx + c = 0$:

$x^2 - 2x - 3 = 0$.

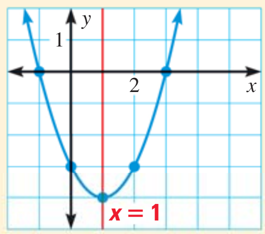
Graph the related quadratic function $y = x^2 - 2x - 3$, as shown.

The x-intercepts of the graph are -1 and 3.

So, the solutions of $x^2 - 2x = 3$ are -1 and 3.

Check the solution algebraically.

$(-1)^2 - 2(-1) \stackrel{?}{=} 3 \rightarrow 1 + 2 = 3$ $\qquad$ $(3)^2 - 2(3) \stackrel{?}{=} 3 \rightarrow 9 - 6 = 3$ ✓



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry.

1. $y = x^2 - 6x + 8$ **2.** $y = -x^2 - 4x + 2$ **3.** $y = 2x^2 - x - 1$

4. $y = 3x^2 - 9x + 2$ **5.** $y = \frac{1}{2}x^2 - x + 3$ **6.** $y = -4x^2 + 6x - 5$

EXAMPLE 2 for Exs. 7–18

Solve the quadratic equation by graphing. Check solutions algebraically.

7. $x^2 = x + 6$ **8.** $4x + 4 = -x^2$ **9.** $2x^2 = -8$ **10.** $3x^2 + 2 = 14$

11. $-x^2 + 4x - 5 = 0$ **12.** $2x - x^2 = -15$ **13.** $\frac{1}{4}x^2 = 2x$ **14.** $x^2 + 3x = 4$

15. $x^2 + 8 = 6x$ **16.** $x^2 = 9x - 1$ **17.** $-25 = x^2 + 10x$ **18.** $x^2 + 6x = 0$

7 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

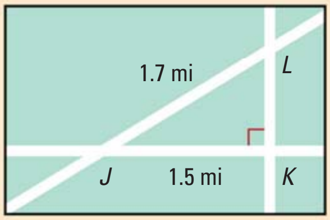
If you have difficulty solving a multiple choice question directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

You ride your bike at an average speed of 10 miles per hour. How long does it take you to ride one time around the triangular park shown in the diagram?

Ⓐ 0.1 h   Ⓑ 0.2 h

Ⓒ 0.3 h   Ⓓ 0.4 h



METHOD 1

SOLVE DIRECTLY The park is a right triangle. Use the Pythagorean Theorem to find KL. Find the perimeter of $\triangle JKL$. Then find how long it takes to ride around the park.

STEP 1 **Find** KL. Use the Pythagorean Theorem.

$$JK^2 + KL^2 = JL^2$$
$$1.5^2 + KL^2 = 1.7^2$$
$$2.25 + KL^2 = 2.89$$
$$KL^2 = 0.64$$
$$KL = 0.8$$

STEP 2 **Find** the perimeter of $\triangle JKL$.

$$P = JK + JL + KL$$
$$= 1.5 + 1.7 + 0.8$$
$$= 4 \text{ mi}$$

STEP 3 **Find** the time t (in hours) it takes you to go around the park.

$$\text{Rate} \times \text{Time} = \text{Distance}$$
$$(10 \text{ mi/h}) \cdot t = 4 \text{ mi}$$
$$t = 0.4 \text{ h}$$

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method is to find how far you can travel in the given times to eliminate choices that are not reasonable.

STEP 1 **Find** how far you will travel in each of the given times. Use the formula $rt = d$.

Choice A: $0.1(10) = 1$ mi

Choice B: $0.2(10) = 2$ mi

Choice C: $0.3(10) = 3$ mi

Choice D: $0.4(10) = 4$ mi

The distance around two sides of the park is $1.5 + 1.7 = 3.2$ mi. But you need to travel around all three sides, which is longer.

Since $1 < 3.2$, $2 < 3.2$, and $3 < 3.2$. You can eliminate choices A, B, and C.

STEP 2 **Check** that D is the correct answer. If the distance around the park is 4 miles, then

$$KL = 4 - JK - JL$$
$$= 4 - 1.5 - 1.7 = 0.8 \text{ mi.}$$

Apply the Converse of the Pythagorean Theorem.

$$0.8^2 + 1.5^2 \stackrel{?}{=} 1.7^2$$
$$0.64 + 2.25 \stackrel{?}{=} 2.89$$
$$2.89 = 2.89 \checkmark$$

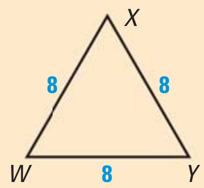
The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

Problem 2

What is the height of $\triangle WXY$?

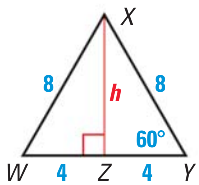
(A) 4 (B) $4\sqrt{3}$

(C) 8 (D) $8\sqrt{3}$



Method 1

SOLVE DIRECTLY Draw altitude $\overline{XZ}$ to form two congruent 30°-60°-90° triangles.



Let h be the length of the longer leg of $\triangle XZY$. The length of the shorter leg is 4.

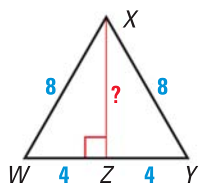
$$\text{longer leg} = \sqrt{3} \cdot \text{shorter leg}$$

$$h = 4\sqrt{3}$$

The correct answer is B. (A) (B) (C) (D)

Method 2

ELIMINATE CHOICES Another method is to use theorems about triangles to eliminate incorrect choices. Draw altitude $\overline{XZ}$ to form two congruent right triangles.



Consider $\triangle XZW$. By the Triangle Inequality, $XW < WZ + XZ$. So, $8 < 4 + XZ$ and $XZ > 4$. You can eliminate choice A. Also, XZ must be less than the hypotenuse of $\triangle XWZ$. You can eliminate choices C and D.

The correct answer is B. (A) (B) (C) (D)

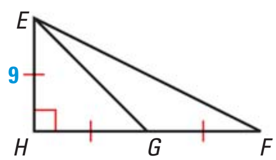
PRACTICE

Explain **why you can eliminate the highlighted answer choice.**

1. In the figure shown, what is the length of $\overline{EF}$?

(A) 9 (B) $9\sqrt{2}$

(C) 18 (D) $9\sqrt{5}$



2. Which of the following lengths are side lengths of a right triangle?

(A) 2, 21, 23 (B) 3, 4, 5 (C) 9, 16, 18 (D) 11, 16, 61

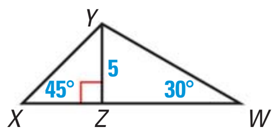
3. In $\triangle PQR$, $PQ = QR = 13$ and $PR = 10$. What is the length of the altitude drawn from vertex Q?

(A) 10 (B) 11 (C) 12 (D)

7 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

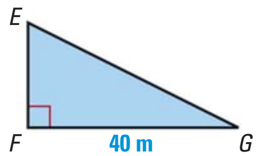
1. Which expression gives the correct length for XW in the diagram below?



Ⓐ $5 + 5\sqrt{2}$ Ⓑ $5 + 5\sqrt{3}$

Ⓒ $5\sqrt{3} + 5\sqrt{2}$ Ⓓ $5 + 10$

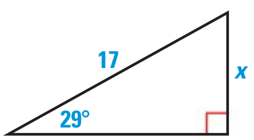
2. The area of $\triangle EFG$ is 400 square meters. To the nearest tenth of a meter, what is the length of side $\overline{EG}$?



Ⓐ 10.0 meters Ⓑ 20.0 meters

Ⓒ 44.7 meters Ⓓ 56.7 meters

3. Which expression can be used to find the value of x in the diagram below?



Ⓐ $\tan 29° = \frac{x}{17}$ Ⓑ $\cos 29° = \frac{x}{17}$

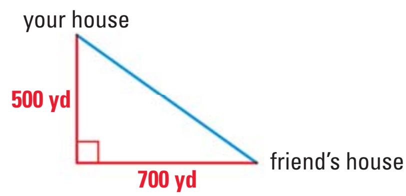
Ⓒ $\tan 61° = \frac{x}{17}$ Ⓓ $\cos 61° = \frac{x}{17}$

4. A fire station, a police station, and a hospital are not positioned in a straight line. The distance from the police station to the fire station is 4 miles. The distance from the fire station to the hospital is 3 miles. Which of the following could *not* be the distance from the police station to the hospital?

Ⓐ 1 mile Ⓑ 2 miles

Ⓒ 5 miles Ⓓ 6 miles

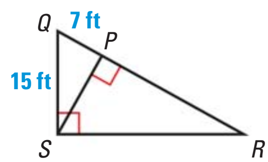
5. It takes 14 minutes to walk from your house to your friend's house on the path shown in red. If you walk at the same speed, about how many minutes will it take on the path shown in blue?



Ⓐ 6 minutes Ⓑ 8 minutes

Ⓒ 10 minutes Ⓓ 13 minutes

6. Which equation can be used to find QR in the diagram below?



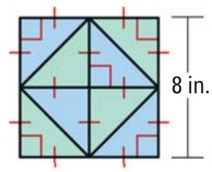
Ⓐ $\frac{QR}{15} = \frac{15}{7}$

Ⓑ $\frac{15}{QR} = \frac{QR}{8}$

Ⓒ $QR = \sqrt{15^2 + 27^2}$

Ⓓ $\frac{QR}{7} = \frac{7}{15}$

7. Stitches are sewn along the black line segments in the potholder shown below. There are 10 stitches per inch. Which is the closest estimate of the number of stitches used?

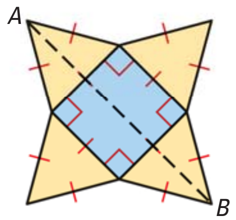


Ⓐ 480 Ⓑ 550

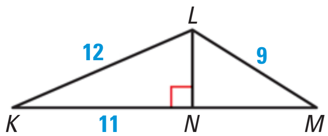
Ⓒ 656 Ⓓ 700

GRIDDED ANSWER

8. A design on a T-shirt is made of a square and four equilateral triangles. The side length of the square is 4 inches. Find the distance (in inches) from point A to point B. Round to the nearest tenth.

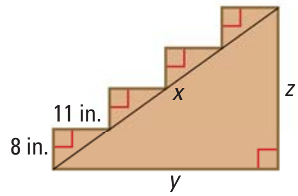


9. Use the diagram below. Find KM to the nearest tenth of a unit.

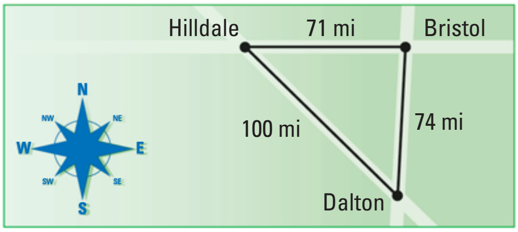


SHORT RESPONSE

10. The diagram shows the side of a set of stairs. In the diagram, the smaller right triangles are congruent. *Explain* how to find the lengths x, y, and z.



11. You drive due north from Dalton to Bristol. Next, you drive from Bristol to Hilldale. Finally, you drive from Hilldale to Dalton. Is Hilldale due west of Bristol? *Explain*.

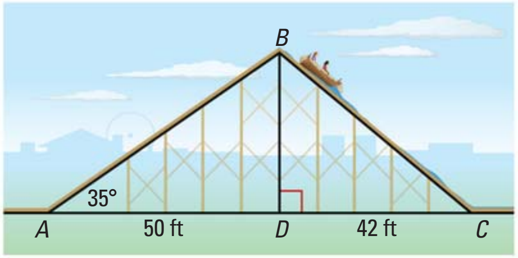


EXTENDED RESPONSE

12. The design for part of a water ride at an amusement park is shown. The ride carries people up a track along ramp $\overline{AB}$. Then riders travel down a water chute along ramp $\overline{BC}$.

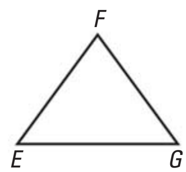
a. How high is the ride above point D? *Explain*.

b. What is the total distance from point A to point B to point C? *Explain*.



13. A formula for the area A of a triangle is *Heron's Formula*. For a triangle with side lengths EF, FG, and EG, the formula is

$$A = \sqrt{s(s - EF)(s - FG)(s - EG)}, \text{ where } s = \frac{1}{2}(EF + FG + EG).$$



a. In $\triangle EFG$ shown, $EF = FG = 15$, and $EG = 18$. Use Heron's formula to find the area of $\triangle EFG$. Round to the nearest tenth.

b. Use the formula $A = \frac{1}{2}bh$ to find the area of $\triangle EFG$. Round to the nearest tenth.

c. Use Heron's formula to *justify* that the area of an equilateral triangle with side length x is $A = \frac{x^2}{4}\sqrt{3}$.